Proportion Word Problems

A **proportion** is mathematical statement showing that two ratios are equal.

Proportions can be written two ways:

Fraction Form: $\dfrac{a}{b} = \dfrac{c}{d}$ **OR**   **Column Form:** $a : b :: c : d$

To solve word problems using proportions:

1. Represent the unknown quantity by the variable x.

2. Set up a ratio using the given rate.

3. Set up a ratio involving the variable x.

4. Form a proportion by setting the two ratios from steps 2 and 3 equal to each other. Include the units of the quantities when you write the proportion. Be sure that the same units occupy corresponding positions in the two ratios of the proportion.

CORRECT	INCORRECT
The units occupy corresponding positions in the 2 ratios.	The units **DO NOT** occupy corresponding positions in the 2 ratios
$\dfrac{\text{miles}}{\text{hours}} = \dfrac{\text{miles}}{\text{hours}}$	$\dfrac{\text{miles}}{\text{hours}} = \dfrac{\text{hours}}{\text{miles}}$
miles: hours :: miles: hours	miles: hours :: hours: miles
$\dfrac{\text{wins}}{\text{games}} = \dfrac{\text{wins}}{\text{games}}$	$\dfrac{\text{wins}}{\text{games}} = \dfrac{\text{games}}{\text{wins}}$
wins: games :: wins: games	wins: games :: games: wins
$\dfrac{\text{dollars}}{\text{ounces}} = \dfrac{\text{dollars}}{\text{ounces}}$	$\dfrac{\text{dollars}}{\text{ounces}} = \dfrac{\text{ounces}}{\text{dollars}}$
dollars: ounces :: dollars: ounces	dollars: ounces :: ounces: dollars

5. If you have a **column form** rewrite the proportion in **fraction form**

6. Once the quantities have been correctly entered into the proportion by using the units as a guide, drop the units. Cross-multiply to solve for x.

7. Write a sentence for the answer.

Now, let's see how this procedure can be applied to solve some actual proportion word problems.

Ex 1: A man used 10 gallons of gasoline on a 180-mile trip. How many gallons of gasoline will he use on a 450-mile trip?

1. Let x represent the number of gallons of gasoline to be used on the 450 mile trip.

2. Set up a ratio using the given rate. $\dfrac{10 \text{ gallons}}{180 \text{ miles}}$

3. Set up a ratio involving x. $\dfrac{x \text{ gallons}}{450 \text{ miles}}$

4. Form a proportion using the ratios from steps 2 and 3:

$$\frac{10 \text{ gallons}}{180 \text{ miles}} = \frac{x \text{ gallons}}{450 \text{ miles}}$$

5. Since you have a ratio form continue to next step

<u>Notice that the same units occupy corresponding positions in the proportion.</u>

6. Drop the units: $\dfrac{10}{180} = \dfrac{x}{450}$

Cross-multiply: $180\,(x) = 10\,(450)$

Solve for x: $180x = 4500$

$$\frac{180x}{180} = \frac{4500}{180}$$

$$x = 25$$

7.

7. On a 450-mile trip, the man will use 25 gallons of gas.

Ex 2: A baseball team wins seven of its first twelve games. How many games would you expect the team to win out of its thirty-six games if the team continues to play with the same degree of success?

1. Let x represent the number of games the baseball team expects to win out of its thirty-six games.

2. Set up a ratio using the given rate: $7 \text{ wins} : 12 \text{ games}$

3. Set up a ratio involving x: $x \text{ wins} : 36 \text{ games}$

4. Form a proportion using the ratios from steps 2 and 3:

$$7 \text{ wins} : 12 \text{ games} :: x \text{ wins} : 36 \text{ games}$$

5. Since you have a column form rewrite the proportion in fraction form:

$$\frac{7 \text{ wins}}{12 \text{ games}} = \frac{x \text{ wins}}{36 \text{ games}}$$

Notice that the same units occupy corresponding positions in the proportion.

6. Drop the units:

$$\frac{7}{12} = \frac{x}{36}$$

Cross-multiply: $12(x) = 7(36)$

Solve for x: $12x = 252$

$$\frac{12x}{12} = \frac{252}{12}$$

$$x = 21$$

7. The team can expect to win 21 out of its 36 games.

Ex 3: The property tax on a $60,000 home is $1,500. At this rate what will the property tax be on a home worth $75,000?

1. Let x represent the property tax on a $75,000 home.

2. Set up a ratio using the given rate: $\dfrac{\$1,500 \text{ tax}}{\$60,000 \text{ home}}$

3. Set up a ratio involving x: $\dfrac{\$ x \text{ tax}}{\$75,000 \text{ home}}$

4. Form a proportion using the ratios from steps 2 and 3: $\dfrac{\$1,500 \text{ tax}}{\$60,000 \text{ home}} = \dfrac{\$ x \text{ tax}}{\$75,000 \text{ home}}$

5. Since you have a ratio form continue to next step

Notice that the same units occupy corresponding positions in the proportion.

6. Drop the units:
$$\frac{1,500}{60,000} = \frac{x}{75,000}$$

Cross-multiply: $60,000(x) = 1500(75,000)$

Solve for x: $60,000x = 112,500,000$

$$\frac{60,000x}{60,000} = \frac{112,500,000}{60,000}$$
$$x = 1,875$$

7. The property tax on a home worth $75,000 will be $1,875.

Ex 4: If a piece of steel 12 feet long weighs 168 pounds, how much will a piece of steel 20 feet long weigh?

1. Let x represent the number of pounds that the 20-foot long piece of steel weighs.

2. Set up a ratio using the given rate: 12 feet : 168 pounds

3. Set up a ratio involving x: 20 feet : x pounds

4. Form a proportion using the ratios from steps 2 and 3:

   12 feet : 168 pounds :: 20 feet : x pounds

5. Since you have a column form rewrite the proportion in fraction form

$$\frac{12 \text{ feet}}{168 \text{ pounds}} = \frac{20 \text{ feet}}{x \text{ pounds}}$$

Notice that the same units occupy corresponding positions in the proportion.

6. Drop the units:
$$\frac{12}{168} = \frac{20}{x}$$

Cross-multiply $12(x) = 20(168)$

Solve for x: $12x = 3,360$
$$\frac{12x}{12} = \frac{3,360}{12}$$
$$x = 280$$

7. A piece of steel 20 feet long will weigh 280 pounds.

Ex 5: How many gallons of paint will you have to purchase to cover 5,500 square feet of a surface if 2 gallons of paint will cover 700 square feet?

1. Let x represent the number of gallons of paint necessary to cover 5500 square feet.

2. Set up a ratio using the given rate: $\dfrac{2 \text{ gallons}}{700 \text{ square feet}}$

3. Set up a ratio involving x: $\dfrac{x \text{ gallons}}{5{,}500 \text{ square feet}}$

4. Form a proportion using the ratios from steps 2 and 3: $\dfrac{2 \text{ gallons}}{700 \text{ square feet}} = \dfrac{x \text{ gallons}}{5{,}500 \text{ square feet}}$

5. Since you have a ratio form continue to next step

Notice that the same units occupy corresponding positions in the proportion.

6. Drop the units: $\dfrac{2}{700} = \dfrac{x}{5{,}500}$

   Cross-multiply: $x(700) = 2(5{,}500)$

   Solve for x:
   $$700x = 11{,}000$$
   $$\frac{700x}{700} = \frac{11{,}000}{700}$$
   $$x = 15\frac{5}{7}$$

7. You will have to purchase 16 gallons of paint to cover 5,500 square feet.

Ex 6: A man 6 feet tall casts a shadow 4 feet long. What is the length of a shadow cast at the same time by a statue that is 18 inches high?

1. Let x feet represent the length of the of the shadow cast by the statue.

2. Set up a ratio using the given rate: $\dfrac{6 \text{ feet tall}}{4\text{-foot shadow}}$

3. Set up a ratio involving x: $\dfrac{18 \text{ inches tall}}{x\text{-foot shadow}}$

4. Form a proportion using the ratios from steps 2 and 3: $\dfrac{6 \text{ feet tall}}{4\text{-foot shadow}} = \dfrac{18 \text{ inches tall}}{x\text{-foot shadow}}$

We need to change 6 feet to 72 inches so that the units are in agreement throughout the proportion as well as occupying corresponding positions. $\dfrac{72 \text{ inches}}{4 \text{ feet}} = \dfrac{18 \text{ inches}}{x \text{ feet}}$

    5. Since you have a ratio form continue to next step

<u>Notice that the same units occupy corresponding positions in the proportion.</u>

    6. Drop the units: $\dfrac{72}{4} = \dfrac{18}{x}$

         Cross-multiply: $x(72) = 18(4)$

         Solve for x: $72x = 72$

$$\frac{72x}{72} = \frac{72}{72}$$

$$x = 1$$

    7. The statue casts a 1-foot shadow.

Ex 7: The dosage of a certain medication is 2 ounces for every 50 pounds of body weight. How many ounces of the medication are required for a person who weighs 175 pounds?

    1. Let x represent the number of ounces of the medication required for a 175-pound person.

    2. Set up a ratio using the given rate: 2 ounces of medication : 50 pounds of body weight

    3. Set up a ratio involving x: x ounces of medication : 175 pounds of body weight

    4. Form a proportion using the ratios from steps 2 and 3:

$$2 \text{ ounces} : 50 \text{ pounds} :: x \text{ ounces} : 175 \text{ pounds}$$

    5. Since you have a column form rewrite the proportion in fraction form

$$\frac{2 \text{ ounces}}{50 \text{ pounds}} = \frac{x \text{ ounces}}{175 \text{ pounds}}$$

<u>Notice that the same units occupy corresponding positions in the proportion.</u>

6. Drop the units: $\dfrac{2}{50} = \dfrac{x}{175}$

         Cross-multiply: $50(x) = 2(175)$

Solve for x:
$$50x = 350$$
$$\frac{50x}{50} = \frac{350}{50}$$
$$x = 7$$

7. A person who weighs 175 pounds requires 7 ounces of the medication.

Note: you can solve the ***column form*** directly instead of rewriting the proportion in ***fraction form*** as shown in the example below.

Ex 8: A patient was administered 48 mg of certain medication over an 8-hour period. At that rate, how much medication will the patient receive in a 22-hour period?

1. Let x represent the amount of medication over a 22-hour period.

2. Set up a ratio using the given rate: $48\,mg$ of medication : 8 hours

3.   Set up a ratio involving x: $x\,mg$ of medication : 22 hours

4. Form a proportion using the ratios from steps 2 and 3:

$$48\,mg \text{ of medication} : 8 \text{ hours} :: x\,mg \text{ of medication} : 22 \text{ hours}$$

Notice that the same units occupy corresponding positions in the proportion.

5. Drop the units: $48 : 8 :: x : 22$

6.   Multiply the outer of the proportion: 48(22)
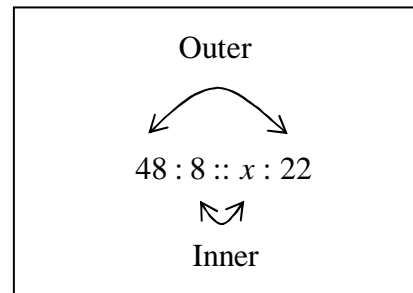
   Multiply the inner of the proportion: 8(x)

   Set: Outer = Inner $\Longrightarrow$ $48(22) = 8(x)$

   Solve for x: $48(22) = 8(x)$
$$8(x) = 48(22)$$
$$\frac{8x}{8} = \frac{1056}{8}$$
$$x = 132$$

Outer

$48 : 8 :: x : 22$

Inner

7. The patient received in 22-hour 132 mg of medication.

Exercises: Set up a proportion to solve each of the following word problems.

1. A car averages 12 gallons of gasoline for each 420 miles driven. How much gasoline will it need for a 3,360-mile trip?

2. A $9 sales tax is charged for a $200 purchase. At this rate, what is the sales tax for a $300 purchase?

3. A life insurance policy costs $6.88 for every $1,000 of insurance. At this rate, what is the cost for $17,500 worth of life insurance?

4. A cabinet maker used $\frac{3}{4}$ gallons of varnish on 10 cabinet doors. At this rate, how many quarts of varnish are required for 25 cabinet doors?

5. A stock investment of 50 shares paid a dividend of $125. At this rate, what dividend would be paid for 325 shares of stock?

6. A biologist estimates that the human body contains 90 pounds of water for every 100 pounds of body weight. At this rate, estimate the number of pounds of water in a child who weighs 80 pounds.

7. If 21 sandwiches are needed at an outing for 14 people, how many sandwiches would be needed for 30 people?

8. If a pole 18 feet high casts a shadow 20 feet long, how long a shadow would a pole 27 feet high cast?

9. A doctor's prescription calls for 1/5 ounce of a particular ingredient for every 20 pounds of body weight. How many ounces of this ingredient would be needed by someone weighing 175 pounds?

10. A farm containing 36.5 acres sells for $5,840. What would a farm of 124 acres sell for at the same rate?

Solutions to the odd-numbered exercises and answers to the even-numbered exercises:

1. $\dfrac{12\text{ gallons}}{420\text{ miles}} = \dfrac{x\text{ gallons}}{3,360\text{ miles}}$

$\qquad 420(x) = 12(3,360)$

$\qquad 420x = 40,320$

$\qquad \dfrac{420x}{420} = \dfrac{40,320}{420}$

$\qquad x = 96$

96 gallons of gas will be needed for a 3,360-mile trip.

2. The sales tax on a $300

   purchase is $13.50.

3. $\$6.88\,\text{cost} : \$1,000\,\text{insurance} :: \$x\,\text{cost} : \$17,500\,\text{insurance}$

$\qquad \dfrac{\$6.88\,\text{cost}}{\$1,000\,\text{insurance}} = \dfrac{\$x\,\text{cost}}{\$17,500\,\text{insurance}}$

$\qquad \dfrac{6.88}{1,000} = \dfrac{x}{17,500}$

$\qquad 1000(x) = 6.88(17,500)$

$\qquad 000x = 120,400$

$\qquad \dfrac{1000x}{1000} = \dfrac{120,400}{1000}$

$\qquad \dfrac{1000x}{1000} = \dfrac{120,400}{1000}$

$\qquad x = 120.4$

The cost for $17,500 worth of insurance is $120.40.

4. 7.5 quarts of varnish are

   required for 25 cabinets.

   (Hint: Change 3/4 gallons to quarts.)

5. $\dfrac{50\text{ shares}}{\$125} = \dfrac{325\text{ shares}}{\$x}$

$\qquad 50(x) = 125(325)$

$\qquad 50x = 40,625$

$\qquad \dfrac{50x}{50} = \dfrac{40,625}{50}$

$\qquad x = 812.5$

A $812.50 dividend would be paid for 325 shares of stock.

6. A child weighing 80 pounds would

   contain about 72 pounds of water.

7. $\dfrac{21 \text{ sandwiches}}{14 \text{ people}} = \dfrac{x \text{ sandwiches}}{30 \text{ people}}$

$$14(x) = 21(30)$$
$$14x = 630$$
$$\dfrac{14x}{14} = \dfrac{630}{14}$$
$$x = 45$$

45 sandwiches would be needed for 30 people.

9. $\dfrac{\dfrac{1}{5} \text{ ounce}}{20 \text{ pounds}} = \dfrac{x \text{ ounces}}{175 \text{ pounds}}$

$$20(x) = \dfrac{1}{5}(175)$$
$$20x = 35$$
$$\dfrac{20x}{20} = \dfrac{35}{20}$$
$$x = 1.75$$

Someone weighing 175 pounds would need
1.75 ounces of the ingredient in the prescription.

8. The shadow cast by a pole 27 feet high

would be 30 feet long.

10. A 124-acre farm would sell

for $19,840.