

# 2.1 Modeling with Expressions



Resource Locker

**Essential Question:** How do you interpret algebraic expressions in terms of their context?

## Explore Interpreting Parts of an Expression

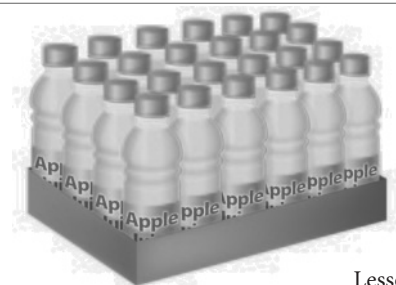
An **expression** is a mathematical phrase that contains operations, numbers, and/or variables. The **terms** of an expression are the parts that are being added. A **coefficient** is the numerical factor of a variable term. There are both *numerical expressions* and *algebraic expressions*. A **numerical expression** contains only numbers while an **algebraic expression** contains at least one variable.

- (A) Identify the terms and the coefficients of the expression  $8p + 2q + 7r$ .  
terms: \_\_\_\_\_ ; coefficients: \_\_\_\_\_
- (B) Identify the terms and coefficients of the expression  $18 - 2x - 4y$ . Since the expression involves \_\_\_\_\_ rather than addition, rewrite the expression as the \_\_\_\_\_ of the terms:  
 $18 - 2x - 4y =$  \_\_\_\_\_. So, the terms of the expression are \_\_\_\_\_  
and the coefficients are \_\_\_\_\_.
- (C) Identify the terms and coefficients in the expression  $2x + 3y - 4z + 10$ . Since the expression involves both \_\_\_\_\_ and addition, rewrite the expression as the \_\_\_\_\_ of the terms:  $2x + 3y - 4z + 10 =$  \_\_\_\_\_. So, the terms of the expression are \_\_\_\_\_ and the coefficients are \_\_\_\_\_.

Tickets to an amusement park are \$60 for adults and \$30 for children. If  $a$  is the number of adults and  $c$  is the number of children, then the cost for  $a$  adults and  $c$  children is  $60a + 30c$ .

- (D) What are the terms of the expression? \_\_\_\_\_
- (E) What are the factors of  $60a$ ? \_\_\_\_\_
- (F) What are the factors of  $30c$ ? \_\_\_\_\_
- (G) What are the coefficients of the expression? \_\_\_\_\_
- (H) Interpret the meaning of the two terms of the expression. \_\_\_\_\_

The price of a case of juice is \$15.00. Fred has a coupon for 20 cents off each bottle in the case. The expression to find the final cost of the case of juice is  $15 - 0.2b$ , wherein  $b$  is the number of bottles.



- (I) What are the terms of the expression? \_\_\_\_\_

- J** What are the factors of each term? \_\_\_\_\_ is the only factor of the \_\_\_\_\_ term and \_\_\_\_\_ and \_\_\_\_\_ are the factors of the \_\_\_\_\_ term.
- K** Do both terms have coefficients? Explain. \_\_\_\_\_  
 \_\_\_\_\_ What are the coefficients? \_\_\_\_\_
- L** What does the expression  $15 - 0.2b$  mean in the given situation?  
 \_\_\_\_\_  
 \_\_\_\_\_

**Reflect**

1. Sally identified the terms of the expression  $9a + 4b - 18$  as  $9a$ ,  $4b$ , and  $18$ . Explain her error.  
 \_\_\_\_\_
2. What is the coefficient of  $b$  in the expression  $b + 10$ ? Explain. \_\_\_\_\_

**Explain 1 Interpreting Algebraic Expressions in Context**

In many cases, real-world situations and algebraic expressions can be related. The coefficients, variables, and operations represent the given real-world context.

**Interpret the algebraic expression corresponding to the given context.**

**Example 1**

- A** Curtis is buying supplies for his school. He buys  $p$  packages of crayons at \$1.49 per package and  $q$  packages of markers at \$3.49 per package. What does the expression  $1.49p + 3.49q$  represent?
- Interpret the meaning of the term  $1.49p$ . What does the coefficient 1.49 represent?
- The term  $1.49p$  represents the cost of  $p$  packages of crayons. The coefficient represents the cost of one package of crayons, \$1.49.
- Interpret the meaning of the term  $3.49q$ . What does the coefficient 3.49 represent?
- The term  $3.49q$  represents the cost of  $q$  packages of markers. The coefficient represents the cost of one package of markers, \$3.49.
- Interpret the meaning of the entire expression.
- The expression  $1.49p + 3.49q$  represents the total cost of  $p$  packages of crayons and  $q$  packages of markers.
- B** Jill is buying ink jet paper and laser jet paper for her business. She buys 8 more packages of ink jet paper than  $p$  packages of laser jet paper. Ink jet paper costs \$6.95 per package and laser jet paper costs \$8 per package. What does the expression  $8p + 6.95(p + 8)$  represent?
- Interpret the meaning of the first term,  $8p$ . What does the coefficient 8 represent?
- The term  $8p$  represents the cost of \_\_\_\_\_. The coefficient represents \_\_\_\_\_, \$8.
- Interpret the meaning of the second expression,  $6.95(p + 8)$ . What do the factors 6.95 and  $(p + 8)$  represent?

The term  $6.95(p + 8)$  represents \_\_\_\_\_. 6.95 represents the cost of \_\_\_\_\_.  $(p + 8)$  represents \_\_\_\_\_ that Jill bought.

Interpret the expression  $8p + 6.95(p + 8)$ .

The expression represents \_\_\_\_\_ that Jill bought.

### Your Turn

Interpret the algebraic expression corresponding to the given context.

- George is buying watermelons and pineapples to make fruit salad. He buys  $w$  watermelons at \$4.49 each and  $p$  pineapples at \$5 each. What does the expression  $4.49w + 5p$  represent?
- Sandi buys 5 fewer packages of pencils than  $p$  packages of pens. Pencils costs \$2.25 per package and pens costs \$3 per package. What does the expression  $3p + 2.25(p - 5)$  represent?

## Explain 2 Comparing Algebraic Expressions

Given two algebraic expressions involving two variables, we can compare whether one is greater or less than the other. We can denote the inequality between the expressions by using  $<$  or  $>$  symbols. If the expressions are the same, or **equivalent expressions**, we denote this equality by using  $=$ .

Suppose  $x$  and  $y$  give the populations of two different cities where  $x > y$ . Compare the expressions and tell which of the given pair is greater.

### Example 1

(A)  $x + y$  and  $2x$

The expression  $2x$  is greater.

- Putting the lesser population,  $y$ , together with the greater population,  $x$ , gives a population that is less than double the greater population.

(B)  $\frac{x}{y}$  and  $\frac{y}{x}$

Since  $x > y$ ,  $\frac{x}{y}$  will be \_\_\_\_\_ than 1 and  $\frac{y}{x}$  will be \_\_\_\_\_ than 1.

So  $\frac{x}{y}$  \_\_\_\_\_  $\frac{y}{x}$ .

### Your Turn

Suppose  $x$  and  $y$  give the populations of two different cities where  $x > y$  and  $y > 0$ . Compare the expressions and tell which of the given pair is greater.

5.  $\frac{x}{x + y}$  and  $\frac{x + y}{x}$

6.  $2(x + y)$  and  $(x + y)^2$

## Explain 3 Modeling Expressions in Context

The table shows some words and phrases associated with the four basic arithmetic operations. These words and phrases can help you translate a real-world situation into an algebraic expression.

Operation	Words	Examples
Addition	the sum of, added to, plus, more than, increased by, total, altogether, and	1. A number increased by 2 2. The sum of $n$ and 2 3. $n + 2$
Subtraction	less than, minus, subtracted from, the difference of, take away, taken from, reduced by	1. The difference of a number and 2 2. 2 less than a number 3. $n - 2$
Multiplication	times, multiplied by, the product of, percent of	1. The product of 0.6 and a number 2. 60% of a number 3. $0.6n$
Division	divided by, division of, quotient of, divided into, ratio of,	1. The quotient of a number and 5 2. A number divided by 5 3. $n \div 5$ or $\frac{n}{5}$

**Example 3** Write an algebraic expression to model the given context. Give your answer in simplest form.

**A** the price of an item plus 6% sales tax

Price of an item      +      6% sales tax  
 $p$                       +               $0.06p$

The algebraic expression is  $p + 0.06p$ , or  $1.06p$ .

**B** the price of a car plus 8.5% sales tax

The price of a car    +      \_\_\_\_\_  
\_\_\_\_\_                      +      \_\_\_\_\_

The algebraic expression is \_\_\_\_\_,  
or \_\_\_\_\_.

### Reflect

7. Use the Distributive Property to show why  $p + 0.06p = 1.06p$ .

\_\_\_\_\_

8. What could the expression  $3(p + 0.06p)$  represent? Explain.

\_\_\_\_\_

\_\_\_\_\_

### Your Turn

Write an algebraic expression to model the given context. Give your answer in simplest form.

9. the number of gallons of water in a tank, that already has 300 gallons in it, after being filled at 35 gallons per minute for  $m$  minutes

\_\_\_\_\_

10. the original price  $p$  of an item less a discount of 15%

\_\_\_\_\_



7. Lorenzo buys 3 shirts at  $s$  dollars apiece and 2 pairs of pants at  $p$  dollars a pair. What does the expression  $3s + 2p$  represent?
8. If a car travels at a speed of 25 mi/h for  $t$  hours, then travels 45 mi/h for  $m$  hours, what does the expression  $25t + 45m$  represent?
9. The price of a sandwich is \$1.50 more than the price of a smoothie, which is  $d$  dollars. What does the expression  $d + 1.5$  represent?
10. A bicyclist travels 1 mile in 5 minutes. If  $m$  represents minutes, what does the expression  $\frac{m}{5}$  represent?
11. What are the factors of the expression  $(y - 2)(x + 3)$ ?
12. **Explain the Error** A student wrote that there are two terms in the expression  $3p - (7 - 4q)$ . Explain the student's error.
13. Yolanda is buying supplies for school. She buys  $n$  packages of pencils at \$1.40 per package and  $m$  pads of paper at \$1.20 each. What does each term in the expression  $1.4n + 1.2m$  represent? What does the entire expression represent?
14. Chris buys  $p$  pairs of pants and 4 more shirts than pairs of pants. Shirts cost \$18 each and pair of pants cost \$25 each. What does each term in the expression  $25p + 18(p + 4)$  represent? What does the entire expression represent?



Suppose  $a$  and  $b$  give the populations of two states where  $a > b$ . Compare the expressions and tell which of the given pair is greater or if the expressions are equal.

15.  $\frac{b}{a+b}$  and 0.5

16.  $a + 13c$  and  $b + 13c$ , where  $c$  is the population of a third state

17.  $\frac{a-b}{2}$  and  $a - \frac{b}{2}$

18.  $a + b$  and  $2b$

19.  $5(a + b)$  and  $(a + b)5$

**Write an algebraic expression to model the given context. Give your answer in simplest form.**

20. the price  $s$  of a pair of shoes plus 5% sales tax.      21. the original price  $p$  of an item less a discount of 20%

22. the price  $h$  of a recently bought house plus 10% property tax

23. the principal amount  $P$  originally deposited in a bank account plus 0.3% interest

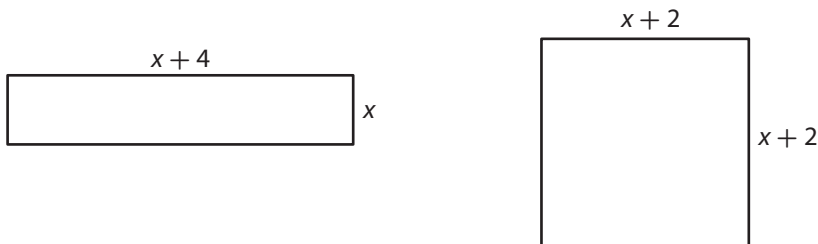


24. Match each statement with the algebraic expression that models it.

- A. the price of a winter coat and a 20% discount \_\_\_\_\_  $x + 0.02x = 1.02x$
- B. the base salary of an employee and a 2% salary increase \_\_\_\_\_  $x - 0.20x = 0.80x$
- C. the cost of groceries and a 2% discount with coupons \_\_\_\_\_  $x + 0.20x = 1.20x$
- D. the number of students attending school last year and a 20% increase from last year \_\_\_\_\_  $x - 0.02x = 0.98x$

**H.O.T. Focus on Higher Order Thinking**

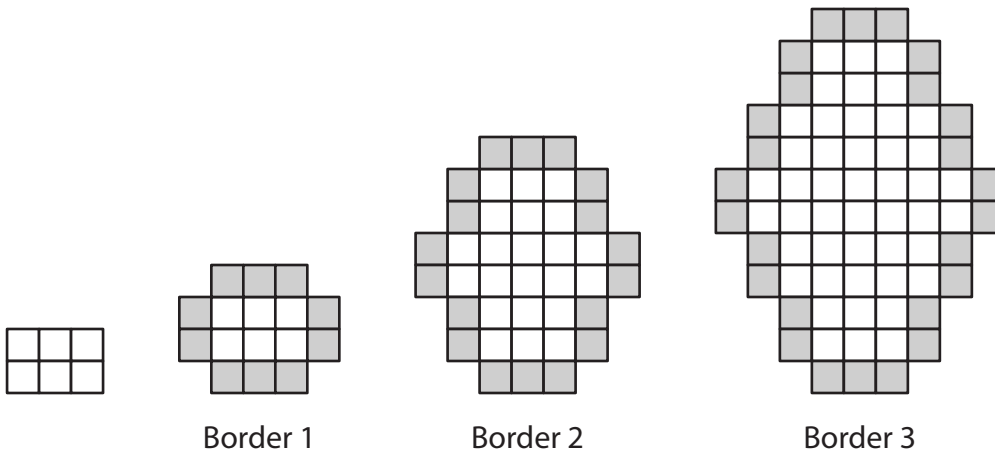
25. **Critique Reasoning** A student is given the rectangle and the square shown. The student states that the two figures have the same perimeter. Is the student correct? Explain your reasoning.



26. **Multi-Step** Yon buys tickets to a concert for himself and a friend. There is a tax of 6% on the price of the tickets and an additional booking fee of \$20 for the transaction. Write an algebraic expression to represent the price per person. Simplify the expression if possible.



- 27. Persevere in Problem Solving** Jerry is planting white daisies and red tulips in his garden and he wants to choose a pattern in which the tulips surround the daisies. He uses tiles to generate patterns starting with two rows of three daisies. He surrounds these daisies with a border of tulips. The design continues as shown.



- a. Jerry writes the expression  $8(b - 1) + 10$  for the number of tulips in each border, wherein  $b$  is the border number and  $b \geq 1$ . Explain why Jerry's expression is correct.
- b. Elaine wants to start with two rows of four daisies. Her reasoning is that Jerry started with two rows of three daisies and his expression was  $8(b - 1) + 10$ , so if she starts with two rows of four daisies, her expression will be  $10(b - 1) + 10$ . Is Elaine's statement correct? Explain.

# Lesson Performance Task

Becky and Michele are both shopping for a new car at two different dealerships. Dealership A is offering \$500 cash back on any purchase, while Dealership B is offering \$1000 cash back. The tax rate is 5% at Dealership A but 8% at Dealership B. Becky wants to buy a car that is \$15,000, and Michele is planning to buy a car that costs \$20,000. Use algebraic expressions to help you answer the following questions.



- a. At which dealership will Becky get the better deal? How much does she save?
- b. At which dealership will Michele get the better deal? How much does she save?
- c. What generalization can you make that would help any shopper know which dealership has the better deal? [Hint: At what price point would the two deals be equal?]

## 2.2 Creating and Solving Equations



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**Essential Question:** How do you use an equation to model and solve a real-world problem?

### Explore **Creating Equations from Verbal Descriptions**

You can use what you know about writing algebraic expressions to write an equation that represents a real-world situation.

Suppose Cory and his friend Walter go to a movie. Each of their tickets costs the same amount, and they share a frozen yogurt that costs \$5.50. The total amount they spend is \$19.90. How can you write an equation that describes the situation?

**A Identify the important information.**

The word \_\_\_\_\_ tells you that the relationship describes an equation.

The word *total* tells you that the operation involved in the relationship is \_\_\_\_\_.

What numerical information do you have? \_\_\_\_\_

\_\_\_\_\_

What is the unknown quantity? \_\_\_\_\_

**B Write a verbal description.**

Choose a name for the variable. In this case, use  $c$  for \_\_\_\_\_.

The verbal description is: Twice the cost of \_\_\_\_\_ plus the cost of \_\_\_\_\_ equals \_\_\_\_\_.

**C To write an equation, write a numerical or \_\_\_\_\_ expression for each quantity and insert an equal sign in the appropriate place. An equation is: \_\_\_\_\_.**

**Reflect**

- How can you use a verbal model to write an equation for the situation described?

2. Could you write a different equation to describe the situation? Explain your reasoning.

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## Explain 1 Creating and Solving Equations Involving the Distributive Property

When you create an equation to model a real-world problem, your equation may involve the Distributive Property. When you solve a real-world problem, you should always check that your answer makes sense.

**Example 1** Write and solve an equation to solve each problem.

- A** Aaron and Alice are bowling. Alice's score is twice the difference of Aaron's score and 5. The sum of their scores is 320. Find each student's bowling score.

Write a verbal description of the basic situation.

The sum of Aaron's score and Alice's score is 320.

Choose a variable for the unknown quantity and write an equation to model the detailed situation.

Let  $a$  represent Aaron's score. Then  $2(a - 5)$  represents Alice's score.

$$a + 2(a - 5) = 320$$

Solve the equation for  $a$ .

$$a + 2(a - 5) = 320$$

$$a + 2a - 10 = 320 \quad \text{Distributive Property}$$

$$3a - 10 = 320$$

$$3a - 10 + 10 = 320 + 10 \quad \text{Addition Property of Equality}$$

$$3a = 330$$

$$\frac{3a}{3} = \frac{330}{3} \quad \text{Division Property of Equality}$$

$$a = 110$$

So, Aaron's score is 110 and Alice's score is  $2(a - 5) = 2(110 - 5) = 2(105) = 210$ .

Check that the answer makes sense.

$110 + 210 = 320$ , so the answer makes sense.

- B** Mari, Carlos, and Amanda collect stamps. Carlos has five more stamps than Mari, and Amanda has three times as many stamps as Carlos. Altogether, they have 100 stamps. Find the number of stamps each person has.

Write a verbal description of the basic situation.



Choose a variable for the unknown quantity and write an equation to model the detailed situation.

Let  $s$  represent the number of stamps Mari has. Then Carlos has \_\_\_\_\_ stamps, and Amanda has \_\_\_\_\_ stamps.

$$s + \boxed{\phantom{000}} + 3(\boxed{\phantom{000}}) = \boxed{\phantom{000}}$$

Solve the equation for  $s$ .

$$s + \boxed{\phantom{000}} + 3(\boxed{\phantom{000}}) = \boxed{\phantom{000}}$$

$$s + s + 5 + 3s + \boxed{\phantom{000}} = \boxed{\phantom{000}}$$

Distributive Property

$$\boxed{\phantom{000}} + \boxed{\phantom{000}} = \boxed{\phantom{000}}$$

Combine like terms

$$\boxed{\phantom{000}}s = \boxed{\phantom{000}}$$

Subtraction Property of Equality

$$s = \boxed{\phantom{000}}$$

Division Property of Equality

So, Mari has \_\_\_\_\_ stamps, Carlos has \_\_\_\_\_ stamps, and Amanda has \_\_\_\_\_ stamps.

Check that the answer makes sense.

$$\boxed{\phantom{000}} + \boxed{\phantom{000}} + \boxed{\phantom{000}} = \boxed{\phantom{000}}$$

; the answer makes sense.

$$\boxed{\phantom{000}} = \boxed{\phantom{000}}$$

### Reflect

3. Would a fractional answer make sense in this situation?

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4. **Discussion** What might it mean if a check revealed that the answer to a real-world problem did not make sense?

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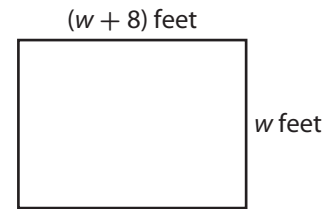


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**Your Turn**

Write and solve an equation to solve the problem.

5. A rectangular garden is fenced on all sides with 256 feet of fencing. The garden is 8 feet longer than it is wide. Find the length and width of the garden.



## Explain 2 Creating and Solving Equations with Variables on Both Sides

In some equations, variables appear on both sides. You can use the properties of equality to collect the variable terms so that they all appear on one side of the equation.

**Example 2** Write and solve an equation to solve each problem.

- A** Janine has job offers at two companies. One company offers a starting salary of \$28,000 with a raise of \$3000 each year. The other company offers a starting salary of \$36,000 with a raise of \$2000 each year. In how many years would Janine's salary be the same with both companies? What will the salary be?

Write a verbal description of the basic situation.

Let  $n$  represent the number of years it takes for the salaries to be equal.

Base Salary A plus \$3000 per year raise = Base Salary B + \$2000 per year raise

$$28,000 + 3000n = 36,000 + 2,000n$$

$$28,000 + 3000n - 2000n = 36,000 + 2,000n - 2000n \quad \text{Subtraction Property of Equality}$$

$$28,000 + 1000n = 36,000 \quad \text{Combine like terms.}$$

$$28,000 + 1000n - 28,000 = 36,000 - 28,000 \quad \text{Subtraction Property of Equality}$$

$$1000n = 8000$$

$$\frac{1000n}{1000} = \frac{8000}{1000} \quad \text{Division Property of Equality}$$

$$n = 8$$

$$28,000 + 3,000(8) = 36,000 + 2,000(8)$$

$$52,000 = 52,000$$

In 8 years, the salaries offered by both companies will be \$52,000.

- B** One moving company charges \$800 plus \$16 per hour. Another moving company charges \$720 plus \$21 per hour. At what number of hours will the charge by both companies be the same? What is the charge?

Write a verbal description of the basic situation. Let  $t$  represent the number of hours that the move takes.

Moving Charge A plus \$16 per hour = Moving Charge B plus \$21 per hour

$$800 + \boxed{\phantom{00}} t = 720 + \boxed{\phantom{00}} t$$

$$800 + \boxed{\phantom{00}} t - \boxed{\phantom{00}} t = 720 + \boxed{\phantom{00}} t - \boxed{\phantom{00}} t \quad \text{Subtraction Property of Equality}$$

$$800 = 720 + \boxed{\phantom{00}} t$$

$$800 - 720 = 720 + \boxed{\phantom{00}} t - 720 \quad \text{Subtraction Property of Equality}$$

$$\boxed{\phantom{00}} = \boxed{\phantom{00}} t$$

$$\frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}} = \frac{\boxed{\phantom{00}} t}{\boxed{\phantom{00}}}$$

Division Property of Equality

$$t = \boxed{\phantom{00}}$$

The charges are the same for a job that takes \_\_\_\_\_ hours.

Substitute the value 16 in the original equation.

$$800 + 16t = 720 + 21t$$

$$800 + 16(\boxed{\phantom{00}}) = 720 + 21(\boxed{\phantom{00}})$$

$$800 + \boxed{\phantom{00}} = 720 + \boxed{\phantom{00}}$$

$$\boxed{\phantom{00}} = \boxed{\phantom{00}}$$

After \_\_\_\_\_ hours, the moving charge for both companies will be \_\_\_\_\_.

**Reflect**

6. Suppose you collected the variable terms on the other side of the equal sign to solve the equation. Would that affect the solution?

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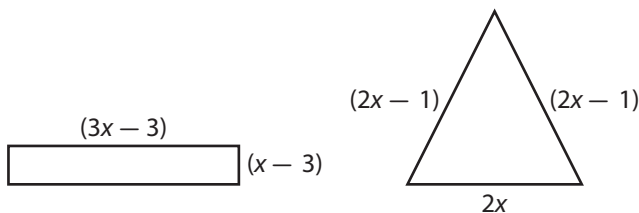


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**Your Turn**

Write and solve an equation to solve each problem.

7. Claire bought just enough fencing to enclose either a rectangular garden or a triangular garden, as shown. The two gardens have the same perimeter. How many feet of fencing did she buy?



8. A veterinarian is changing the diets of two animals, Simba and Cuddles. Simba currently consumes 1200 Calories per day. That number will increase by 100 Calories each day. Cuddles currently consumes 3230 Calories a day. That number will decrease by 190 Calories each day. The patterns will continue until both animals are consuming the same number of Calories each day. In how many days will that be? How many Calories will each animal be consuming each day then?

**Explain 3 Constructing Equations from an Organized Table**

You can use a table to organize information and see relationships.

**Example 3 Construct and solve an equation to solve the problem.**

Kim works 4 hours more each day than Jill does, and Jack works 2 hours less each day than Jill does. Over 2 days, the number of hours Kim works is equal to the difference of 4 times the number of hours Jack works and the number of hours Jill works. How many hours does each person work each day?

**Analyze Information**

Identify the important information.

- Kim works  hours more per day than Jill does.
- Jack works  hours less per day than Jill does.

**Formulate a Plan**

Make a table using the information given. Let  $x$  be the number of hours Jill works in one day.

	Hours Worked Per Day	Hours Worked Over 2 Days
Kim		
Jill		
Jack		

Over 2 days, the number of hours Kim works is equal to the difference of 4 times the number of hours Jack works and the number of hours Jill works.





## Solve

$$2(x + 4) = 4 \cdot 2(x - 2) - 2x$$

$$2(x + 4) = \square(x - 2) - 2x$$

Simplify.

$$2x + \square = 8x - \square - 2x$$

Distributive Property

$$2x + 8 = \square x - 16$$

$$2x + 8 + \square = 6x - 16 + \square$$

Addition Property of Equality

$$2x + \square = 6x$$

$$2x + 24 - \square x = 6x - \square x$$

Subtraction Property of Equality

$$24 = \square x$$

$$\frac{24}{\square} = \frac{\square x}{\square}$$

Division Property of Equality

$$\square = x$$

Jill works  hours per day, Kim works  hours per day, and Jack works  hours per day.



## Justify and Evaluate

Substitute  $x = 6$  into the original equation.

$$2(6 + 4) = 4 \cdot 2(6 - 2) - 2x$$

$$2(\square) = 8(\square) - 2(\square)$$

$$\square = \square$$

### Your Turn

Write and solve an equation to solve the problem.

9. Lisa is 10 centimeters taller than her friend Ian. Ian is 14 centimeters taller than Jim. Every month, their heights increase by 2 centimeters. In 7 months, the sum of Ian's and Jim's heights will be 170 centimeters more than Lisa's height. How tall is Ian now?

## Elaborate

10. How can you use properties to solve equations with variables on both sides?

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11. How is a table helpful when constructing equations?

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12. When solving a real-world problem to find a person's age, would a negative solution make sense? Explain.

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13. **Essential Question Check-In** How do you write an equation to represent a real-world situation?

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## Evaluate: Homework and Practice



- Online Homework
- Hints and Help
- Extra Practice

Write an equation for each description.

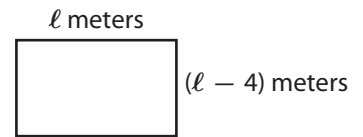
1. The sum of 14 and a number is equal to 17.
2. A number increased by 10 is 114.
3. The difference between a number and 12 is 20.
4. Ten times the sum of half a number and 6 is 8.
5. Two-thirds a number plus 4 is 7.
6. Tanmayi wants to raise \$175 for a school fundraiser. She has raised \$120 so far. How much more does she need to reach her goal?
7. Hector is visiting a cousin who lives 350 miles away. He has driven 90 miles. How many more miles does he need to drive to reach his cousin's home?
8. The length of a rectangle is twice its width. The perimeter of the rectangle is 126 feet.

**Write and solve an equation for each situation.**

- 9.** In one baseball season, Peter hit twice the difference of the number of home runs Alice hit and 6. Altogether, they hit 18 home runs. How many home runs did each player hit that season?



- 10.** The perimeter of a parallelogram is 72 meters. The width of the parallelogram is 4 meters less than its length. Find the length and the width of the parallelogram.



- 11.** One month, Ruby worked 6 hours more than Isaac, and Svetlana worked 4 times as many hours as Ruby. Together they worked 126 hours. Find the number of hours each person worked.
- 12.** In one day, Annie traveled 5 times the sum of the number of hours Brian traveled and 2. Together they traveled 20 hours. Find the number of hours each person traveled.

- 13.** Xian and his cousin Kai both collect stamps. Xian has 56 stamps, and Kai has 80 stamps. The boys recently joined different stamp-collecting clubs. Xian's club will send him 12 new stamps per month. Kai's club will send him 8 new stamps per month. After how many months will Xian and Kai have the same number of stamps? How many stamps will each have?



- 14.** Kenya plans to make a down payment plus monthly payments in order to buy a motorcycle. At one dealer she would pay \$2,500 down and \$150 each month. At another dealer, she would pay \$3,000 down and \$125 each month. After how many months would the total amount paid be the same for both dealers? What would that amount be?

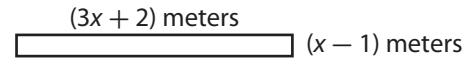
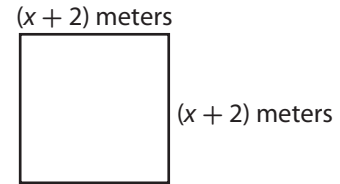
- 15.** Community Gym charges a \$50 membership fee and a \$55 monthly fee. Workout Gym charges a \$200 membership fee and a \$45 monthly fee. After how many months will the total amount of money paid to both gyms be the same? What will the amount be?
- 16.** Tina is saving to buy a notebook computer. She has two options. The first option is to put \$200 away initially and save \$10 every month. The second option is to put \$100 away initially and save \$30 every month. After how many months would Tina save the same amount using either option? How much would she save with either option?

Use the table to answer each question.

	Starting Salary	Yearly Salary Increase
Company A	\$24,000	\$3000
Company B	\$30,000	\$2400
Company C	\$36,000	\$2000

- 17.** After how many years are the salaries offered by Company A and Company B the same?
- 18.** After how many years are the salaries offered by Company B and Company C the same?
- 19.** Paul started work at Company B ten years ago at the salary shown in the table. At the same time, Sharla started at Company C at the salary shown in the table. Who earned more during the last year? How much more?
- 20.** George's page contains twice as many typed words as Bill's page and Bill's page contains 50 fewer words than Charlie's page. If each person can type 60 words per minute, after one minute, the difference between twice the number of words on Bill's page and the number of words on Charlie's page is 210. How many words did Bill's page contain initially? Use a table to organize the information.

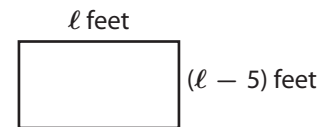
- 21. Geometry** Sammie bought just enough fencing to border either a rectangular plot or a square plot, as shown. The perimeters of the plots are the same. How many meters of fencing did she buy?



**H.O.T. Focus on Higher Order Thinking**

- 22. Justify Reasoning** Suppose you want to solve the equation  $2a + b = 2a$ , where  $a$  and  $b$  are nonzero real numbers. Describe the solution to this equation. Justify your description.

- 23. Multi-step** A patio in the shape of a rectangle, is fenced on all sides with 134 feet of fencing. The patio is 5 feet less wide than it is long.



- a. What information can be used to solve the problem? How can you find the information?

- b. Describe how to find the area of the patio. What is the area of the patio?

- 24. Explain the Error** Kevin and Brittany write an equation to represent the following relationship, and both students solve their equation. Who found the correct equation and solution? Why is the other person incorrect?

5 times the difference of a number and 20 is the same as half the sum of 4 more than 4 times a number.

Kevin:

$$5(x - 20) = \frac{1}{2}(4x + 4)$$

$$5x - 100 = 2x + 2$$

$$3x - 100 = 2$$

$$3x = 102$$

$$x = 34$$

Brittany:

$$5(20 + x) = \frac{1}{2}(4x + 4)$$

$$100 - 5x = 2x + 2$$

$$100 - 7x = 2$$

$$-7x = -98$$

$$x = 14$$

**25. What If?** Alexa and Zack are solving the following problem.

The number of miles on Car A is 50 miles more than the number of miles on Car B, and the number of miles on Car B is 30 miles more than the number of miles on Car C. All the cars travel 50 miles in 1 hour. After 1 hour, twice the number of miles on Car A is 70 miles less than 3 times the number of miles on Car C. How many miles were there on Car B initially?

Alexa assumes there are  $m$  miles on Car B. Zack assumes there are  $m$  miles on Car C. Will Zack's answer be the same as Alexa's answer? Explain.

## Lesson Performance Task

Stacy, Oliver, and Jivesh each plan to put a certain amount of money into their savings accounts that earn simple interest of 6% per year. Stacy puts \$550 more than Jivesh, and Oliver puts in 2 times as much as Jivesh. After a year, the amount in Stacy's account is 2 times the sum of \$212 and the amount in Oliver's account. How much does each person initially put into his or her account? Who had the most money in his or her account after a year? Who had the least? Explain.

# 2.3 Solving for a Variable



Resource Locker

**Essential Question:** How do you rewrite formulas and literal equations?

## Explore Rearranging Mathematical Formulas

**Literal Equations** are equations that contain two or more variables. There are many literal equations in the form of math, science, and engineering formulas. These formulas may seem like they can only be solved for the variable that is isolated on one side of the formula. By using inverse operations and the properties of equality, a formula can be rearranged so any variable in the formula can be isolated. It is no different than how equations are solved by using inverse operations and the properties of equality.

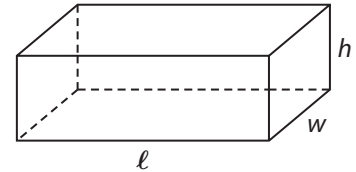
How can you solve the equation  $42 = 6x$ ?

**A**  $\frac{42}{\square} = \frac{6x}{\square}$       What is the reason for dividing? \_\_\_\_\_

Why divide by  $\square$ ? \_\_\_\_\_

$\square = x$       By rearranging the equation  $x$  was isolated and the solution was found.

The mathematical formula for the volume of a rectangular prism,  $B = Vh$  or  $V = \ell wh$ , is a literal equation.  $V$  represents volume,  $\ell$  represents length,  $w$  represents width, and  $h$  represents height. Using inverse operations, the formula can be rearranged to solve for any one of the variables that might be unknown. Like solving for  $x$ , a formula can be rearranged to isolate a variable.



**B** In the formula  $V = \ell wh$ , the variable  $h$  needs to be isolated.

The operation of \_\_\_\_\_ is used in the formula. The inverse

operation, \_\_\_\_\_, should be used to isolate  $\square$ .

**C**  $\frac{V}{\square} = \frac{\ell wh}{\square}$

$\frac{V}{\square} = \square$

The formula rearranged in this way can easily produce the height of the rectangular prism, when the volume, length, and width are known.

### Reflect

- Using the formula for a rectangular prism, rewrite the formula to solve for  $\ell$ .

## Explain 1 Rearranging Scientific Formulas

Use inverse operations to isolate the unknown variable in a scientific formula.

The formula for density is  $D = \frac{m}{V}$ . Lead has a very high density of  $11,340 \text{ kg/m}^3$ . Plastic foam has a very low density of  $75 \text{ kg/m}^3$ . The formula for density can be rearranged to solve for  $V$ , volume or  $m$ , mass.

### Example 1

- A** A sinker on a fishing line is made of lead and has a volume of  $0.000015 \text{ m}^3$ . What is the mass of the sinker?

The density formula can be rearranged to isolate  $m$ , the mass. The values for volume and density can then be substituted into the formula to find the mass.

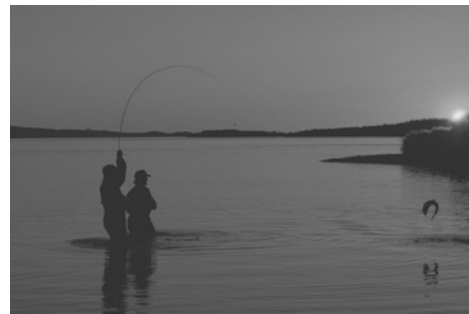
$$D = \frac{m}{V}$$

$$DV = \left(\frac{m}{V}\right)V$$

$$DV = m$$

$$(11,340 \text{ kg/m}^3)(0.000015 \text{ m}^3) = m$$

$$0.17 \text{ kg} \approx m$$



- B** The design for a life preserver requires  $0.3$  kilogram of plastic foam to provide proper buoyancy. What is the volume of the plastic foam required?

Rearrange the density formula to isolate  $V$ .

$$D = \frac{m}{V}$$

$$(D)V = \frac{m}{V} \square$$

$$DV = \square$$

$$\frac{DV}{\square} = \frac{m}{\square}$$

$$V = \square$$

Now substitute the given values.

$$V = \frac{\square}{\square}$$

$$V = \square \text{ m}^3$$

### Your Turn

- 2.** For altitudes up to  $36,000$  feet, the relationship between ground temperature and atmospheric temperature can be described by the formula  $t = -0.0035a + g$ , in which  $t$  is the atmospheric temperature in degrees Fahrenheit,  $a$  is the altitude, in feet, at which the atmospheric temperature is measured, and  $g$  is the ground temperature in degrees Fahrenheit. Determine the altitude in feet when  $t$  is  $-27.5^\circ\text{F}$  and  $g$  is  $60^\circ\text{F}$ .



## Explain 2 Rearranging Literal Equations

Using inverse operations to rearrange literal equations can be applied to any formula. The interest formula,  $I = prt$ , is another example of a literal equation. In the formula,  $I$  represents interest,  $p$  the principal or the initial amount to which interest will be applied,  $r$  the rate at which interest will be paid, and  $t$  is the time in years.

### Example 2

- A** Find the number of years used in the calculation of a \$1000 loan at an interest rate of 5% with interest totaling \$600.

Solve the formula for  $t$ .

$$I = prt$$

$$\frac{I}{pr} = \frac{prt}{pr}$$

$$\frac{I}{pr} = t$$

Substitute the given values. Since the interest rate is 5%,  $r = 0.05$ .

$$\frac{\$600}{\$1000 \cdot 0.05} = t$$

$$12 = t$$

So the length of time for the loan is 12 years.

- B** Determine the interest rate for a \$2000 loan that will be paid off in 4 years with interest totaling \$640. In order to find the interest rate, solve the formula for  $r$ .

$$I = prt$$

$$\frac{I}{pr} = \frac{prt}{pr}$$

$$\frac{\boxed{\phantom{000}}}{\boxed{\phantom{000}} \cdot \boxed{\phantom{000}}} = r$$

Now substitute the values and simplify.

$$\frac{\boxed{\phantom{000}}}{(\boxed{\phantom{000}})(\boxed{\phantom{000}})} = r$$

$$0.08 = r$$

So the interest rate is \_\_\_\_\_% per year.

### Your Turn

- 3.** The formula  $y = mx + b$  is the slope-intercept form of the equation of a line. Solve the equation for  $m$ .

## Elaborate

4. **Discussion** What could be a reason for isolating a variable in a literal equation?

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5. Describe a situation in which a formula could be used more easily if it were rearranged. Include the formula in your description.

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6. **Essential Question Check-In** How do you isolate a variable?

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## Evaluate: Homework and Practice



- Online Homework
- Hints and Help
- Extra Practice

Solve for the indicated variable in each mathematical formula.

1.  $C = 2\pi r$  for  $r$

2.  $A = \frac{1}{2}bh$  for  $b$

3.  $y = mx + b$  for  $x$

4.  $A = \frac{1}{2}(a + b)h$  for  $h$

5.  $V = \pi r^2 h$  for  $h$

6.  $SA = 2\pi r^2 + 2\pi rh$  for  $h$

Solve for the indicated variable in each scientific formula.

7.  $d = rt$  for  $t$

8.  $PV = nRT$  for  $T$

9.  $A = \frac{FV - OV}{T}$  for  $OV$

10.  $C = \frac{Wtc}{1000}$  for  $W$

Solve for the indicated variable in each literal equation.

11.  $2p + 5r = q$  for  $p$

12.  $-10 = xy + z$  for  $x$

13.  $\frac{a}{b} = c$  for  $b$

14.  $\frac{h-4}{j} = k$  for  $j$

15.  $\frac{x}{5} - g = a$  for  $x$

16.  $5p + 9c = p$  for  $c$

17.  $\frac{2}{5}(z + 1) = y$  for  $z$

18.  $g\left(h + \frac{2}{3}\right) = 1$  for  $h$

19.  $a(n - 3) + 8 = bn$  for  $n$

20. Which is a possible way to rewrite the equation  $y = 3x + 3b$  to solve for  $b$ ?

A.  $b = \frac{y - 3x}{3}$

C.  $b = \frac{y - 3}{3x}$

B.  $b = 3(y - 3x)$

D.  $b = x(y - 3)$

21. **Sports** To find a baseball pitcher's earned run average (ERA), you can use the formula  $Ei = 9r$ , in which  $E$  represents ERA,  $i$  represents the number of innings pitched, and  $r$  represents the number of earned runs allowed. Solve the equation for  $E$ . What is a pitcher's ERA if he allows 5 earned runs in 18 innings pitched?



22. **Meteorology** For altitudes up to 36,000 feet, the relationship between ground temperature and atmospheric temperature can be described by the formula  $t = -0.0035a + g$ , in which  $t$  is the atmospheric temperature in degrees Fahrenheit,  $a$  is the altitude, in feet, at which the atmospheric temperature is measured, and  $g$  is the ground temperature in degrees Fahrenheit. Solve the equation for  $a$ . If the atmospheric temperature is  $-65.5^\circ\text{F}$  and the ground temperature is  $57^\circ\text{F}$ , what is the altitude?

**H.O.T. Focus on Higher Order Thinking**

**23. Explain the Error** A student was asked to use the formula for the perimeter of a rectangle,  $P = 2\ell + 2w$ , to solve for  $\ell$ . The student came up with an answer,  $P - 2w = 2\ell$ . What error did the student make? Explain. Then solve for  $\ell$ .

**24. Multi-Step** The formula  $c = 5p + 215$  relates  $c$ , the total cost in dollars of hosting a birthday party at a skating rink, to  $p$ , the number of people attending. If Allie's parents are willing to spend \$300 for a party, how many people can attend?



**25. Multi-Step** The formula for the area of a triangle is  $A = \frac{1}{2}bh$ , in which  $b$  represents the length of the base and  $h$  represents the height. If a triangle has an area of  $192 \text{ mm}^2$  and the height is 12 mm, what is the measure of the base?

## Lesson Performance Task

The following table shows the average low temperatures in Fahrenheit for the city of Boston for several months during the year. The formula  $F = \frac{9}{5}C + 32$  allows you to determine the temperature in Fahrenheit when given the temperature in Celsius.

Month	Temperature in Fahrenheit	Temperature in Celsius
January	22°	
April	41°	
July	65°	
October	47°	
December	28°	

- Use the information given to determine the average low temperatures in Celsius.
- Would it ever be possible for the temperature in Celsius to have a greater value than the temperature in Fahrenheit? Explain why or why not.

# 2.4 Creating and Solving Inequalities



Resource Locker

**Essential Question:** How do you write and solve an inequality that represents a real-world situation?

## Explore Creating Inequalities from Verbal Descriptions

An **inequality** is a statement that compares two expressions that are not strictly equal by using one of the following inequality signs.

Symbol	Meaning
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to
$\neq$	is not equal to

You have probably seen a sign at an amusement park saying something like, “You must be at least 48 inches tall to ride this ride.” This statement could be written as  $h \geq 48$  in., where  $h$  represents the height of a person allowed to ride.

Nora is planning a birthday party for her little sister, Colleen. Nora’s budget will allow her to spend no more than \$50 for party supplies. Eight children, including Colleen, will attend the party, and Nora wants to determine how much she could spend on party favors for each child. She will also purchase a cake for \$10. Write an inequality that represents the situation, and find possible solutions.



- A** First, let  $c$  represent the cost of a party favor for each child. Write an expression for the total cost of the party as a function of  $c$ .

$$\square c + \square$$

- B** Which inequality symbol should be used to represent the phrase “no more than”? \_\_\_\_\_

- C** Write the inequality that represents Nora’s budget goal.

$$8c + 10 \leq \square$$

- D** Suppose Nora finds party favors that cost \$4 each. Use a value of 4 for  $c$  and check to see if this inequality is true.

$$8 \cdot 4 + 10 \stackrel{?}{\leq} 50$$

$$\square \stackrel{?}{\leq} 50$$

E Is the inequality true? \_\_\_\_\_

F Could Nora buy \$6 party favors for all of her guests without going over budget?

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**Reflect**

1. Why does an inequality represent Nora's budget calculation better than an equation?

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2. The solution set of an inequality consists of all values that make the statement true. Describe the whole dollar amounts that are in the solution set for this situation.

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**Explain 1** **Creating and Solving Inequalities Involving the Distributive Property**

You may need to use the Distributive Property before you can solve an inequality.

**Distributive Property**

If  $a$ ,  $b$ , and  $c$  are real numbers, then  $a(b + c) = ab + ac$ .

The inequality sign must be reversed when multiplying or dividing both sides of an inequality by a negative number.

**Example 1**

A Trina is buying 12 shirts for the drama club. She will choose a style for the blank shirts and then pay an additional charge of \$2.75 for each shirt to have the club logo. If Trina cannot spend more than \$99, how much can she spend on each blank shirt? Write and solve an inequality to find the possible cost of each blank shirt.

Let  $s$  represent the cost of each blank shirt.

Write an inequality to represent the situation.

$$12(s + 2.75) \leq 99$$

Use the Distributive Property.

$$12s + 33 \leq 99$$

Subtraction Property of Inequality

$$12s + 33 - 33 \leq 99 - 33$$

Simplify.

$$12s \leq 66$$

Division Property of Inequality

$$\frac{12s}{12} \leq \frac{66}{12}$$

Simplify.

$$s \leq 5.5$$

Check your answer.

Since  $s \leq 5.5$ , check a smaller number.

$$12(5 + 2.75) \stackrel{?}{\leq} 99$$

Trina can order blank shirts that cost \$5.50 or less.

$$93 \stackrel{?}{\leq} 99 \text{ true}$$

- B** Sergio needs to buy gifts for 8 friends. He wants to give the same gift to all his friends and he plans to have the gifts wrapped for an additional charge of \$1.50 each. If Sergio spends at least \$70, he will receive free shipping on his order. Write and solve an inequality to determine how much Sergio needs to spend on each gift in order to receive free shipping.

Let  $g$  be the cost of one gift.

Write an inequality to represent the situation.

$$\square (g + \square) \geq 70$$

Use the Distributive Property.

$$\square g + \square \geq 70$$

Subtraction Property of Inequality

$$\square g + 12 - \square \geq 70 - \square$$

Simplify.

$$8g \geq \square$$

Division Property of Inequality

$$\frac{8g}{\square} \geq \frac{58}{\square}$$

Simplify.

$$g \geq \square$$

Check your answer.

Since  $g \geq 7.25$ , check a larger number.

$$8(8 + 1.50) \stackrel{?}{\geq} 70$$

Sergio must spend at least  on each gift.

$$\square \stackrel{?}{\geq} 70$$

### Reflect

- 3. Discussion** Why is the first step in solving the inequality to use the Distributive Property instead of working inside the parentheses?

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### Your Turn

- 4.** Zachary is planning to send a video game to each of his two brothers. If he buys the same game for both brothers and pays \$4.75 to ship each game, how much can he spend on each game without spending more than \$100? Write and solve an inequality for this situation.

**Solve each inequality.**

**5.**  $\frac{4}{3}(6x + 9) < 4$

**6.**  $-2\left(\frac{1}{4}x + 2\right) \geq 5$



## Explain 2

# Creating and Solving Inequalities with Variables on Both Sides

Some inequalities have variable terms on both sides of the inequality symbol. You can solve these inequalities the same way you solved equations with variables on both sides. Use the properties of inequality to collect all the variable terms on one side and all the constant terms on the other side.

### Example 2

- A** The *Daily Info* charges a fee of \$650 plus \$80 per week to run an ad. The *People's Paper* charges \$145 per week. For how many weeks must an ad run for the total cost at the *Daily Info* to be less expensive than the cost at the *People's Paper*? Let  $w$  be the number of weeks the ad runs in the paper.

Write an inequality to represent the situation.

$$650 + 80w < 145w$$

Subtraction Property of Inequality

$$650 + 80w - 80w < 145w - 80w$$

Simplify.

$$650 < 65w$$

Division Property of Inequality

$$\frac{650}{65} < \frac{65w}{65}$$

Simplify.

$$10 < w$$

The total cost at the *Daily Info* is less than the cost at the *People's Paper* if the ad runs for more than 10 weeks.

- B** The Home Cleaning Company charges \$312 to power-wash the siding of a house plus \$12 for each window. Power Clean charges \$36 per window, and the price includes power-washing the siding. How many windows must a house have to make the total cost from The Home Cleaning Company less expensive than Power Clean? Let  $w$  be the number of windows.

Write an inequality to represent the situation.

$$\boxed{\phantom{000}} + 12w < 36w$$

Subtraction Property of Inequality

$$312 + 12w - \boxed{\phantom{000}} < 36w - \boxed{\phantom{000}}$$

Simplify.

$$312 < \boxed{\phantom{000}}$$

Division Property of Inequality

$$\frac{312}{\boxed{\phantom{000}}} < \frac{24w}{\boxed{\phantom{000}}}$$

Simplify.

$$\boxed{\phantom{000}} < w$$

A house must have more than 13 windows for The Home Cleaning Company to be less expensive than Power Clean.

### Reflect

7. How would the final inequality change if you divided by  $-24$  in the next to last step?
- 

### Your Turn

8. The school band will sell pizzas to raise money for new uniforms. The supplier charges \$100 plus \$4 per pizza. The band members sell the pizzas for \$7 each. Write and solve an inequality to find how many pizzas the band members will have to sell to make a profit?



## Elaborate

9. Which inequality symbol would you use to represent the following words or phrases? Can you come up with more examples?
- a. at most                      b. farther than                      c. younger than                      d. up to
- 
10. **Discussion** How are the steps to solving an inequality similar to those for solving an equation? How are they different?
- 
- 
- 
11. **Essential Question Check-In** How can you write an inequality that represents a real-world situation?
- 
- 

## Evaluate: Homework and Practice



- Online Homework
- Hints and Help
- Extra Practice

Write an inequality that represents the description, and then solve.

- |   |  |
|---|--|
| 1. Max has more than 5 carrots (number of carrots Max has = $c$ ).  | 2. Brigitte is shorter than 5 feet (Brigitte's height = $h$ ).   |
| 3. Twice a number ( $x$ ) is less than 10.  | 4. Six more than five times a number ( $x$ ) is at least twenty-one.   |
| 5. Dave has \$15 to spend on an \$8 book and two birthday cards ( $c$ ) for his friends. How much can he spend on each card if he buys the same card for each friend? | 6. Toni can carry up to 18 lb in her backpack. Her lunch weighs 1 lb, her gym clothes weigh 2 lb, and her books ( $b$ ) weigh 3 lb each. How many books can she carry in her backpack? |

**Solve each inequality.**

**7.**  $3(x - 2) > -3$

**8.**  $5 + 5(x + 4) \leq 20$

**9.**  $3 + \frac{1}{2}(3 - x) < -7$

**10.**  $3(x + 6) - 2(x + 2) \geq 10$

**11.**  $5(3 - x) - 4(2 - 3x) > 2$

**12.**  $\frac{1}{2}(4x - 2) - \frac{2}{3}(6x + 9) \leq 4$

**13.**  $x + 1 > -5(7 - 2x)$

**14.**  $\frac{5}{3}(6x + 3) \leq 2x - 7$

**15.**  $2x \leq -\frac{2}{3}(4x + 4)$

**16.**  $\frac{1}{2}(-2x - 10) > 3(4 - 6x)$

**17.**  $-5 - 3x \geq 2(10 + 2x) + 3$

**18.**  $-3(9x + 20) \geq 15x - 20$

**19.**  $8\left(\frac{1}{4}x - 3\right) + 24 < 4(x + 5)$

**20.**  $6x - 2(x + 2) > 2 - 3(x + 3)$

**21. Physics** A crane cable can support a maximum load of 15,000 kg. If a bucket has a mass of 2,000 kg and gravel has a mass of 1,500 kg for every cubic meter, how many cubic meters of gravel ( $g$ ) can be safely lifted by the crane?

**22.** Find the solution set of each inequality below, and then determine which inequalities have the same solution set as  $\frac{1}{3}(-5x - 3) < 14$ .

a.  $\frac{1}{3}(5x + 3) > -14$

b.  $\frac{2}{5}(10x + 20) > 44$

c.  $-\frac{2}{5}(10x + 20) < -44$

d.  $-\frac{1}{3}(5x + 3) < 14$

e.  $\frac{2}{5}(10x - 20) > -44$

f.  $\frac{1}{3}(5x + 3) < -14$

**H.O.T. Focus on Higher Order Thinking**

**23. Explain the Error** Sven is trying to find the maximum amount of time he can spend practicing the five scales of piano music he is supposed to be working on. He has 60 minutes to practice piano and would like to spend at least 35 minutes playing songs instead of practicing scales. So, Sven sets up the following inequality, where  $t$  is the number of minutes he spends on each scale, and solves it.

$$\begin{aligned}60 - 5t &\leq 35 \\ -5t &\leq -25 \\ t &\geq 5\end{aligned}$$

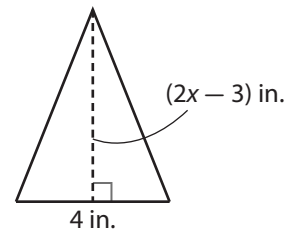
Sven has concluded that he should spend 5 minutes or more on each scale. Is this correct? If not, what mistake did he make? Then solve for the correct answer.

- 24. Critical Thinking** Anika wants to determine the maximum number of tulip bulbs ( $t$ ) she can purchase if each bulb costs \$1.50. She will also need to purchase separate pots for each bulb at \$1.25 each and a bag of potting soil for \$10.00. Set up an inequality to determine how many tulip bulbs Anika can purchase without spending more than \$20.00, and solve it. Can Anika buy exactly enough bulbs and pots to spend the full \$20.00? Explain. Can you think of a better inequality to describe the answer?



- 25. Geometry** The area of the triangle shown is no more than 10 square inches.

a. Write an inequality that can be used to find  $x$ .



b. Solve the inequality from part a.

c. What is the maximum height of the triangle?

## Lesson Performance Task

When planning an airplane route, the trip planner must be careful to consider the range the aircraft can travel and the availability of airports to stop and refuel.

A large commercial jet airplane burns approximately 5 gallons of fuel per mile flown, plus about 8500 gallons of fuel per trip to reach cruising altitude. With a useable fuel capacity of 50,000 gallons (there is additional fuel reserved for emergencies), describe the acceptable distance for which an airline could establish a non-stop flight.

For flights originating from Chicago, determine which cities could be safely reached by a non-stop flight.

City	Distance from Chicago (miles)
Bangalore, India	8530
Jakarta, Indonesia	9810
Johannesburg, South Africa	8680
Moscow, Russia	4970
Paris, France	4130
Perth, Australia	10,970
Tokyo, Japan	6300

# 2.5 Creating and Solving Compound Inequalities



Resource Locker

**Essential Question:** How can you solve a compound inequality and graph the solution set?

## Explore Truth Tables and Compound Statements

A **compound statement** is formed by combining two or more simple statements. A compound statement can be true or false. A compound statement involving **AND** is true when *both* simple statements are true. A compound statement involving **OR** is true when *either* one simple statement *or both* are true.

**A** Complete the truth table.

<i>P</i>	<i>Q</i>	<i>P</i> True or False?	<i>Q</i> True or False?	<i>P AND Q</i> True or False?
A dog is a mammal.	Red is a color.			
A dog is a mammal.	Red is not a color.			
A dog is a fish.	Red is a color.			
A dog is a fish.	Red is not a color.			

**B** *P AND Q* is true when \_\_\_\_\_.

**C** Complete the truth table.

<i>P</i>	<i>Q</i>	<i>P</i> True or False?	<i>Q</i> True or False?	<i>P OR Q</i> True or False?
1 is an odd number.	2 is an even number.			
1 is an odd number.	2 is an odd number.			
1 is an even number.	2 is an even number.			
1 is an even number.	2 is an odd number.			

**D** *P OR Q* is true when \_\_\_\_\_.

### Reflect

1. Give two simple statements *P* and *Q* for which *P AND Q* is false and *P OR Q* is true.

\_\_\_\_\_

\_\_\_\_\_



## Explain 1

# Solving Compound Inequalities Involving AND

Combining two or more simple inequalities forms a **compound inequality**. The graph of a compound inequality involving **AND** is the **intersection**, or the overlapping region, of the simple inequality graphs.

Compound Inequalities: AND		
Words	Algebra	Graph
All real numbers greater than 2 <b>AND</b> less than 6	$x > 2$ <b>AND</b> $x < 6$ $2 < x < 6$	
All real numbers greater than or equal to 2 <b>AND</b> less than or equal to 6	$x \geq 2$ <b>AND</b> $x \leq 6$ $2 \leq x \leq 6$	

**Example 1** Solve each compound inequality and graph the solutions.

**A**  $4 \leq x + 2 \leq 8$

$4 \leq x + 2$       **AND**       $x + 2 \leq 8$

Write the compound inequality using AND.

$4 - 2 \leq x + 2 - 2$        $x + 2 - 2 \leq 8 - 2$

Subtract 2 from both sides of each simple inequality.

$2 \leq x$        $x \leq 6$

Simplify.

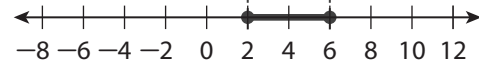
Graph  $2 \leq x$ .



Graph  $x \leq 6$ .



Graph the intersection by finding where the two graphs overlap.



**B**  $-5 \leq 2x + 3 < 9$

$-5 - \square \leq 2x + 3 - \square < 9 - \square$

Subtract  $\square$  from each part of the inequality.

$\square \leq 2x < \square$

Simplify.

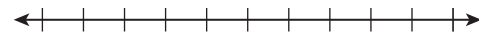
$\frac{-8}{\square} \leq \frac{2x}{\square} < \frac{6}{\square}$

Divide each part of the inequality by  $\square$ .

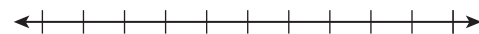
$\square \leq x < \square$

Simplify.

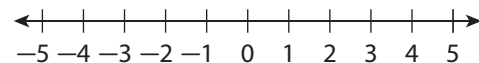
Graph  $-4 \leq x$ .



Graph  $x < 3$ .



Graph the intersection by finding where the two graphs overlap.



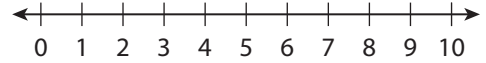
**Reflect**

2. **Discussion** Explain why  $2 \leq x \leq 6$  can be considered the *short method* for writing the **AND** compound inequality  $x \geq 2$  **AND**  $x \leq 6$ .
- 

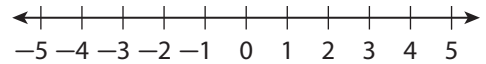
**YourTurn**

Solve each compound inequality and graph the solutions.

3.  $-2 < x - 3 < 5$



4.  $-10 < 3x + 2 \leq 8$



**Explain 2 Solving Compound Inequalities Involving OR**

The graph of a compound inequality involving **OR** is the **union**, or the combined region, of the simple inequality graphs.

Compound Inequalities: OR		
Words	Algebra	Graph
All real numbers less than 2 <b>OR</b> greater than 6	$x < 2$ <b>OR</b> $x > 6$	
All real numbers less than or equal to 2 <b>OR</b> greater than or equal to 6	$x \leq 2$ <b>OR</b> $x \geq 6$	

**Example 2** Solve each compound inequality and graph the solutions.

**A**  $-4 + x > 1$  OR  $-4 + x < -3$

$-4 + x > 1$  OR  $-4 + x < -3$

Write the compound inequality using OR.

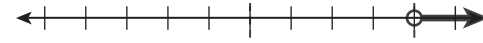
$-4 + 4 + x > 1 + 4$      $-4 + 4 + x < -3 + 4$

Add 4 to both sides of each simple inequality.

$x > 5$                                    $x < 1$

Simplify.

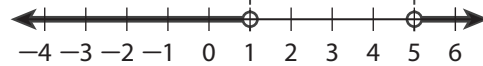
Graph  $x > 5$ .



Graph  $x < 1$ .



Graph the union by combining the graphs.



**B**  $2x \leq 6$  OR  $3x > 12$

$2x \leq 6$  OR  $3x > 12$

Write the compound inequality using OR.

$\frac{2x}{\square} \leq \frac{6}{\square}$  OR  $\frac{3x}{\square} > \frac{12}{\square}$

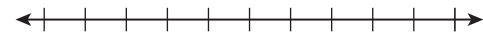
Divide the first simple inequality by  $\square$ .

$x \leq \square$  OR  $x > \square$

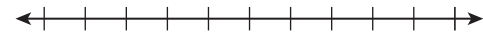
Divide the second simple inequality by  $\square$ .

Simplify.

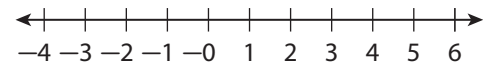
Graph  $x \leq 3$ .



Graph  $x > 4$ .



Graph the union by combining the graphs.



**Reflect**

**5. Critical Thinking** What kind of compound inequality has no solution?

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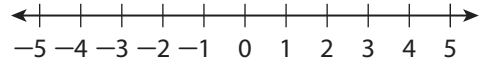
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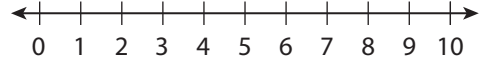
**Your Turn**

Solve each compound inequality and graph the solutions.

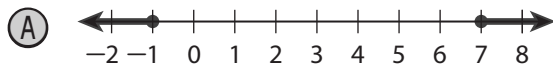
6.  $x - 5 \geq -2$  OR  $x - 5 \leq -6$



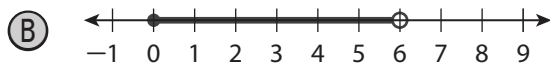
7.  $4x - 1 < 15$  OR  $8x \geq 48$

**Explain 3 Creating Compound Inequalities From Graphs**

Given a number line graph with a solution set graphed, you can create a compound inequality to fit the graph.

**Example 3** Write the compound inequality shown by each graph.

The shaded portion of the graph is not between two values, so the compound inequality involves OR.

On the left, the graph shows an arrow pointing left from  $-1$  and a solid circle, so use  $\leq$ .The inequality is  $x \leq -1$ .On the right, the graph shows an arrow pointing right from  $7$  and a solid circle, so use  $\geq$ .The inequality is  $x \geq 7$ .The compound inequality is  $x \leq -1$  OR  $x \geq 7$ .The graph is shaded between the values  and , so the compound inequality involves AND.The graph is shaded to the right/left of  and the circle is open/solid, so use the inequality symbol \_\_\_\_\_.

The inequality is \_\_\_\_\_.

The graph is shaded to the right/left of  and the circle is open/solid, so use the inequality symbol \_\_\_\_\_.

The inequality is \_\_\_\_\_.

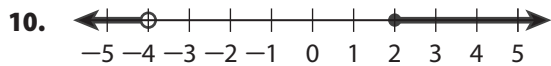
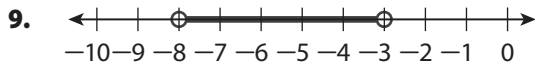
The compound inequality is \_\_\_\_\_.

**Reflect**

8. What is a *short method* to write the compound inequality  $x \geq 0$  AND  $x < 6$ ?
- 

**Your Turn**

Write the compound inequality shown by each graph.

**Explain 4**

## Expressing Acceptable Levels with Compound Inequalities

You can express quality-controls levels in real-world problems using compound inequalities.

**Example 4** Write a compound inequality to represent the indicated quality-control level, and graph the solutions.

- (A) The recommended pH level for swimming pool water is between 7.2 and 7.6, inclusive.

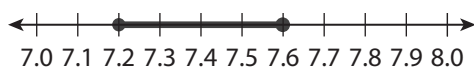
Let  $p$  be the pH level of swimming pool water.

7.2 is less than or equal to pH level is less than or equal to 7.6

$7.2 \leq p \leq 7.6$

The compound inequality is  $7.2 \leq p \leq 7.6$ .

Graph the solutions.



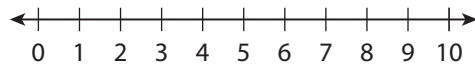
- B** The recommended free chlorine level for swimming pool water is between 1.0 and 3.0 parts per million, inclusive.

Let  $c$  be the free chlorine level in the pool.

is less than or equal to free chlorine level is less than or equal to   
   $c$

The compound inequality is   $\leq c \leq$  .

Graph the solutions.



**Reflect**

- 11. Discussion** What does the phrase “between 7.2 and 7.6, inclusive” mean?

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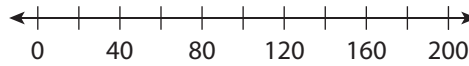


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**Your Turn**

Write a compound inequality to represent the indicated quality-control level, and graph the solutions.

- 12.** The recommended alkalinity level for swimming pool water is between 80 and 120 parts per million, inclusive.



**Elaborate**

- 13.** Explain the difference between graphing a compound inequality involving **AND** and graphing a compound inequality involving **OR**.

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- 14.** How can you tell whether a compound inequality involves **AND** or **OR** from looking at its graph?

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- 15. Essential Question Check-In** Explain how to find the solutions of a compound inequality.

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# Evaluate: Homework and Practice



- Online Homework
- Hints and Help
- Extra Practice

Complete the truth tables.

1.

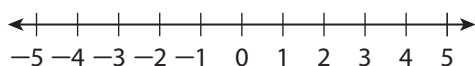
$P$	$Q$	$P$ True or False?	$Q$ True or False?	$P$ AND $Q$ True or False?
An apple is a fruit.	A carrot is a vegetable.			
An apple is a fruit.	A carrot is a fruit.			
An apple is a vegetable.	A carrot is a vegetable.			
An apple is a vegetable.	A carrot is a fruit.			

2.

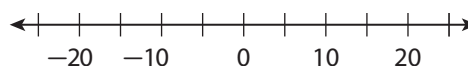
$P$	$Q$	$P$ True or False?	$Q$ True or False?	$P$ OR $Q$ True or False?
Blue is a color.	Five is a number.			
Blue is a color.	Five is a color.			
Blue is a number.	Five is a number.			
Blue is a number.	Five is a color.			

Solve each compound inequality and graph the solutions.

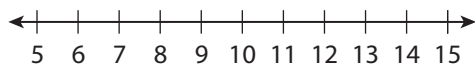
3.  $-3 < 3x \leq 9$



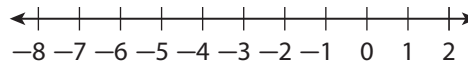
4.  $0 \leq 2x - 10 \leq 20$



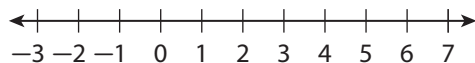
5.  $x - 5 < 3$  OR  $x - 5 \geq 8$



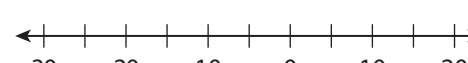
6.  $1 \leq x + 7 < 7$



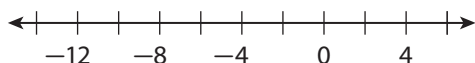
7.  $4x + 3 < -5$  OR  $4x + 3 > 23$



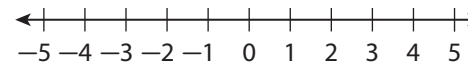
8.  $\frac{x}{5} - 2 \leq -6$  OR  $8x + 1 \geq 41$



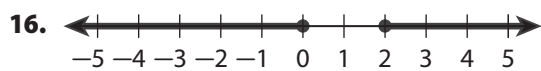
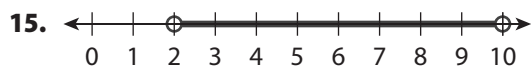
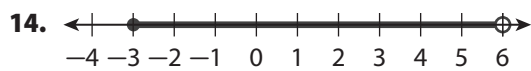
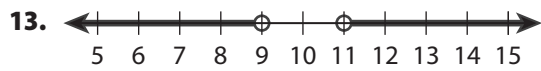
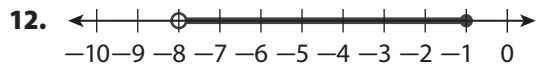
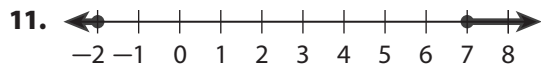
9.  $-6 < \frac{x - 12}{4} < -2$



10.  $x + 7 \leq 7$  OR  $5 + 2x > 7$



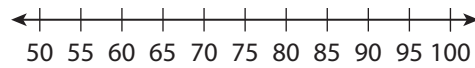
Write the compound inequality shown by each graph.



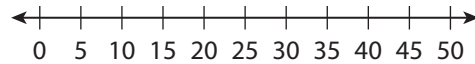
Write a compound inequality to show the levels that are within each range. Then graph the solutions.



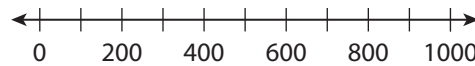
- 17. Biology** An iguana needs to live in a warm environment. The temperature in a pet iguana's cage should be between 70 °F and 95 °F, inclusive.



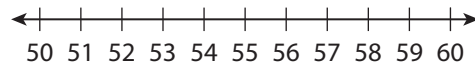
- 18. Meteorology** One layer of Earth's atmosphere is called the stratosphere. At one point above Earth's surface, the stratosphere extends from an altitude of 16 kilometers to an altitude of 50 kilometers.



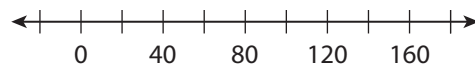
- 19. Music** A typical acoustic guitar has a range of three octaves. When the guitar is tuned to "concert pitch," the range of frequencies for those three octaves is between 82.4 Hertz and 659.2 Hertz inclusive.



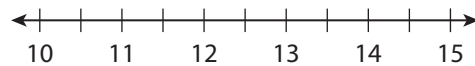
- 20. Transportation** The cruise-control function on Georgina's car should keep the speed of the car within 3 miles per hour of the set speed. The set speed is 55 miles per hour.



- 21. Chemistry** Water is not a liquid if its temperature is above 100 °C or below 0 °C.

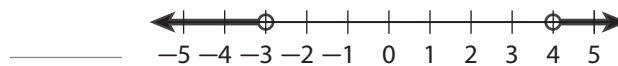


- 22. Sports** The ball used in a soccer game may not weigh more than 16 ounces or less than 14 ounces at the start of the match. After 1.5 ounces of air were added to a ball, the ball was approved for use in a game.

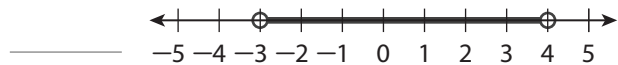


- 23.** Match the compound inequalities with the graphs of their solutions.

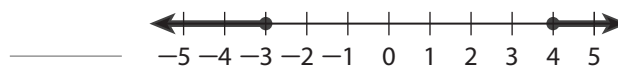
A.  $-26 < 6x - 8 < 16$



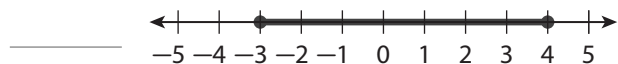
B.  $-\frac{1}{3} \leq \frac{x+2}{3} \leq 2$



C.  $4x + 1 < -11$  OR  $\frac{x}{2} - 5 > -3$



D.  $\frac{x-6}{3} \leq -3$  OR  $2x + 8 \geq 16$



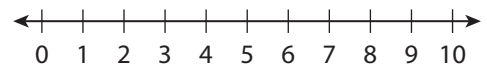
**H.O.T. Focus on Higher Order Thinking**



**24. Multi-Step** Jenna's band is going to record a CD at a recording studio. They will pay \$225 to use the studio for one day and \$80 per hour for sound technicians. Jenna has \$200 and can reasonably expect to raise up to an additional \$350 by taking pre-orders for the CDs.

- a. Explain how the inequality  $200 \leq 225 + 80n \leq 550$  can be used to find the number of hours Jenna and her band can afford to use the studio and sound technicians.
  
  
  
  
  
  
  
  
  
  
- b. Solve the inequality. Are there any numbers in the solution set that are not reasonable in this situation?
  
  
  
  
  
  
  
  
  
  
- c. Suppose Jenna raises \$350 in pre-orders. How much more money would she need to raise if she wanted to use the studio and sound technicians for 6 hours?

**25. Explain the Error** A student solves the compound inequality  $15 \leq 2x + 5 \leq 17$  and finds the solutions of the compound inequality to be all real numbers. Explain and correct the student's mistake. Graph the actual solutions to back up your answer.



**26. Communicate Mathematical Thinking** Describe the solutions of the compound inequalities.

$x > 9$  AND  $x < 9$

$x < 9$  OR  $x > 9$

$x \geq 9$  AND  $x \leq 9$

$x \leq 9$  OR  $x \geq 9$

# Lesson Performance Task

The table gives the melting point and boiling point of various elements. Write a compound inequality for each element to show the temperature range of the element in its liquid state. Graph the solutions of each. Suppose you were to set the temperature of each element to its melting point and increase the temperature of each element at the same rate. Which element will remain liquid for the longest amount of time? Which element will reach its boiling point first? Explain.

Element	Melting Point ( $^{\circ}\text{C}$ )	Boiling Point ( $^{\circ}\text{C}$ )
Gold	1064	2856
Copper	1085	2562
Iron	1538	2861
Lead	327	1749
Aluminum	660	2519

Gold	
Copper	
Iron	
Lead	
Aluminum	

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