Name ____________________ Class ____________ Date ________

1.1 Solving Equations

Essential Question: How do you solve an equation in one variable?

Resource Locker

Explore Solving Equations by Guess-and-Check or by Working Backward

An **equation** is a mathematical sentence that uses the equal sign = to show two expressions are equivalent. The expressions can be numbers, variables, constants, or combinations thereof.

There are many ways to solve an equation. One way is by using a method called *guess-and-check*. A guess-and-check method involves guessing a value for the variable in an equation and checking to see if it is the solution by substituting the value in the equation. If the resulting equation is a true statement, then the value you guessed is the solution of the equation. If the equation is not a true statement, then you adjust the value of your guess and try again, continuing until you find the solution.

Another way to solve an equation is by *working backward*. In this method, you begin at the end and work backward toward the beginning.

Solve the equation $x - 6 = 4$ using both methods.

Use the *guess-and-check* method to find the solution of the equation $x - 6 = 4$.

(A) Guess 11 for x.

$x - 6 = 4$

$\boxed{} - 6 \stackrel{?}{=} 4$

$\boxed{} \stackrel{?}{=} 4$

Is 11 the solution of $x - 6 = 4$? ____________

(B) The value 11 is too high.

Guess 10 for x.

$x - 6 = 4$

$\boxed{} - 6 \stackrel{?}{=} 4$

$\boxed{} \stackrel{?}{=} 4$

Is 10 the solution of $x - 6 = 4$? ____________

(C) Use the *working backward* method to find the solution of the equation $x - 6 = 4$.

$4 + 6 = \boxed{}$ Is this the value of x before taking away 6?

$\boxed{} - 6 \stackrel{?}{=} 4$ ____________

Reflect

1. **Discussion** Which method of solving do you think is more efficient? Explain your answer.

____

____

Explain 1 Solving One-Variable Two-Step Equations

A **solution of an equation** is a value for the variable that makes the equation true. To determine the solution of an equation, you will use the Properties of Equality.

Properties of Equality		
Words	**Numbers**	**Algebra**
Addition Property of Equality You can add the same number to both sides of an equation, and the statement will still be true.	$3 = 3$ $3 + 2 = 3 + 2$ $5 = 5$	$a = b$ $a + c = b + c$
Subtraction Property of Equality You can subtract the same number from both sides of an equation, and the statement will still be true.	$7 = 7$ $7 - 5 = 7 - 5$ $2 = 2$	$a = b$ $a - c = b - c$
Multiplication Property of Equality You can multiply both sides of an equation by the same number, and the statement will still be true.	$3 = 3$ $3 \cdot 4 = 3 \cdot 4$ $12 = 12$	$a = b$ $a \cdot c = b \cdot c$
Division Property of Equality You can divide both sides of an equation by the same nonzero number, and the statement will still be true.	$15 = 15$ $\frac{15}{3} = \frac{15}{3}$ $5 = 5$	$a = b$ $\frac{a}{c} = \frac{b}{c}$, where $c \neq 0$

Example 1 **Solve the equation by using Properties of Equality.**

(A) $3x - 2 = 6$

Use the Addition Property of Equality. $3x - 2 + 2 = 6 + 2$

Combine like terms. $3x = 8$

Now use the Division Property of Equality. $\frac{3x}{3} = \frac{8}{3}$

Simplify. $x = \frac{8}{3}$

(B) $\frac{1}{2}z + 4 = 10$

Use the Subtraction Property of Equality. $\frac{1}{2}z + 4 - \boxed{} = 10 - \boxed{}$

Combine like terms. $\frac{1}{2}z = \boxed{}$

Now use the Multiplication Property of Equality to multiply each side by 2. $2 \cdot \frac{1}{2}z = 2 \cdot \boxed{}$

Simplify. $z = \boxed{}$

Reflect

2. **Discussion** What is the goal when solving a one-variable equation?

Your Turn

Solve the equation by using Properties of Equality.

3. $5x - 10 = 20$

4. $\frac{1}{3}x + 9 = 21$

Explain 2 Solving Equations to Define a Unit

One useful application of algebra is to use an equation to determine what a unit of measure represents. For instance, if a person uses the unit of time "score" in a speech and there is enough information given, you can solve an equation to find the quantity that a "score" represents.

Example 2 **Solve an equation to determine the unknown quantity.**

(A) In 1963, Dr. Martin Luther King, Jr., began his famous "I have a dream" speech with the words "Five score years ago, a great American, in whose symbolic shadow we stand, signed the Emancipation Proclamation." The proclamation was signed by President Abraham Lincoln in 1863. But how long is a score? We can use algebra to find the answer.

Let s represent the quantity (in years) represented by a score.

s = number of years in a score

Calculate the quantity in years after President Lincoln signed the Emancipation Proclamation.

$$1963 - 1863 = 100$$

Dr. Martin Luther King, Jr. used "five score" to describe this length of time. Write the equation that shows this relationship.

$$5s = 100$$

Use the Division Property of Equality to solve the equation.

$$\frac{5s}{5} = \frac{100}{5}$$

$$s = 20$$

A score equals 20 years.

(B) An airplane descends in altitude from 20,000 feet to 10,000 feet. A gauge at Radar Traffic Control reads that the airplane's altitude drops 1.8939 miles. How many feet are in a mile?

Let m represent the quantity (in feet) represented by a mile.

m = number of feet in a mile

Calculate the quantity in feet of the descent. $20{,}000 - \boxed{} = \boxed{}$

A gauge described this quantity as 1.8939 miles. Write the equation that shows this relationship.

$$1.8939m = \boxed{}$$

Use the Division Property of Equality to solve the equation.

$$\frac{1.8939m}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

Round to the nearest foot. $m \approx \boxed{}$

There are 5280 feet in a mile.

Your Turn

Solve an equation to determine the unknown quantity.

5. An ostrich that is 108 inches tall is 20 inches taller than 4 times the height of a kiwi. What is the height of a kiwi in inches?

6. An emu that measures 60 inches in height is 70 inches less than 5 times the height of a kakapo. What is the height of a kakapo in inches?

Elaborate

7. How do you know which operation to perform first when solving an equation?

8. How can you create an equivalent equation by using the Properties of Equality?

9. When a problem involves more than one unit for a characteristic (such as length), how can you tell which unit is more appropriate to report the answer in?

10. **Essential Question Check-In** Describe each step in a solution process for solving an equation in one variable.

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Use the *guess-and-check* method to find the solution of the equation. Show your work.

1. $2x + 5 = 19$

Use the *working backward* method to find the solution of the equation. Show your work.

2. $4y - 1 = 7$

Solve each equation using the Properties of Equality. Check your solutions.

3. $4a + 3 = 11$

4. $8 = 3r - 1$

5. $42 = -2d + 6$

6. $3x + 0.3 = 3.3$

7. $15y + 31 = 61$

8. $9 - c = -13$

9. $\frac{x}{6} + 4 = 15$

10. $\frac{1}{3}y + \frac{1}{4} = \frac{5}{12}$

11. $\frac{2}{7}m - \frac{1}{7} = \frac{3}{14}$

12. $15 = \frac{a}{3} - 2$

13. $4 - \frac{m}{2} = 10$

14. $\frac{x}{8} - \frac{1}{2} = 6$

Justify each step.

15. $2x - 5 = -20$

$2x = -15$ ____________________

$x = -\frac{15}{2}$ ____________________

16. $\frac{x}{3} - 7 = 11$

$\frac{x}{3} = 18$ ____________________

$x = 6$ ____________________

17. $\frac{9x}{4} = -9$

$9x = -36$ ____________________

$x = -4$ ____________________

18. In 2003, the population of Zimbabwe was about 12.6 million people, which is 1 million more than 4 times the population in 1950. Write and solve an equation to find the approximate population *p* of Zimbabwe in 1950.

19. Julio is paid 1.4 times his normal hourly rate for each hour he works over 30 hours in a week. Last week he worked 35 hours and earned $436.60. Write and solve an equation to find Julio's normal hourly rate, *r*. Explain how you know that your answer is reasonable.

20. The average weight of the top 5 fish caught at a fishing tournament was 12.3 pounds. Some of the weights of the fish are shown in the table.

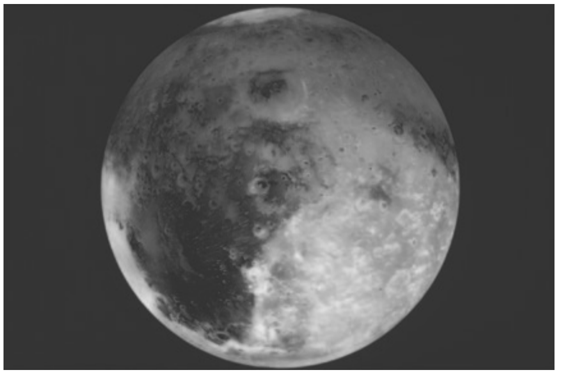
Top 5 Fish	
Caught by	**Weight (lb)**
Wayne S.	
Carla P.	12.8
Deb N.	12.6
Vincente R.	11.8
Armin G.	9.7

What was the weight of the heaviest fish?

21. Paul bought a student discount card for the bus. The card allows him to buy daily bus passes for \$1.50. After one month, Paul bought 15 passes and spent a total of \$29.50. How much did he spend on the student discount card?

22. Jennifer is saving money to buy a bike. The bike costs \$245. She has \$125 saved, and each week she adds \$15 to her savings. How long will it take her to save enough money to buy the bike?

23. **Astronomy** The radius of Earth is 6378.1 km, which is 2981.1 km greater than the radius of Mars. Find the radius of Mars.



24. Maggie's brother is 3 years younger than twice her age. The sum of their ages is 24. How old is Maggie?

H.O.T. Focus on Higher Order Thinking

25. Analyze Relationships One angle of a triangle measures 120°. The other two angles are congruent. Write and solve an equation to find the measure of the congruent angles.

26. Explain the Error Find the error in the solution, and then solve correctly.

$$9x + 18 + 3x = 1$$

$$9x + 18 = -2$$

$$9x = -20$$

$$x = -\frac{20}{9}$$

27. Check for Reasonableness Marietta was given a raise of \$0.75 per hour, which gave her a new wage of \$12.25 per hour. Write and solve an equation to determine Marietta's hourly wage before her raise. Show that your answer is reasonable.

Lesson Performance Task

The formula $p = 8n - 30$ gives the profit p when a number of items n are each sold at \$8 and expenses totaling \$30 are subtracted.

a. If the profit is \$170.00, how many items were bought?

b. If the same number of items were bought but the expenses changed to \$40, would the profit increase or decrease, and by how much? Explain.

Name ______________________ Class ______________ Date __________

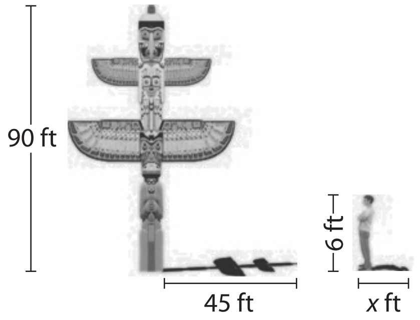
1.2 Modeling Quantities

Essential Question: How can you use rates, ratios, and proportions to solve real-world problems?

Resource Locker

Explore Using Ratios and Proportions to Solve Problems

Ratios and *proportions* are very useful when solving real-world problems. A **ratio** is a comparison of two numbers by division. An equation that states that two ratios are equal is called a **proportion**.



A totem pole that is 90 feet tall casts a shadow that is 45 feet long. At the same time, a 6-foot-tall man casts a shadow that is x feet long.

The man and the totem pole are both perpendicular to the ground, so they form right angles with the ground. The sun shines at the same angle on both, so similar triangles are formed.

(A) Write a ratio of the man's height to the totem pole's height. $\frac{\square}{\square}$

(B) Write a ratio of the man's shadow to the totem pole's shadow. $\frac{\square}{\square}$

(C) Write a proportion.

$$\frac{\text{man's height}}{\text{pole's height}} = \frac{\text{man's shadow}}{\text{pole's shadow}} \qquad \frac{\square}{\square} = \frac{\square}{\square}$$

(D) Solve the proportion by ______________ both sides by 45.

(E) Solve the proportion to find the length of the man's shadow in feet. $x = \square$

Reflect

1. **Discussion** What is another ratio that could be written for this problem? Use it to write and solve a different proportion to find the length of the man's shadow in feet.

____

____

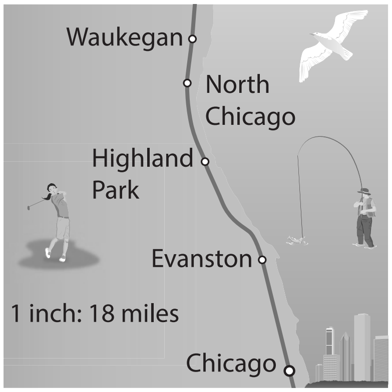
2. **Discussion** Explain why your new proportion and solution are valid.

____

____

Explain 1 Using Scale Drawings and Models to Solve Problems

A **scale** is the ratio of any length in a *scale drawing* or *scale model* to the corresponding actual length. A drawing that uses a scale to represent an object as smaller or larger than the original object is a **scale drawing**. A three-dimensional model that uses a scale to represent an object as smaller or larger than the actual object is called a **scale model**.



Example 1 **Use the map to answer the following questions.**

(A) The actual distance from Chicago to Evanston is 11.25 mi. What is the distance on the map?

Write the scale as a fraction. $\dfrac{\text{map}}{\text{actual}} \begin{matrix}\rightarrow \\ \rightarrow\end{matrix} \dfrac{1 \text{ in.}}{18 \text{ mi}}$

Let d be the distance on the map. $\dfrac{1}{18} = \dfrac{d}{11.25}$

Multiply both sides by 11.25. $\dfrac{11.25}{18} = d$

$0.625 = d$

The distance on the map is about 0.625 in.

(B) The actual distance between North Chicago and Waukegan is 4 mi. What is this distance on the map? Round to the nearest tenth.

Write the scale as a fraction. $\dfrac{\text{map}}{\text{actual}} \begin{matrix}\rightarrow \\ \rightarrow\end{matrix} \dfrac{\boxed{}}{\boxed{}}$

Let d be the distance on the map. $\dfrac{\boxed{}}{\boxed{}} = \dfrac{\boxed{}}{\boxed{}}$

Multiply both sides by 4. $\dfrac{\boxed{}}{\boxed{}} = d$

$\boxed{} \approx d$

The distance on the map is about ________ in.

Your Turn

3. A scale model of a human heart is 196 inches long. The scale is 32 to 1. How many inches long is the actual heart? Round your answer to the nearest whole number.

Explain 2 Using Dimensional Analysis to Convert Measurements

Dimensional analysis is a method of manipulating unit measures algebraically to determine the proper units for a quantity computed algebraically. The comparison of two quantities with different units is called a **rate**. The ratio of two equal quantities, each measured in different units, is called a **conversion factor**.

Example 2 **Use dimensional analysis to convert the measurements.**

(A) A large adult male human has about 12 pints of blood. Use dimensional analysis to convert this quantity to gallons.

Step 1 Convert pints to quarts.

Multiply by a conversion factor whose first quantity is quarts and whose second quantity is pints.

$$12 \text{ pt} \cdot \frac{1 \text{ qt}}{2 \text{ pt}} = 6 \text{ qt}$$

12 pints is 6 quarts.

Step 2 Convert quarts to gallons.

Multiply by a conversion factor whose first quantity is gallons and whose second quantity is quarts.

$$6 \text{ qt} \cdot \frac{1 \text{ gal}}{4 \text{ qt}} = \frac{6}{4} \text{ gal} = 1\frac{1}{2} \text{ gal}$$

A large adult male human has about $1\frac{1}{2}$ gallons of blood.

(B) The length of a building is 720 in. Use dimensional analysis to convert this quantity to yards.

Step 1 Convert inches to feet.

Multiply by a conversion factor whose first quantity is feet and whose second quantity is inches.

$$720 \text{ in.} \cdot \frac{\boxed{} \text{ ft}}{\boxed{} \text{ in.}} = \boxed{} \text{ ft}$$

720 inches is $\boxed{}$ feet.

Step 2 Convert feet to yards.

Multiply by a conversion factor whose first quantity is yards and whose second quantity is feet.

$$\boxed{} \text{ ft} \cdot \frac{\boxed{} \text{ yd}}{\boxed{} \text{ ft}} = \boxed{} \text{ yd}$$

$\boxed{}$ feet is $\boxed{}$ yards.

Therefore, 720 inches is ________ yards.

Your Turn

Use dimensional analysis to convert the measurements. Round answers to the nearest tenth.

4. 7500 seconds ≈ ________ hours

5. 3 feet ≈ ________ meters

6. 4 inches ≈ ________ yards

Explain 3 Using Dimensional Analysis to Convert and Compare Rates

Use dimensional analysis to determine which rate is greater.

Example 3 **During a cycling event for charity, Amanda traveled 105 kilometers in 4.2 hours and Brenda traveled at a rate of 0.2 mile per minute. Which girl traveled at a greater rate? Use 1 mi = 1.61 km.**

(A) Convert Amanda's rate to the same units as Brenda's rate.
Set up conversion factors so that both kilometers and hours cancel.

$$\frac{x \text{ miles}}{\text{minute}} \approx \frac{105 \text{ km}}{4.2 \text{ h}} \cdot \frac{1 \text{ mi}}{1.61 \text{ km}} \cdot \frac{1 \text{ h}}{60 \text{ min}}$$

$$\approx \frac{105 \text{ mi}}{4.2 \cdot 1.61 \cdot 60 \text{ min}}$$

$$\approx 0.2588 \text{ mi/min}$$

Amanda traveled approximately 0.26 mi/min.

Amanda traveled faster than Brenda.

(B) A box of books has a mass of 4.10 kilograms for every meter of its height. A box of magazines has a mass of 3 pounds for every foot of its height. Which box has a greater mass per unit of height? Use 1 lb = 0.45 kg and 1 m = 3.28 ft. Round your answer to the nearest tenth.

Convert the mass of the box of books to the same units as the mass of the box of magazines. Set up conversion factors so that both kilograms and pounds cancel.

$$\frac{x \text{ lb}}{\text{ft}} \approx \frac{4.10 \text{ kg}}{1 \text{ m}} \cdot \frac{\boxed{} \text{ lb}}{\boxed{} \text{ kg}} \cdot \frac{\boxed{} \text{ m}}{\boxed{} \text{ ft}} \approx \frac{\boxed{} \text{ lb}}{\boxed{} \text{ ft}} \approx \boxed{} \text{ lb/ft}$$

The box of ____________ has a greater mass per unit of height.

Reflect

7. Why is it important to convert rates to the same units before comparing them?

____

____

____

Your Turn

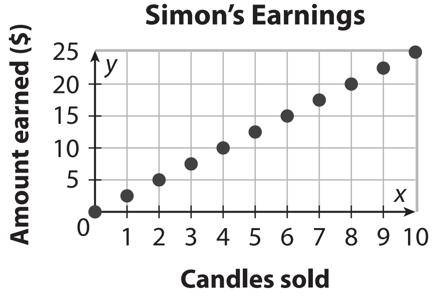
Use dimensional analysis to determine which rate is greater.

8. Alan's go-kart travels 1750 feet per minute, and Barry's go-kart travels 21 miles per hour. Whose go-kart travels faster? Round your answer to the nearest tenth.

Explain 4 Graphing a Proportional Relationship

To graph a proportional relationship, first find the unit rate, then create scales on the *x*- and *y*-axes and graph points.

Example 4 **Simon sold candles to raise money for the school dance. He raised a total of $25.00 for selling 10 candles. Find the unit rate (amount earned per candle). Then graph the relationship.**



(A) Find the unit rate. $\frac{\text{Amount earned}}{\text{Candles sold}}: \frac{25}{10} = \frac{x}{1}$

$$2.5 = x$$

The unit rate is $2.50 per candle.

Using this information, create scales on the *x*- and *y*-axes.

The *x*-axis will represent the candles sold, since this is the independent variable.

The *y*-axis will represent the amount earned, since this is the dependent variable.

The origin represents what happens when Simon sells 0 candles. The school gets $0.

Simon sold a total of 10 candles, so the *x*-axis will need to go from 0 to 10.

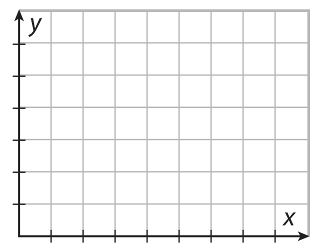
Since the school gets a total of $25 from Simon, the *y*-axis will need to go from 0 to 25.

Plot points on the graph to represent the amount of money the school earns for the different numbers of candles sold.

A local store sells 8 corn muffins for a total of $6.00. Find the unit rate. Then graph the points.

(B) Find the unit rate. $\frac{\text{Amount earned}}{\text{Muffins sold}}: \frac{\boxed{}}{\boxed{}} = \frac{x}{1}$

$$\boxed{} = x$$



The unit rate is ________ per muffin.

Using this information, create scales on the *x*- and *y*-axes.

The *x*-axis will represent the ____________________, since this is the independent variable.

The *y*-axis will represent the ____________________, since this is the dependent variable.

The origin in this graph represents what happens when ____________________.

The *x*-axis will need to go from ________ to ________.

The *y*-axis will need to go from ________ to ________.

Plot points on the graph to represent the earnings from the different numbers of muffins sold.

Reflect

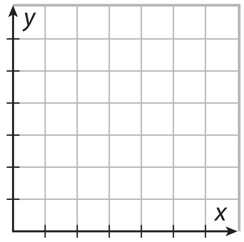
9. In Example 4A, Simon raised a total of $25.00 for selling 10 candles. If Simon raised $30.00 for selling 10 candles, would the unit rate be higher or lower? Explain.

____

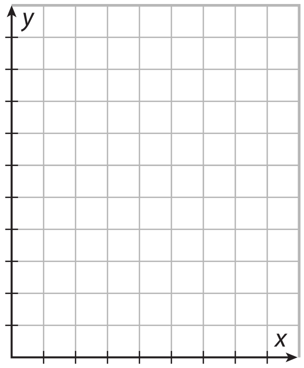
Your Turn

Find the unit rate, create scales on the x- and y-axes, and then graph the function.

10. Alex drove 135 miles in 3 hours at a constant speed.



11. Max wrote 10 pages of his lab report in 4 hours.



Elaborate

12. Give three examples of proportions. How do you know they are proportions? Then give three nonexamples of proportions. How do you know they are not proportions?

13. If a scale is represented by a ratio less than 1, what do we know about the actual object? If a scale is represented by a ratio greater than 1, what do we know about the actual object?

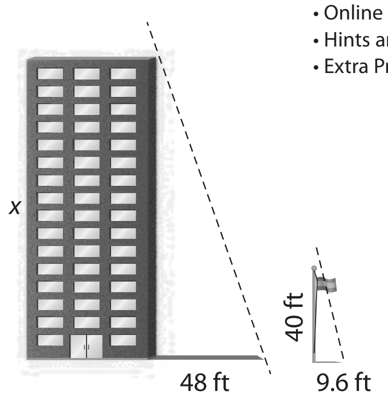
14. How is dimensional analysis useful in calculations that involve measurements?

15. **Essential Question Check In** How is finding the unit rate helpful before graphing a proportional relationship?

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. **Represent Real-World Problems** A building casts a shadow 48 feet long. At the same time, a 40-foot-tall flagpole casts a shadow 9.6 feet long. What is the height of the building?



Use the table to answer questions 2–4. Select the best answer. Assume the shadow lengths were measured at the same time of day.

2. The flagpole casts an 8-foot shadow, as shown in the table. At the same time, the oak tree casts a 12-foot shadow. How tall is the oak tree?

Object	Length of Shadow (ft)	Height (ft)
Flagpole	8	20
Oak tree	12	
Goal post	18	
Fence		6.5

3. How tall is the goal post?

4. What is the length of the fence's shadow?

5. **Decorating** A particular shade of paint is made by mixing 5 parts red paint with 7 parts blue paint. To make this shade, Shannon mixed 12 quarts of blue paint with 8 quarts of red paint. Did Shannon mix the correct shade? Explain.

6. **Geography** The scale on a map of Virginia shows that 1 inch represents 30 miles. The actual distance from Richmond, VA, to Washington, D.C., is 110 miles. On the map, how many inches are between the two cities?

7. Sam is building a model of an antique car. The scale of his model to the actual car is 1:10. His model is $18\frac{1}{2}$ inches long. How long is the actual car?

8. **Archaeology** Stonehenge II in Hunt, Texas, is a scale model of the ancient construction in Wiltshire, England. The scale of the model to the original is 3 to 5. The Altar Stone of the original construction is 4.9 meters tall. Write and solve a proportion to find the height of the model of the Altar Stone.

For 9–11, tell whether each scale *reduces*, *enlarges*, or *preserves* the size of an actual object.

9. 1 m to 25 cm

10. 8 in. to 1 ft

11. 12 in. to 1 ft

12. **Analyze Relationships** When a measurement in inches is converted to centimeters, will the number of centimeters be greater or less than the number of inches? Explain.

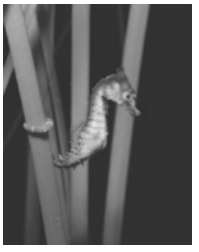
Use dimensional analysis to convert the measurements.

13. Convert 8 milliliters to fluid ounces. Use 1 mL $\approx$ 0.034 fl oz.

14. Convert 12 kilograms to pounds. Use 1 kg $\approx$ 2.2 lb.

15. Convert 950 US dollars to British pound sterling. Use 1 US dollar = 0.62 British pound sterling.

16. The dwarf sea horse *Hippocampus zosterae* swims at a rate of 52.68 feet per hour. Convert this speed to inches per minute.



Use dimensional analysis to determine which rate is greater.

17. Tortoise A walks 52.0 feet per hour and tortoise B walks 12 inches per minute. Which tortoise travels faster? Explain.

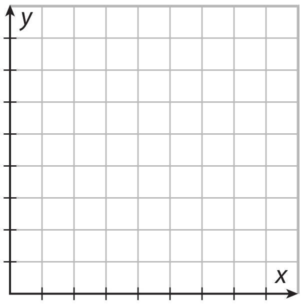
18. The pitcher for the Robins throws a baseball at 90.0 miles per hour. The pitcher on the Bluebirds throws a baseball 121 feet per second. Which pitcher throws a baseball faster? Explain.

19. For a science experiment Marcia dissolved 1.0 kilogram of salt in 3.0 liters of water. For a different experiment, Bobby dissolved 2.0 pounds of salt in 7.0 pints of water. Which person made a more concentrated salt solution? Explain. Use 1 L = 2.11 pints. Round your answer to the nearest hundredth.

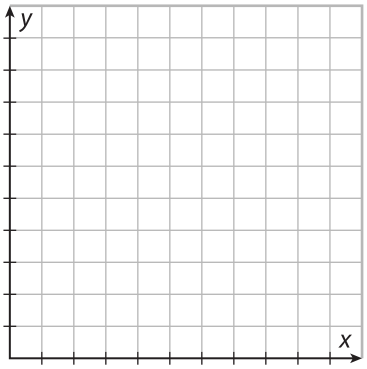
20. Will a stand that can hold up to 40 pounds support a 21-kilogram television? Explain. Use 2.2 lb = 1 kg.

Find the unit rate, create scales on the *x*- and *y*-axes, and then graph the function.

21. Brianna bought a total of 8 notebooks and got 16 free pens.

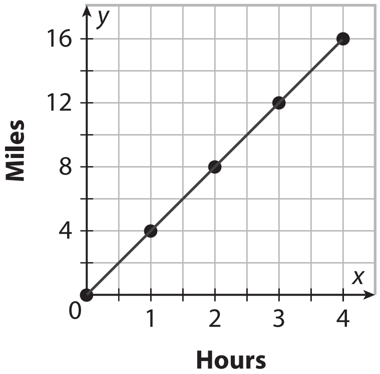


22. Mason sold 10 wristbands and made a total of 5 dollars.

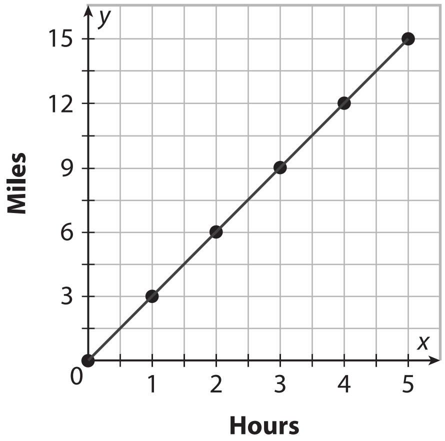


23. Match each graph to the data it goes with. Explain your reasoning.

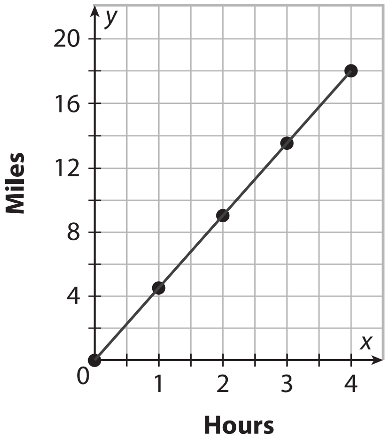
A.



B.



C.



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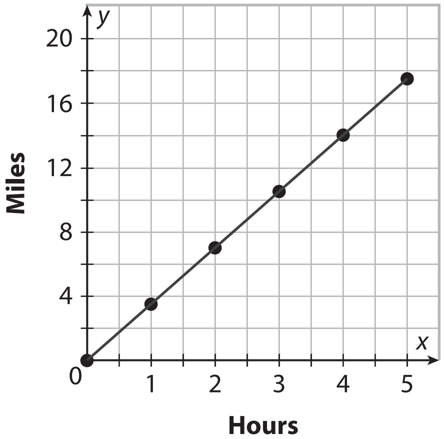


Mike walks 3 miles per hour for 5 hours.

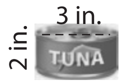
Brad walks 3.5 miles per hour for 5 hours.

Jesse walks 4 miles per hour for 4 hours.

Josh walks 4.5 miles per hour for 4 hours.

H.O.T. Focus on Higher Order Thinking

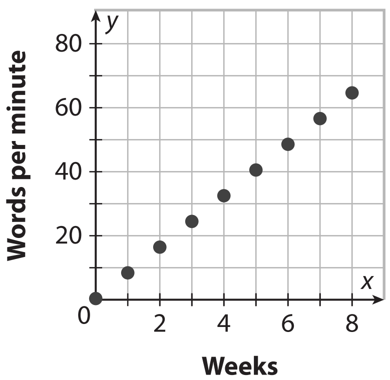
24. Multi-Step A can of tuna has a shape similar to the shape of a large water tank. The can of tuna has a diameter of 3 inches and a height of 2 inches. The water tank has a diameter of 6 yards. What is the height of the water tank in both inches and yards?

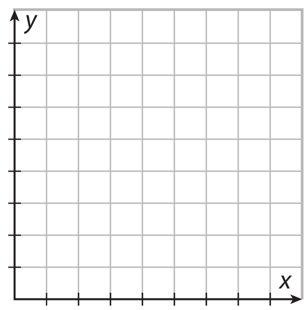


25. Represent Real-World Problems Write a real-world scenario in which 12 fluid ounces would need to be converted into liters. Then make the conversion. Use 1 fl oz = 0.0296 L. Round your answer to the nearest tenth.

26. Find the Error The graph shown was given to represent this problem. Find the error(s) in the graph and then create a correct graph to represent the problem.

Jamie took an 8-week keyboarding class. At the end of each week, she took a test to find the number of words she could type per minute and found out she improved the same amount each week. Before Jamie started the class, she could type 25 words per minute, and by the end of week 8, she could type 65 words per minute.





Lesson Performance Task

The Wright Flyer was the first successful powered aircraft. A model was made to display in a museum with the length of 35 cm and a wingspan of about 66.9 cm. The length of the actual plane was 21 ft 1 in., and the height was 2.74 m. Compare the length, height, and wingspan of the model to the actual plane and explain why any errors may occur. (Round any calculations to the nearest whole number.)

Name ____________________ Class ____________ Date ________

1.3 Reporting with Precision and Accuracy

Resource Locker

Essential Question: How do you use significant digits when reporting the results of calculations involving measurement?

Explore Comparing Precision of Measurements

Numbers are values without units. They can be used to compute or to describe measurements. Quantities are real-word values that represent specific amounts. For instance, 15 is a number, but 15 grams is a quantity.

Precision is the level of detail of a measurement, determined by the smallest unit or fraction of a unit that can be reasonably measured.

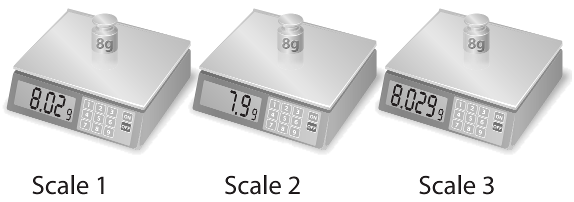
Accuracy is the closeness of a given measurement or value to the actual measurement or value. Suppose you know the actual measure of a quantity, and someone else measures it. You can find the accuracy of the measurement by finding the absolute value of the difference of the two.

(A) Complete the table to choose the more precise measurement.

Measurement 1	Measurement 2	Smaller Unit	More Precise Measurement
4 g	4.3 g		
5.71 oz	5.7 oz		
4.2 m	422 cm		
7 ft 2 in.	7.2 in.		

(B) Eric is a lab technician. Every week, he needs to test the scales in the lab to make sure that they are accurate. He uses a standard mass that is exactly 8.000 grams and gets the following results.

Scale	Mass
Scale 1	8.02 g
Scale 2	7.9 g
Scale 3	8.029 g



Complete each statement:

The measurement for Scale __________ is the most precise

because it measures to the nearest __________, which is smaller than the smallest unit measured on the other two scales.

(C) Find the accuracy of each of the measurements in Step B.

Scale 1: Accuracy $= |8.000 - \underline{\hspace{3em}}| = \underline{\hspace{3em}}$

Scale 2: Accuracy $= |8.000 - \underline{\hspace{3em}}| = \underline{\hspace{3em}}$

Scale 3: Accuracy $= |8.000 - \underline{\hspace{3em}}| = \underline{\hspace{3em}}$

Complete each statement: the measurement for Scale ________, which is ________ grams,

is the most accurate because ____________________.

Reflect

1. **Discussion** Given two measurements of the same quantity, is it possible that the more precise measurement is not the more accurate? Why do you think that is so?

____

____

____

Explain 1 Determining Precision of Calculated Measurements

As you have seen, measurements are reported to a certain precision. The reported value does not necessarily represent the actual value of the measurement. When you measure to the nearest unit, the actual length can be 0.5 unit less than the measured length or less than 0.5 unit greater than the measured length. So, a length reported as 4.5 centimeters could actually be anywhere between 4.45 centimeters and 4.55 centimeters, but not including 4.55 centimeters. It cannot include 4.55 centimeters because 4.55 centimeters reported to the nearest tenth would round *up* to 4.6 centimeters.

Example 1 **Calculate the minimum and maximum possible areas. Round your answers to the nearest square centimeter.**

(A) The length and width of a book cover are 28.3 centimeters and 21 centimeters, respectively.

Find the range of values for the actual length and width of the book cover.

Minimum length $= (28.3 - 0.05)$ cm and maximum length $= (28.3 + 0.05)$ cm, so $28.25 \text{ cm} \leq \text{length} < 28.35 \text{ cm}$.

Minimum width $= (21 - 0.5)$ cm and maximum width $= (21 + 0.5)$ cm, so $20.5 \text{ cm} \leq \text{width} < 21.5 \text{ cm}$.

Find the minimum and maximum areas.

Minimum area = minimum length · minimum width

$= 28.25 \text{ cm} \cdot 20.5 \text{ cm} \approx 579 \text{ cm}^2$

Maximum area = maximum length · maximum width

$= 28.35 \text{ cm} \cdot 21.5 \text{ cm} \approx 610 \text{ cm}^2$

So $579 \text{ cm}^2 \leq \text{area} < 610 \text{ cm}^2$.

(B) The length and width of a rectangle are 15.5 centimeters and 10 centimeters, respectively.

Find the range of values for the actual length and width of the rectangle.

Minimum length $= (15.5 - \_\_\_\_\_\_\_\_)$ cm and maximum length $= (15.5 + \_\_\_\_\_\_\_\_)$ cm,

so $\_\_\_\_\_\_\_\_ \leq$ length $< \_\_\_\_\_\_\_\_$.

Minimum width $= (10 - \_\_\_\_\_\_\_\_)$ cm and maximum width $= (10 + \_\_\_\_\_\_\_\_)$ cm,

so $\_\_\_\_\_\_\_\_ \leq$ width $< \_\_\_\_\_\_\_\_$.

Find the minimum and maximum areas.

Minimum area = minimum length · minimum width

$= \_\_\_\_\_\_\_\_$ cm $\cdot$ $\_\_\_\_\_\_\_\_$ cm $\approx$ $\_\_\_\_\_\_\_\_$ cm^2

Maximum area = maximum length · maximum width

$= \_\_\_\_\_\_\_\_$ cm $\cdot$ $\_\_\_\_\_\_\_\_$ cm $\approx$ $\_\_\_\_\_\_\_\_$ cm^2

So $\_\_\_\_\_\_\_\_$ $cm^2 \leq$ area $< \_\_\_\_\_\_\_\_$ cm^2.

Reflect

2. How do the ranges of the lengths and widths of the books compare to the range of the areas? What does that mean in terms of the uncertainty of the dimensions?

Your Turn

Calculate the minimum and maximum possible areas. Round your answers to the nearest whole square unit.

3. Sara wants to paint a wall. The length and width of the wall are 2 meters and 1.4 meters, respectively.

4. A rectangular garden plot measures 15 feet by 22.7 feet.

Explain 2 Identifying Significant Digits

Significant digits are the digits in measurements that carry meaning about the precision of the measurement.

Identifying Significant Digits	
Rule	**Examples**
All nonzero digits are significant.	55.98 has 4 significant digits. 115 has 3 significant digits.
Zeros between two other significant digits are significant.	102 has 3 significant digits. 0.4000008 has 7 significant digits.
Zeros at the end of a number to the right of a decimal point are significant.	3.900 has 4 significant digits. 0.1230 has 4 significant digits.
Zeros to the left of the first nonzero digit in a decimal are *not* significant.	0.00035 has 2 significant digits. 0.0806 has 3 significant digits.
Zeros at the end of a number without a decimal point are assumed to be *not* significant.	60,600 has 3 significant digits. 77,000,000 has 2 significant digits.

Example 2 **Determine the number of significant digits in a given measurement.**

(A) 6040.0050 m

Significant Digits Rule	Digits	Count
Nonzero digits:	6 0 4 0 . 0 0 5 0	3
Zeros between two significant digits:	6 0 4 0 . 0 0 5 0	4
End zeros to the right of a decimal:	6 0 4 0 . 0 0 5 0	1
	Total	8

So, 6040.0050 m has 8 significant digits.

(B) 710.080 cm

Significant Digits Rule	Digits	Count
Nonzero digits:	7 1 0 . 0 8 0	
Zeros between two significant digits:	7 1 0 . 0 8 0	
End zeros to the right of a decimal:	7 1 0 . 0 8 0	
	Total	

710.080 cm has ______ significant digit(s).

Reflect

5. **Critique Reasoning** A student claimed that 0.045 and 0.0045 m have the same number of significant digits. Do you agree or disagree?

Your Turn

Determine the number of significant digits in each measurement.

6. 0.052 kg

7. 10,000 ft

8. 10.000 ft

Explain 3 Using Significant Digits in Calculated Measurements

When performing calculations with measurements of different precision, the number of significant digits in the solution may differ from the number of significant digits in the original measurements. Use the rules from the following table to determine how many significant digits to include in the result of a calculation.

Rules for Significant Digits in Calculated Measurements	
Operation	**Rule**
Addition or Subtraction	The sum or difference must be rounded to the same place value as last significant digit of the least precise measurement.
Multiplication or Division	The product or quotient must have no more significant digits than the least precise measurement.

Example 3 **Find the perimeter and area of the given object. Make sure your answers have the correct number of significant digits.**

(A) A rectangular swimming pool measures 22.3 feet by 75 feet.

Find the perimeter of the swimming pool using the correct number of significant digits.

$$\begin{aligned} \text{Perimeter} &= \text{sum of side lengths} \\ &= 22.3 \text{ ft} + 75 \text{ ft} + 22.3 \text{ ft} + 75 \text{ ft} \\ &= 194.6 \text{ ft} \end{aligned}$$

The least precise measurement is 75 feet. Its last significant digit is in the ones place. So round the sum to the ones place. The perimeter is 195 ft.

Find the area of the swimming pool using the correct number of significant digits.

$$\begin{aligned} \text{Area} &= \text{length} \cdot \text{width} \\ &= 22.3 \text{ ft} \cdot 75 \text{ ft} = 1672.5 \text{ ft}^2 \end{aligned}$$

The least precise measurement, 75 feet, has two significant digits, so round the product to a number with two significant digits. The area is 1700 ft^2.

(B) A rectangular garden plot measures 21 feet by 25.2 feet.

Find the perimeter of the garden using the correct number of significant digits.

$$\text{Perimeter} = \text{sum of side lengths}$$
$$= \boxed{} + \boxed{} + \boxed{} + \boxed{} = \boxed{}$$

The least precise measurement is ________. Its last significant digit is in the ones place. So round the sum to the ________ place. The perimeter is ________.

Find the area of the garden using the correct number of significant digits.

$$\text{Area} = \text{length} \cdot \text{width}$$
$$= \boxed{} \cdot \boxed{} = \boxed{}$$

The least precise measurement, ________ has ________ significant digit(s), so round to a number with ________ significant digit(s). The area is ________.

Reflect

9. In the example, why did the area of the garden and the swimming pool each have two significant digits?

10. Is it possible for the perimeter of a rectangular garden to have more significant digits than its length or width does?

Your Turn

Find the perimeter and area of the given object. Make sure your answers have the correct number of significant digits.

11. A children's sandbox measures 7.6 feet by 8.25 feet.

12. A rectangular door measures 91 centimeters by 203.2 centimeters.

Explain 4 Using Significant Digits in Estimation

Real-world situations often involve estimation. Significant digits play an important role in making reasonable estimates.

A city is planning a classic car show. A section of road 820 feet long will be closed to provide a space to display the cars in a row. In past shows, the longest car was 18.36 feet long and the shortest car was 15.1 feet long. Based on that information, about how many cars can be displayed in this year's show?

Analyze Information

- Available space: ______
- Length of the longest car: ______
- Length of the shortest car: ______

Formulate a Plan

The word *about* indicates that your answer will be a(n) ______.

Available Space = Number of Cars · ______

Find the number of longest cars and the number of shortest cars, and then use the average.

Solve

Longest:

$820 \approx L \cdot \boxed{}$

$L \approx \dfrac{820}{\boxed{}} = \boxed{}$

So, at least about ______ cars can be displayed.

Shortest:

$820 \approx S \cdot \boxed{}$

$S \approx \dfrac{820}{\boxed{}} = \boxed{}$

So, at most about ______ cars can be displayed.

$$\text{Estimated number of cars} \approx \frac{L + S}{2} = \frac{\boxed{} + \boxed{}}{2} = \boxed{}$$

The number of cars must be rounded to ______ significant digits.

So, the club can estimate that a minimum of ______ cars and a maximum of ______ cars can be displayed, and on average, ______ cars can be displayed.

Justify and Evaluate

Because the cars will probably have many different lengths, a reasonable estimate is a value between ______.

Reflect

13. In the example, why wouldn't it be wise to use the length of a shorter car?

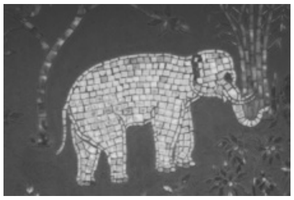
14. **Critical Thinking** How else might the number of cars be estimated? Would you expect the estimate to be the same? Explain.

Your Turn

Estimate the quantity needed in the following situations. Use the correct number of significant digits.

15. Claire and Juan are decorating a rectangular wall of 433 square feet with two types of rectangular pieces of fabric. One type has an area of 9.4 square feet and the other has an area of 17.2 square feet. About how many decorative pieces can Claire and Juan fit in the given area?

16. An artist is making a mosaic and has pieces of smooth glass ranging in area from 0.25 square inch to 3.75 square inches. Suppose the mosaic is 34.1 inches wide and 50.0 inches long. About how many pieces of glass will the artist need?



Elaborate

17. Given two measurements, is it possible that the more accurate measurement is not the more precise? Justify your answer.

18. What is the relationship between the range of possible error in the measurements used in a calculation and the range of possible error in the calculated measurement?

19. **Essential Question Check-In** How do you use significant digits to determine how to report a sum or product of two measurements?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. Choose the more precise measurement from the pair 54.1 cm and 54.16 cm. Justify your answer.

Choose the more precise measurement in each pair.

2. 1 ft; 12 in.   **3.** 5 kg; 5212 g   **4.** 7 m; 7.7 m   **5.** 123 cm; 1291 mm

6. True or False? A scale that measures the mass of an object in grams to two decimal places is more precise than a scale that measures the mass of an object in milligrams to two decimal places. Justify your answer.

7. Every week, a technician in a lab needs to test the scales in the lab to make sure that they are accurate. She uses a standard mass that is exactly 4 g and gets the following results.

Scale 1   Scale 2   Scale 3

a. Which scale gives the most precise measurement?

b. Which scale gives the most accurate measurement?

8. A manufacturing company uses three measuring tools to measure lengths. The tools are tested using a standard unit exactly 7 cm long. The results are as follows.

Measuring Tool	Length
Tool 1	7.033 cm
Tool 2	6.91 cm
Tool 3	7.1 cm

a. Which tool gives the most precise measurement?

b. Which tool gives the most accurate measurement?

Given the following measurements, calculate the minimum and maximum possible areas of each object. Round your answer to the nearest whole square unit.

9. The length and width of a book cover are 22.2 centimeters and 12 centimeters, respectively.

10. The length and width of a rectangle are 19.5 centimeters and 14 centimeters, respectively.

11. Chris is painting a wall with a length of 3 meters and a width of 1.6 meters.

12. A rectangular garden measures 15 feet by 24.1 feet.

Show the steps to determine the number of significant digits in the measurement.

13. 123.040 m

14. 0.00609 cm

Determine the number of significant digits in each measurement.

15. 0.0070 ft

16. 3333.33 g

17. 20,300.011 lb

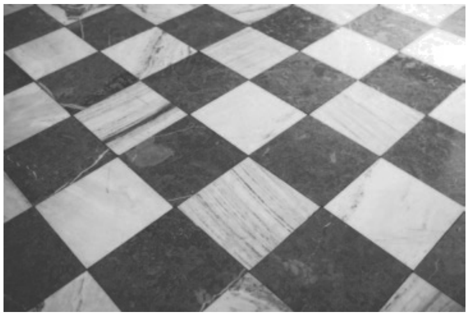
Find the perimeter and area of each garden. Report your answers with the correct number of significant digits.

18. A rectangular garden plot measures 13 feet by 26.6 feet.

19. A rectangular garden plot measures 24 feet by 25.3 feet.

20. Samantha is putting a layer of topsoil on a garden plot. She measures the plot and finds that the dimensions of the plot are 5 meters by 21 meters. Samantha has a bag of topsoil that covers an area of 106 square meters. Should she buy another bag of topsoil to ensure that she can cover her entire plot? Explain.

21. Tom wants to tile the floor in his kitchen, which has an area of 320 square feet. In the store, the smallest tile he likes has an area of 1.1 square feet and the largest tile he likes has an area of 1.815 square feet. About how many tiles can be fitted in the given area?



H.O.T. Focus on Higher Order Thinking

22. Communicate Mathematical Ideas Consider the calculation 5.6 mi $\div$ 9s $=$ 0.62222 mi/s. Why is it important to use significant digits to round the answer?

23. Find the Error A student found that the dimensions of a rectangle were 1.20 centimeters and 1.40 centimeters. He was asked to report the area using the correct number of significant digits. He reported the area as 1.7 cm^2. Explain the error the student made.

24. Make a Conjecture Given two values with the same number of decimal places and significant digits, is it possible for an operation performed with the two values to have a different number of decimal places or significant digits than the original values?

Lesson Performance Task

The sun is an excellent source of electrical energy. A field of solar panels yields 16.22 Watts per square feet. Determine the amount of electricity produced by a field of solar panels that is 305 feet by 620 feet.