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### 8.1 Two-Way Frequency Tables

Essential Question: How can categorical data for two categories be summarized?


## Explore Categorical Data and Frequencies

Data that can be expressed with numerical measurements are quantitative data. In this lesson you will examine qualitative data, or categorical data, which cannot be expressed using numbers. Data describing animal type, model of car, or favorite song are examples of categorical data.
(A) Circle the categorical data variable. Justify your choice.
temperature weight height color
(B) Identify whether the given data is categorical or quantitative.
large, medium, small $\qquad$
$120 \mathrm{ft}^{2}, 130 \mathrm{ft}^{2}, 140 \mathrm{ft}^{2}$ $\qquad$
(C) A frequency table shows how often each item occurs in a set of categorical data. Use the categorical data listed on the left to complete the frequency table.

| Ways Students Get to School |
| :--- |
| bus car walk car car car bus <br> walk walk walk bus bus car <br> bus bus walk bus car bus car |


| Way | Frequency |
| :---: | :---: |
| bus | 8 |
| car | $\square$ |
|  | $\square$ |

## Reflect

1. How did you determine the numbers for each category in the frequency column?
$\qquad$
$\qquad$
2. What must be true about the sum of the frequencies in a frequency table?
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$\qquad$

## Explain 1 Constructing Two-Way Frequency Tables

If a data set has two categorical variables, you can list the frequencies of the paired values in a two-way frequency table.

## Example 1 Complete the two-way frequency table.

(A) A high school's administration asked 100 randomly selected students in the 9th and 10th grades about what fruit they like best. Complete the table.

|  | Preferred Fruit |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Grade | Apple | Orange | Banana | Total |
| 9th | 19 | 12 | 23 |  |
| 10th | 22 | 9 | 15 |  |
| Total |  |  |  |  |

Row totals:
9th: $19+12+23=54$
10th: $22+9+15=46$

Column totals:
Apple: $19+22=41$
Orange: $12+9=21 \quad$ Sum of column totals: $41+21+38=100$
Grand total:
Sum of row totals: $54+46=100$

Banana: $23+15=38 \quad$ Both sums should equal the grand total.

|  | Preferred Fruit |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Grade | Apple | Orange | Banana | Total |
| 9th | 19 | 12 | 23 | 54 |
| 10th | 22 | 9 | 15 | 46 |
| Total | 41 | 21 | 38 | 100 |

(B) Jenna asked some randomly selected students whether they preferred dogs, cats, or other pets. She also recorded the gender of each student. The results are shown in the two-way frequency table below. Each entry is the frequency of students who prefer a certain pet and are a certain gender. For instance, 8 girls prefer dogs as pets. Complete the table.

|  | Preferred Pet |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gender | Dog | Cat | Other | Total |
| Girl | 8 | 7 | 1 | $\square$ |
| Boy | 10 | 5 | 9 | $\square$ |
| Total | $\square$ | $\square$ | $\square$ | $\square$ |

Row totals:
Girl: $8+7+1=\square$
Boy: $10+5+9=$ $\square$

Column totals:
Dog: $8+10=$
Cat: $7+5=$
Other: $1+9=$

Grand total:
Sum of row totals: $16+\square=\square$
Sum of column totals: $18+$ Both sums should equal the grand total.

## Reflect

3. Look at the totals for each row. Was Jenna's survey evenly distributed among boys and girls? Explain.
4. Look at the totals for each column. Which pet is preferred by the most students? Justify your answer.

## Your Turn

Complete the two-way frequency table.
5. Antonio surveyed 60 of his classmates about their participation in school activities and whether they have a part-time job. The results are shown in the two-way frequency table below. Complete the table.

| Activities |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Job | Clubs Only | Sports Only | Both | Neither | Total |  |
| Yes | 12 | 13 | 16 | 4 |  |  |
| No | 3 | 5 | 5 | 2 |  |  |
| Total |  |  |  |  |  |  |

6. Jen surveyed 100 students about whether they like baseball or basketball. Complete the table.

|  | Like Basketball |  |  |
| :---: | :---: | :---: | :---: |
| Like Baseball | Yes | No | Total |
| Yes | 61 | 13 |  |
| No | 16 | 10 |  |
| Total |  |  |  |

## Explain 2 Reading Two-Way Frequency Tables

You can extract information about paired categorical variables by reading a two-way frequency table.

Example 2 Read and complete the two-way frequency table.
(A) Suppose you are given the circled information in the table and instructed to complete the table.

|  | Eat Cereal for Breakfast |  |  |
| :---: | :---: | :---: | :---: |
| Gender | Yes | No | Total |
| Girl | 42 | 12 | 54 |
| Boy | 36 | 10 | 46 |
| Total | 78 | 22 | 100 |

Find the total number of boys by subtracting: $100-54=46$
Find the number of boys who do eat cereal by subtracting: $46-10=36$
Add to find the total number of students who eat cereal and the total number of students who do not eat cereal.
(B) One hundred students were surveyed about which beverage they chose at lunch. Some of the results are shown in the two-way frequency table below. Complete the table.

|  | Lunch Beverage |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gender | Juice | Milk | Water | Total |
| Girl | 10 | $\square$ | 17 | $\square$ |
| Boy | 15 | 24 | 21 | 60 |
| Total | $\square$ | $\square$ | $\square$ | $\square$ |

Find the total number of girls by subtracting: $100-60=$ $\square$
So, the total number of girls is $\square$. The number of girls who do not choose milk is $\square+\square=\square$. Find the number of girls who chose milk by subtracting: $\square-\square=\square$

## Reflect

7. Which lunch beverage is the least preferred? How do you know?

## Your Turn

## Read and complete the two-way frequency table.

8. 100 students were asked what fruit they chose at lunch. The two-way frequency table shows some of the results of the survey. Complete the table.

|  | Lunch Fruit |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gender | Apple | Pear | Banana | Total |
| Girl |  | 17 | 11 | 49 |
| Boy |  | 10 | 16 |  |
| Total |  |  |  |  |

9. 200 high school teachers were asked whether they prefer to use the chalkboard or projector in class. The two-way frequency table shows some of the results of the survey. Complete the table.

|  | Preferred Teaching Aid |  |  |
| :---: | :---: | :---: | :---: |
| Gender | Chalkboard | Projector | Total |
| Female |  | 56 | 99 |
| Male | 44 |  |  |
| Total | 87 | 113 | 200 |

## Elaborate

10. You are making a two-way frequency table of 5 fruit preferences among a survey sample of girls and boys. What are the dimensions of the table you would make? How many entries would you need to fill the table with frequencies and totals?
11. A 3 categories-by- 3 categories two-way frequency table has a row with 2 numbers, and no row or column totals. Can you fill the row?
$\qquad$
$\qquad$
$\qquad$
12. Essential Question Check-In How can you summarize categorical data for 2 categories?

## Evaluate: Homework and Practice

1. Identify whether the given data is categorical or quantitative. gold medal, silver medal, bronze medal $\qquad$

- Online Homework
- Hints and Help
- Extra Practice
$100 \mathrm{~m}, 200 \mathrm{~m}, 400 \mathrm{~m}$

2. A theater company asked its members to bring in canned food for a food drive.

Use the categorical data to complete the frequency table.

Cans Donated to Food Drive
peas corn peas soup corn corn soup soup corn peas peas corn soup peas corn peas corn peas corn soup corn peas soup corn corn

| Cans | Frequency |
| :---: | :---: |
| soup | $\square$ |
| peas | $\square$ |
|  | $\square$ |

## Complete the two-way frequency table.

3. James surveyed some of his classmates about what vegetable they like best.

Complete the table.

|  | Preferred Vegetable |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Grade | Carrots | Green Beans | Celery | Total |
| 9th | 30 | 15 | 24 |  |
| 10th | 32 | 9 | 20 |  |
| Total |  |  |  |  |

4. A high school's extracurricular committee surveyed a randomly selected group of students about whether they like tennis and soccer. Complete the table.

|  | Like Tennis |  |  |
| :---: | :---: | :---: | :---: |
| Like Soccer | Yes | No | Total |
| Yes | 37 | 20 |  |
| No | 16 | 15 |  |
| Total |  |  |  |

5. After a school field trip, Ben surveyed some students about which animals they liked from the zoo. Complete the table.

|  | Preferred Animal at a Zoo |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Grade | Lion | Zebra | Monkey | Total |
| 11th | 9 | 15 | 14 |  |
| 12th | 4 | 17 | 15 |  |
| Total |  |  |  |  |

6. Jill asked some randomly selected students whether they preferred blue, green, or other colors. She also recorded the gender of each student. The results are shown in the two-way frequency table below. Complete the table.

|  | Preferred Color |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gender | Green | Blue | Other | Total |
| Girl | 15 | 3 | 10 |  |
| Boy | 3 | 16 | 6 |  |
| Total |  |  |  |  |

7. Kevin surveyed some students about whether they preferred soccer, baseball, or another sport. He also recorded their gender. Complete the table.

|  | Preferred Sport |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Gender | Soccer | Baseball | Other | Total |  |
| Girl | 33 | 7 | 10 |  |  |
| Boy | 15 | 27 | 7 |  |  |
| Total |  |  |  |  |  |

8. A school surveyed a group of students about whether they like backgammon and chess. They will use this data to determine whether there is enough interest for the school to compete in these games. Complete the table.

|  | Like Backgammon |  |  |
| :---: | :---: | :---: | :---: |
| Like Chess | Yes | No | Total |
| Yes | 10 | 61 |  |
| No | 5 | 3 |  |
| Total |  |  |  |

9. Hugo surveyed some 9th and 10th graders in regard to whether they preferred math, English, or another subject. The results of the survey are in the following table. Complete the table.

|  | Preferred Subject |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Grade | Math | English | Other | Total |
| 9th | 40 | 35 | 20 |  |
| 10th | 41 | 32 | 17 |  |
| Total |  |  |  |  |

10. Luis surveyed some middle school and high school students about the type of music they prefer. Complete the table.

|  | Preferred Music |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| School Level | Country | Pop | Other | Total |
| Middle School | 18 | 13 | 23 |  |
| High School | 7 | 32 | 15 |  |
| Total |  |  |  |  |


11. Natalie surveyed some teenagers and adults on whether they prefer standard cars, vans, or convertibles. Her results are in the following table. Complete the table.

| Preferred Car Type |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Age | Standard | Van | Convertible | Total |  |
| Adults | 10 | 25 | 9 |  |  |
| Teenagers | 11 | 7 | 24 |  |  |
| Total |  |  |  |  |  |

12. Eli surveyed some teenagers and adults on whether they prefer apples, oranges, or bananas. His results are in the following table. Complete the table.

|  | Preferred Fruit |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Age | Apple | Orange | Banana | Total |  |
| Adults | 22 | 12 | 10 |  |  |
| Teenagers | 24 | 9 | 9 |  |  |
| Total |  |  |  |  |  |

200 students were asked to name their favorite science class. The results are shown in the two-way frequency table. Use the table for the following questions.

|  | Favorite Science Class |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gender | Biology | Chemistry | Physics | Total |
| Girl | 42 | 39 | 23 | 104 |
| Boy |  | 45 | 32 |  |
| Total |  |  |  |  |

13. How many boys were surveyed? Explain how you found your answer.
14. Complete the table. How many more girls than boys chose biology as their favorite science class? Explain how you found your answer.

The results of a survey of $\mathbf{1 5 0}$ students about whether they own an electronic tablet or a laptop are shown in the two-way frequency table.

|  | Device |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Gender | Electronic tablet | Laptop | Both | Neither | Total |  |
| Girl | 15 | 54 |  | 9 | 88 |  |
| Boy |  | 35 | 8 | 5 |  |  |
| Total |  |  |  |  |  |  |

15. Complete the table. Do the surveyed students own more laptops or more electronic tablets?
16. Which group had more people answer the survey, boys or students who own an electronic tablet only? Explain.
17. The table shows the results of a survey about students' preferred frozen yogurt flavor. Complete the table, and state the flavors that students preferred the most and the least.

|  | Preferred Flavor |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gender | Vanilla | Mint | Strawberry | Total |
| Girl |  | 15 | 18 | 45 |
| Boy | 17 | 25 |  |  |
| Total |  |  |  | 100 |

18. Teresa surveyed 100 students about whether they like pop music or country music. Out of the 100 students surveyed, 42 like only pop, 34 like only country, 15 like both pop and country, and 9 do not like either pop or country. Complete the two-way frequency table.

| Like Pop |  |  |  |
| :---: | :---: | :---: | :---: |
| Like Country | Yes | No | Total |
| Yes |  |  |  |
| No |  |  |  |
| Total |  |  |  |

19. Forty students in a class at an international high school were surveyed about which non-English language they can speak. Complete the table.

|  | Foreign Language |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gender | Chinese | Spanish | French | Total |
| Girl | 7 | 8 |  |  |
| Boy |  | 6 | 7 | 18 |
| Total |  |  |  |  |

Luis surveyed 100 students about whether they like soccer. The number of girls and the number of boys completing the survey are equal.
20. Complete the table.

|  | Likes Soccer |  |  |
| :---: | :---: | :---: | :---: |
| Gender | Yes | No | Total |
| Girl |  | 20 |  |
| Boy |  | 35 |  |
| Total |  |  | 100 |

21. Twice as many girls like soccer as the number that like tennis. The same number of students like soccer as like tennis. Construct a table containing the tennis data.
22. A group of 200 high school students were asked about their use of email and text messages. The results are shown in the two-way frequency table. Complete the table.

|  | Text Messages |  |  |
| :---: | :---: | :---: | :---: |
| Email | Yes | No | Total |
| Yes | 72 |  | 90 |
| No |  | 45 |  |
| Total |  |  |  |

23. Circle the letter of each data set that is categorical. Select all that apply.
A. $75^{\circ}, 79^{\circ}, 77^{\circ}, 85^{\circ}$
B. apples, oranges, pears
C. male, female
D. blue, green, red
E. 2 feet, 5 feet, 12 feet
F. classical music, country music
G. 1 centimeter, 3 centimeters, 9 centimeters
24. Explain the Error Find the mistake in completing the two-way frequency table for a survey involving 50 students. Then complete the table correctly.

|  | Favorite Foreign Language Class |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gender | Russian | German | Italian | Total |
| Girl | 8 | 8 | 8 | 24 |
| Boy | 42 | 9 | 7 | 58 |
| Total | 50 |  |  |  |

Correct table:

|  | Favorite Foreign Language Class |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gender | Russian | German | Italian | Total |
| Girl | 8 | 8 | 8 | 24 |
| Boy |  | 9 | 7 |  |
| Total |  |  |  |  |

## H.O.T. Focus on Higher Order Thinking

25. Justify Reasoning Charles surveyed 100 boys about their favorite color. Of the 100 boys surveyed, 44 preferred blue, 25 preferred green, and 31 preferred red.
a. Explain why it is not possible to make a two-way frequency table from the given data.
b. Suppose Charles also surveyed some girls. Of the girls surveyed, 30 preferred blue and 43 preferred green. Can Charles make a two-way frequency table now? Can he complete it?
26. Persevere in Problem Solving Shown are two different tables about a survey involving students. Each survey had a few questions about musical preferences. All students answered all questions. Complete the tables. What type of music do the students prefer?

| Likes Classical Music |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gender | Yes | No | Total |  |
| Girl | 21 |  |  |  |
| Boy |  | 22 |  |  |
| Total |  |  | 100 |  |


|  | Likes Blues Music |  |  |
| :---: | :---: | :---: | :---: |
| Gender | Yes | No | Total |
| Girl |  | 15 | 49 |
| Boy |  | 15 |  |
| Total |  |  |  |

## Lesson Performance Task

Two hundred students were asked about their favorite sport. Of the 200 students surveyed, 98 were female. Some of the results are shown in the following two-way frequency table.

|  | Favorite Sport |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Gender | Football | Baseball | Basketball | Soccer | Total |
| Female |  |  | 36 | 12 |  |
| Male | 38 | 19 |  |  |  |
| Total | 64 |  |  | 36 |  |

a. Complete the table.
b. Which sport is the most popular among the students? Which is the least popular? Explain.
c. Which sport is most popular among the females? Which sport is most popular among the males? Explain.
$\qquad$
$\qquad$

### 8.2 Relative Frequency

Essential Question: How can you recognize possible associations and trends between two categories of categorical data?

## Explore Relative Frequencies

To show what portion of a data set each category in a frequency table makes up, you can convert the data to relative frequencies. The relative frequency of a category is the frequency of the category divided by the total of all frequencies.

The frequency table below shows the results of a survey Kenesha conducted at school. She asked 80 randomly selected students whether they preferred basketball, football, or soccer.


| Favorite Sport | Basketball | Football | Soccer | Total |
| :---: | :---: | :---: | :---: | :---: |
| Frequency | 20 | 32 | 28 | 80 |

(A)

Use the frequencies to make a relative frequency table expressed with decimals.

| Favorite Sport | Basketball | Football | Soccer | Total |
| :---: | :---: | :---: | :---: | :---: |
| Relative Frequency | $\frac{20}{80}=0.25$ |  |  | $\frac{80}{80}=\square$ |

(B) Rewrite the relative frequency table using percents instead of decimals.

| Favorite Sport | Basketball | Football | Soccer | Total |
| :---: | :---: | :---: | :---: | :---: |
| Relative Frequency | $25 \%$ |  |  |  |

## Reflect

1. Explain what the numerator and denominator of the ratio $\frac{20}{80}$ refer to in part A .
2. What types of numbers can you use to write relative frequencies?

## Explain 1 Two-Way Relative Frequency Tables

Two types of relative frequencies are found in a relative frequency table:

1. A joint relative frequency is found by dividing a frequency that is not in the Total row or the Total column by the grand total. It tells what portion of the total has both of the two specified characteristics.
2. A marginal relative frequency is found by dividing a row total or a column total by the grand total. It tells what portion of the total has a specified characteristic.

Example 1 Complete a two-way relative frequency table from the data in a two-way frequency table. Identify the joint relative frequencies and the marginal relative frequencies.
(A) For her survey about sports preferences, Kenesha also recorded the gender of each student. The results are shown in the two-way frequency table for Kenesha's data.

|  | Preferred Sport |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gender | Basketball | Football | Soccer | Total |
| Girl | 6 | 12 | 18 | 36 |
| Boy | 14 | 20 | 10 | 44 |
| Total | 20 | 32 | 28 | 80 |

To find the relative frequencies, divide each frequency by the grand total.

| Preferred Sport |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- |
| Gender | Basketball | Football | Soccer | Total |
| Girl | $\frac{6}{80}=0.075$ | $\frac{12}{80}=0.15$ | $\frac{18}{80}=0.225$ | $\frac{36}{80}=0.45$ |
| Boy | $\frac{14}{80}=0.175$ | $\frac{20}{80}=0.25$ | $\frac{10}{80}=0.125$ | $\frac{44}{80}=0.55$ |
| Total | $\frac{20}{80}=0.25$ | $\frac{32}{80}=0.4$ | $\frac{28}{80}=0.35$ | $\frac{80}{80}=1$ |

The joint relative frequencies tell what percent of all those surveyed are in each category:

- $7.5 \%$ are girls who prefer basketball.
- $17.5 \%$ are boys who prefer basketball.
- $15 \%$ are girls who prefer football.
- $25 \%$ are boys who prefer football.
- $22.5 \%$ are girls who prefer soccer.
- $12.5 \%$ are boys who prefer soccer.

The marginal relative frequencies tell what percent of totals has a given single characteristic:

- $25 \%$ prefer basketball.
- $45 \%$ are girls.
- $40 \%$ prefer football.
- $55 \%$ are boys.
- $35 \%$ prefer soccer.
(B) Millie performed a survey of students in the lunch line and recorded which type of fruit each student selected along with the gender of each student. The two-variable frequency data she collected is shown in the table.

|  | Fruit |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Apple | Banana | Orange | Total |
| Girl | 16 | 10 | 14 | 40 |
| Boy | 25 | 13 | 14 | 52 |
| Total | 41 | 23 | 28 | 92 |


|  | Fruit |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Apple | Banana | Orange | Total |
| Girl | $17.4 \%$ |  |  |  |
| Boy | $27.2 \%$ |  |  |  |
| Total | $44.6 \%$ |  |  |  |

The joint relative frequencies:
$\square$ are girls who selected an apple.

- $\square$ are boys who selected an apple.
- $\square$ are girls who selected a banana.
- $\square$ are boys who selected a banana.
- $\square$ are girls who selected an orange.
- $\square$ are boys who selected an orange.

The marginal relative frequencies:

- $\square$ selected an apple.
$\square$ selected a banana.
- 

$\square$ are girls.
-

- $\square$ are boys.
- $\square$ selected an orange.


## Reflect

3. Discussion Explain how you can use joint and marginal relative frequencies to check your relative frequency table.
$\qquad$
$\qquad$

## Your Turn

Use the two-way table of data from another student survey to answer the following questions.

|  | Like Aerobic Exercise |  |  |
| :---: | :---: | :---: | :---: |
| Like Weight Lifting | Yes | No | Total |
| Yes | 7 | 14 | 21 |
| No | 12 | 7 | 19 |
| Total | 19 | 21 | 40 |

4. Find the joint relative frequency of students surveyed who like aerobics exercise but dislike weight lifting.
5. What is the marginal relative frequency of students surveyed who like weight lifting?

## Explain 2 Conditional Relative Frequencies

A conditional relative frequency describes what portion of a group with a given characteristic also has another characteristic. A conditional relative frequency is found by dividing a frequency that is not in the Total row or the Total column by the total for that row or column.

Example 2 Use the joint relative frequencies to calculate the associated conditional relative frequencies and describe what each one means.
(A) Use the data from Example 1A. Find the conditional relative frequency that a a person in Kenesha's survey prefers soccer, given that the person is a girl.

Divide the number of girls who prefer soccer by the total number of girls.
$\frac{\text { Number of girls who prefer soccer }}{\text { Total number of girls }}=\frac{18}{36}=0.5=50 \%$
Half of the girls in the sample prefer soccer.
(B) Use the data from Example 1B. Find the conditional relative frequency that a student in Millie's survey chose an orange, given that the student is a boy.


## Your Turn

Use the data from Your Turn Exercises 4 and 5 after Example 1.
6. What is the conditional relative frequency that a student likes to lift weights, given that the student does not like aerobics?
7. Find the conditional relative frequency that a student likes to lift weights, given that the student likes aerobics.

## Explain 3 Finding Possible Associations

You can analyze two-way frequency tables to locate possible associations or patterns in the data.

## Example 3 Analyze the results of the surveys to determine preferences by gender.

Kenesha is interested in the question, "Does gender influence what type of sport students prefer?" If there is no influence, then the distribution of gender within each sport preference will roughly equal the distribution of gender within the whole group. Analyze the results of Kenesha's survey from Example 1. Determine which sport each gender is more likely to prefer.
(A) Analyze the data about girls that were surveyed.

Step 1: Identify the percent of all students surveyed who are girls.

$$
\frac{36}{80}=0.45=45 \%
$$

Step 2: Determine each conditional relative frequency.

| Basketball | Football | Soccer |
| :--- | :--- | :--- |
| Of the 20 students who <br> prefer basketball, 6 are <br> girls. | Of the 32 students who <br> prefer football, 12 are <br> girls. | Of the 28 students who <br> prefer soccer, 18 are <br> girls. |
| $\frac{6}{20}=0.3=30 \%$ | $\frac{12}{32}=0.375=37.5 \%$ | $\frac{18}{28} \approx 0.643=64.3 \%$ |

Step 3: Interpret the results by comparing each conditional relative frequency to the percent of all students surveyed who are girls, $45 \%$.

| Basketball | Football | Soccer |
| :---: | :---: | :---: |
| $30 \%<45 \%$ | $37.5 \%<45 \%$ | $64.3 \%>45 \%$ |
| Girls are less likely to <br> prefer basketball. | Girls are less likely to <br> prefer football. | Girls are more likely to <br> prefer soccer. |

(B) Analyze the data about boys that were surveyed.

Step 1: Identify the percent of all students surveyed who are boys.

$$
\frac{\square}{80}=0 . \square=\square \%
$$

Step 2: Determine each conditional relative frequency.

| Basketball | Football | Soccer |
| :---: | :---: | :---: |
| Of the 20 students who prefer basketball, $\square$ are boys. $\frac{\square}{20}=0 . \square=\square \%$ | Of the $\qquad$ students who prefer football, are boys. $\square$ | Of the $\qquad$ students who prefer soccer, are boys. $\square$ |

Step 3: Interpret the results by comparing each conditional relative frequency to the percent of all students surveyed who are boys, $\%$.

| Basketball | Football | Soccer |
| :---: | :--- | :--- |
| $70 \%>55 \%$ | $62.5 \%>55 \%$ | $35.7 \%<55 \%$ |
| Boys are more likely to <br> prefer basketball. | Boys are more likely to prefer <br> football. | Boys are less likely to prefer <br> soccer. |

## Reflect

8. Making Connections How can the statement " 6 out of the 20 students who prefer basketball are girls" be stated as a conditional relative frequency?

## Your Turn

9. Analyze the data given in the Your Turn after Example 1 to determine if liking aerobic exercise influences whether a person also likes weight lifting. Explain.

## Elaborate

10. What does it mean to say there is an association between characteristics in a two-way frequency table?
$\qquad$
$\qquad$
11. Essential Question Check-In How can you use two-way frequency data to recognize possible associations between the two categories of categorical data?
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Evaluate: Homework and Practice

Use the table of frequency data for Exercises 1-4.

- Online Homework
- Hints and Help
- Extra Practice

Class Survey of Favorite Colors

| Favorite Color | Red | Orange | Yellow | Green | Blue | Purple | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 2 | 5 | 1 | 6 | 8 | 2 | 24 |

1. Complete the relative frequency table for this data using decimals rounded to the nearest thousandth.

## Class Survey of Favorite Colors

| Favorite Color | Red | Orange | Yellow | Green | Blue | Purple | Total |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Relative <br> Frequency |  |  |  |  |  |  |  |

2. Complete the relative frequency table for this data using percents rounded to the nearest tenth.

Class Survey of Favorite Colors

| Favorite Color | Red | Orange | Yellow | Green | Blue | Purple | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Relative <br> Frequency |  |  |  |  |  |  |  |

3. What is the relative frequency of having blue as a favorite color, expressed as a decimal?
4. Which color is a favorite color with a relative frequency of $25 \%$ ?

The following frequency data shows the number of states, including the District of Columbia, that favored each party in the presidential popular vote in 1976 and in 2012.

| 2012 Election |  |  |  |
| :---: | :---: | :---: | :---: |
| $\mathbf{1 9 7 6}$ Election | Democrat | Republican | Total |
| Democrat | $12=\square$ | $12=\square$ | $24=\square$ |
| Republican | $15=\square$ | $12=\square$ | $27=\square$ |
| Total | $27=\square$ | $24=\square$ | $51=\square$ |

5. Complete the table above with relative frequencies using percents.
6. What percent switched from Democrat in 1976 to Republican in 2012?

What type of frequency is this?
7. What percent voted Republican in 1976? What type of frequency is this?

The results of a survey of 45 students and the foreign language they are studying are shown in the two-way frequency table.

| Language |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Gender | Chinese | French | Spanish | Total |  |
| Girl | 2 | 8 | 15 | 25 |  |
| Boy | 4 | 4 | 12 | 20 |  |
| Total | 6 | 12 | 27 | 45 |  |

8. Fill in the table of two-way relative frequencies using decimals, rounded to the nearest thousandth.

|  | Language |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Gender | Chinese | French | Spanish | Total |
| Girl |  |  |  |  |
| Boy |  |  |  |  |
| Total |  |  |  |  |

9. What fraction of the surveyed students are boys taking Spanish?
10. What fraction of the surveyed students are taking Chinese?

In some states, a driver of a vehicle may not use a handheld cell phone while driving. In one state with this law, 250 randomly selected drivers were surveyed to determine the association between drivers who know the law and drivers who obey the law. The results are shown in the table below.
11. Complete the table of two-way relative frequencies using percents.

|  | Knows the Law |  |  |
| :---: | :---: | :---: | :---: |
| Obeys the Law | Yes | No | Total |
| Yes | $160=$ | $45=$ |  |
| No | $25=$ | $20=$ |  |
| Total |  |  |  |


12. What is the relative frequency of drivers who know and obey the law?
13. What is the relative frequency of drivers who know the law?

Refer to the election data from Exercises 5-7. Answer using percents rounded to the nearest tenth.
14. What is the conditional relative frequency of a state's popular vote being won by the Democrat in 2012, given that it was won by the Democrat in 1976?
15. What is the conditional relative frequency of a state's popular vote being won by the Democrat in 1976, given that it was won by the Democrat in 2012?

Refer to the language data from Exercises 8-10. Answer using decimals rounded to the nearest thousandth.
16. What fraction of girls are studying French?
17. What fraction of Spanish students are boys?

Refer to the cell phone law data from Exercises 11-13. Answer using percents rounded to the nearest tenth.
18. What percent of drivers obey the law despite not knowing the law?
19. What is the conditional relative frequency of drivers who obey the law, given that they know the law?

Use the previously described data to determine whether there are associations between the categories surveyed.
20. Refer to the election data from Exercises 5-7. Is there an association between the party that won the popular vote in a state in 1976 and in 2012?
21. Refer to the language data from Exercises 8-10. Can you use gender to predict a preference for taking Spanish?
22. Refer to the language data from Exercises 8 -10. Is there an association between gender and a preference for French?
23. Refer to the cell phone law data from Exercises 11-13. Most drivers who don't know that it is illegal to operate a cell phone while driving obey the law anyway, presumably out of a general concern for safe driving. Does this mean there is no association between knowledge of the cell phone law and obeying the cell phone law?
24. Multipart Classification Classify each statement as describing a joint, marginal, or conditional relative frequency.
a. In a study on age and driving safety, $33 \%$ of drivers were considered younger and a high accident risk.
b. In a study on age and driving safety, $45 \%$ of older drivers were considered a high accident risk.
c. In a study on age and driving safety, $67 \%$ of drivers were classified as younger.
d. In a pre-election poll, $67 \%$ of the respondents who preferred the incumbent were men.
e. In a pre-election poll, $33 \%$ of women preferred the challenger.
f. In a pre-election poll, $16 \%$ of respondents were men who preferred the challenger.

## H.O.T. Focus on Higher Order Thinking

25. Explain the Error In the survey on gender and fruit selection (Example 1B), Millicent notices that given a preference for oranges, the conditional relative frequencies of a student being a boy or a girl are the same. She concludes that there is no association between gender and orange preference. Explain her error.
26. Communicate Mathematical Ideas Can a joint relative frequency be greater than either of the conditional relative frequencies associated with it? Explain your reasoning.
27. Explain the Error Refer to the cell phone data from Exercises 11-13. Cole found the conditional relative frequency that a driver surveyed does not know the law, given that the driver obeys the law, by dividing 45 by 250. Explain Cole's error.

## Lesson Performance Task

Eighty students were surveyed about playing an instrument. The results are shown in the two-way frequency table.

|  | Play an Instrument |  |  |
| :---: | :---: | :---: | :---: |
| Gender | Yes | No | Total |
| Female | 28 | 17 | 45 |
| Male | 20 | 15 | 35 |
| Total | 48 | 32 | 80 |

a. Complete the two-way relative frequency table for the data.

|  | Play an Instrument |  |  |
| :---: | :---: | :---: | :---: |
| Gender | Yes | No | Total |
| Female |  |  |  |
| Male |  |  |  |
| Total |  |  |  |

b. What percent of the students surveyed play an instrument? What percent of the males surveyed do not play an instrument? Identify what type of frequency each percent is.
c. Is there an association between the sex of a student and whether the student plays an instrument? Explain.

