Name ____________________ Class ____________ Date ________

6.1 Slope-Intercept Form

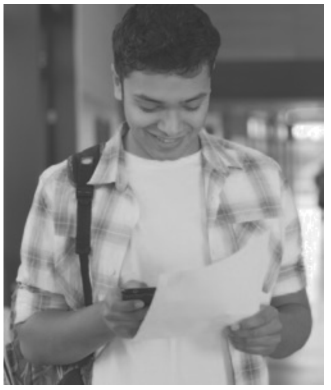
Essential Question: How can you represent a linear function in a way that reveals its slope and *y*-intercept?

Resource Locker

Explore Graphing Lines Given Slope and *y*-intercept

Graphs of linear equations can be used to model many real-life situations. Given the slope and *y*-intercept, you can graph the line, and use the graph to answer questions.

Andrew wants to buy a smart phone that costs \$500. His parents will pay for the phone, and Andrew will pay them \$50 each month until the entire amount is repaid. The loan repayment represents a linear situation in which the amount y that Andrew owes his parents is dependent on the number x of payments he has made.



(A) When $x = 0$, $y =$ ______.

The y-intercept of the graph of the equation that represents the situation is ______

(B) The rate of change in the amount Andrew owes over time is ______ per month.

The slope is ______.

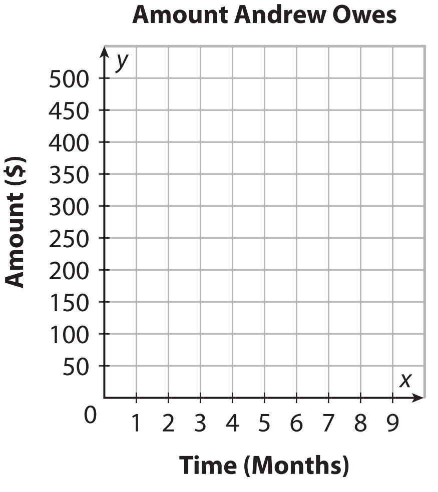
(C) Use the y-intercept to plot a point on the graph of the equation. The y-intercept is ______, so plot the point ________.

(D) Using the definition of slope, plot a second point.

$$\text{Slope} = \frac{\text{Change in } y}{\text{Change in } x} = \frac{\boxed{}}{1} = \boxed{}.$$

Start at the point you plotted. Count ______ units down and ______ unit right and plot another point.

(E) Draw a line through the points you plotted.



Reflect

1. **Discussion** How can you use the same method to find two more points on that same line?

2. How many months will it take Andrew to pay off his loan? Explain your answer.

Explain 1 Creating Linear Equations in Slope-Intercept Form

You can use the slope formula to derive the slope-intercept form of a linear equation.

Consider a line with slope m and y-intercept b.

The slope formula is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Substitute $(0, b)$ for (x_1, y_1) and (x, y) for (x_2, y_2).

$$m = \frac{y - b}{x - 0}$$

$$m = \frac{y - b}{x}$$

$$mx = y - b \qquad \text{Multiply both sides by } x \ (x \neq 0).$$

$$mx + b = y \qquad \text{Add } b \text{ to both sides.}$$

$$y = mx + b$$

Slope-Intercept Form of an Equation

If a line has slope m and y-intercept $(0, b)$, then the line is described by the equation $y = mx + b$.

Example 1 **Write the equation of each line in slope-intercept form.**

(A) Slope is 3, and $(2, 5)$ is on the line.

Step 1: Find the y-intercept.

$y = mx + b$	Write the slope–intercept form.
$5 = 3(2) + b$	Substitute 3 for m, 2 for x, and 5 for y.
$5 = 6 + b$	Multiply.
$5 - 6 = 6 + b - 6$	Subtract 6 from both sides.
$-1 = b$	Simplify.

Step 2: Write the equation.

$y = mx + b$	Write the slope–intercept form.
$y = 3x + (-1)$	Substitute 3 for m and -1 for b.
$y = 3x - 1$	

Ⓑ The line passes through $(0, 5)$ and $(2, 13)$.

Step 1: Use the points to find the slope.

$$m = \frac{\square}{\square}$$

Substitute $(0, 5)$ for (x_1, y_1) and $(\square, \square)$ for (x_2, y_2).

$$m = \frac{\square}{\square} = \frac{\square}{\square} = \square$$

Step 2: Substitute the slope and x- and y-coordinates of either of the points in the equation $y = mx + b$.

$$y = mx + b$$

$$\square = \square(\square) + b$$

$$\square = \square + b$$

$$\square - \square = \square + b - \square$$

$$\square = b$$

Step 3: Substitute ______ for m and ______ for b in the equation $y = mx + b$.

The equation of the line is $\square$.

Your Turn

Write the equation of each line in slope-intercept form.

3. Slope is -1, and $(3, 2)$ is on the line.

4. The line passes through $(1, 4)$ and $(3, 18)$.

Explain 2 Graphing from Slope-Intercept Form

Writing an equation in slope-intercept form can make it easier to graph the equation.

Example 2 **Write each equation in slope-intercept form. Then graph the line.**

Ⓐ $y = 5x - 4$

The equation $y = 5x - 4$ is already in slope-intercept form.

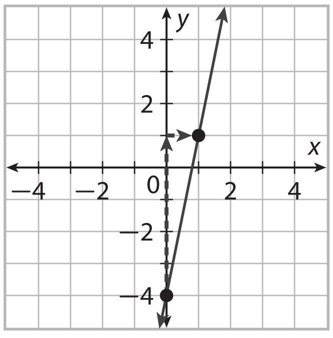
Slope: $m = 5 = \frac{5}{1}$

y-intercept: $b = -4$

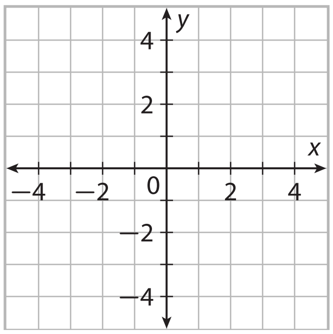
Step 1: Plot $(0, -4)$

Step 2: Count 5 units up and 1 unit to the right and plot another point.

Step 3: Draw a line through the points.



(B) $2x + 6y = 6$



Step 1: Write the equation in slope-intercept form by solving for y.

$2x + 6y - 2x = 6 - \boxed{}$ Slope: $\boxed{}$

$6y = \boxed{} + 6$ y-intercept: $\boxed{}$

$y = \boxed{}\,x + \boxed{}$

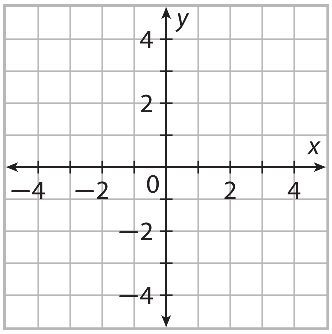
Step 2: Graph the line.

Plot $\left(\boxed{}, \boxed{}\right)$. Move ______ unit down and ______ units to the right to plot a second point. Draw a line through the points.

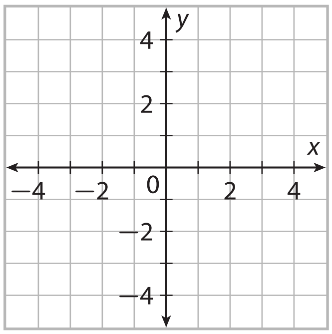
Your Turn

Write each equation in slope-intercept form. Then graph the line.

5. $2x + y = 4$



6. $2x + 3y = 6$



Explain 3 Determining Solutions of Equations in Two Variables

Given a real-world linear situation described by a table, a graph, or a verbal description, you can write an equation in slope-intercept form. You can use that equation to solve problems.

Example 3 **Identify the slope and y-intercept of the graph that represents each linear situation and interpret what they mean. Then write an equation in slope-intercept form and use it to solve the problem.**

(A) For one taxi company, the cost y in dollars of a taxi ride is a linear function of the distance x in miles traveled. The initial charge is \$2.50, and the charge per mile is \$0.35. Find the cost of riding a distance of 10 miles.

The rate of change is \$0.35 per mile, so the slope, m, is 0.35.

The initial cost is the cost to travel 0 miles, \$2.50, so the y-intercept, b, is 2.50.

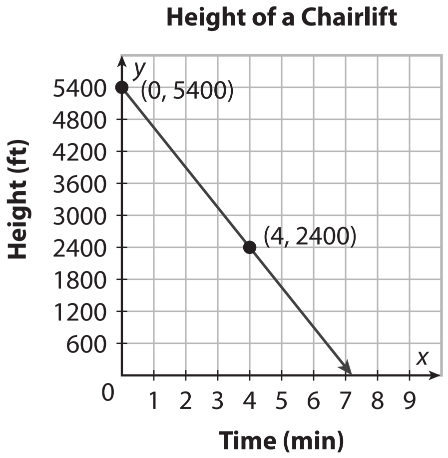
Then an equation is $y = 0.35x + 2.50$. $\quad y = 0.35x + 2.50$

To find the cost of riding 10 miles, substitute 10 for x. $\quad = 0.35(10) + 2.50$

$= 6$

$(6, 10)$ is a solution of the equation, and the cost of riding a distance of 10 miles is \$6.

(B) A chairlift descends from a mountain top to pick up skiers at the bottom. The height in feet of the chairlift is a linear function of the time in minutes since it begins descending as shown in the graph. Find the height of the chairlift 2 minutes after it begins descending.



The graph contains the points (0, ______) and (_____, 2400).

The slope is $\dfrac{\boxed{} - 5400}{\boxed{} - 0} = \boxed{}$.

It represents the rate at which the chairlift ____________.

The graph passes through the point (0, ______), so the y-intercept is ______. It represents the height of the chairlift _____ minutes after it begins descending.

Let x be the time in seconds after the chairlift begins to descend.

Let y be the height of the chairlift in feet.

The equation is __________________.

To find the height after 4 minutes, substitute 4 for x and simplify.

$$\boxed{} = \boxed{}\left(\boxed{}\right) + 5400$$
$$= \boxed{} + 5400$$
$$= \boxed{}$$

__________ is a solution of the equation, and the height of the chairlift 2 minutes after it begins descending is ________ feet.

Reflect

7. In the example involving the taxi, how would the equation change if the cost per mile increased or decreased? How would this affect the graph?

Identify the slope and *y*-intercept of the graph that represents each linear situation and interpret what they mean. Then write an equation in slope-intercept form and use it to solve the problem.

Your Turn

8. A local club charges an initial membership fee as well as a monthly cost. The cost C in dollars is a linear function of the number of months of membership. Find the cost of the membership after 4 months.

Membership Cost	
Time (months)	**Cost ($)**
0	100
3	277
6	454

Elaborate

9. What are some advantages to using slope-intercept form?

10. What are some disadvantages of slope-intercept form?

11. **Essential Question Check-In** When given a real-world situation that can be described by a linear equation, how can you identify the slope and *y*-intercept of the graph of the equation?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

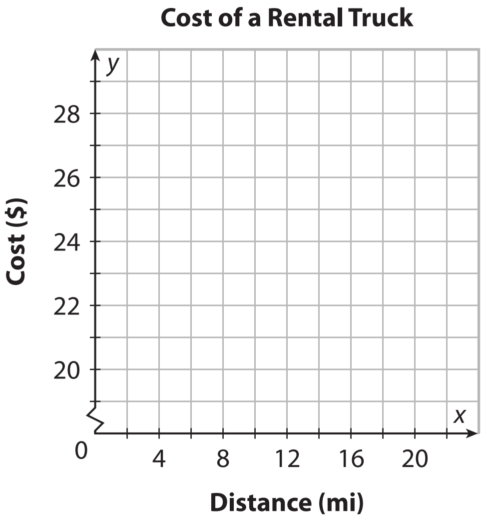
For each situation, determine the slope and *y*-intercept of the graph of the equation that describes the situation.

1. John gets a new job and receives a $500 signing bonus. After that, he makes $200 a day.

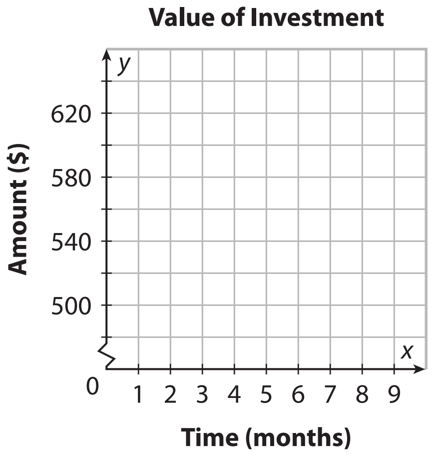
2. Jennifer is 20 miles north of her house, and she is driving north on the highway at a rate of 55 miles per hour.

Sketch a graph that represents the situation.

3. Morwenna rents a truck. She pays \$20 plus \$0.25 per mile.



4. An investor invests \$500 in a certain stock. After the first six months, the value of the stock has increased at a rate of \$20 per month.



Write the equation of each line in slope-intercept form.

5. Slope is 3, and $(1, 5)$ is on the line.

6. Slope is -2, and $(5, 3)$ is on the line.

7. Slope is $\frac{1}{4}$, and $(4, 2)$ is on the line.

8. Slope is 5, and $(2, 6)$ is on the line.

9. Slope is $-\frac{2}{3}$, and $(-6, -5)$ is on the line.

10. Slope is $-\frac{1}{2}$, and $(-3, 2)$ is on the line.

11. Passes through $(5, 7)$ and $(3, 1)$

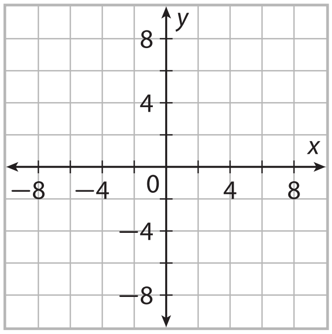
12. Passes through $(-6, 10)$ and $(-3, -2)$

13. Passes through $(6, 6)$ and $(-2, 2)$

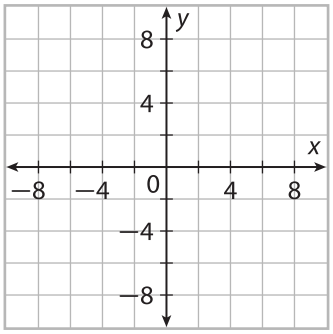
14. Passes through $(-1, -5)$ and $(2, 6)$

Write each equation in slope-intercept form. Identify the slope and *y*-intercept. Then graph the line described by the equation.

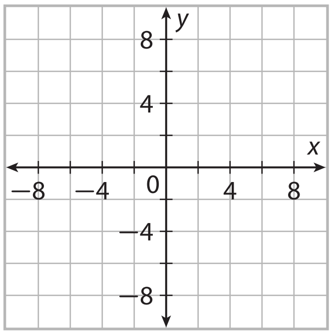
15. $y = 2x + 3$



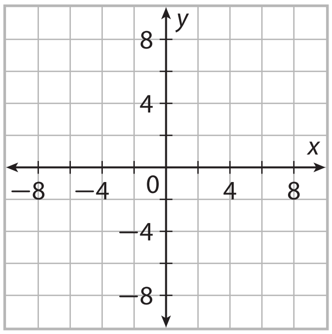
16. $y = -x + 2$



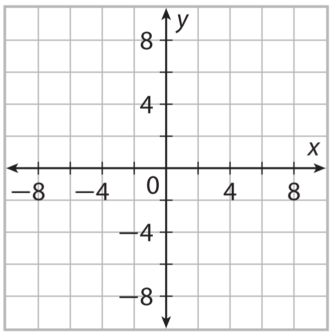
17. $y = \frac{2}{3}x - 4$



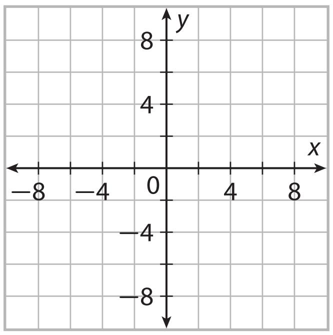
18. $y = -\frac{1}{2}x - 1$



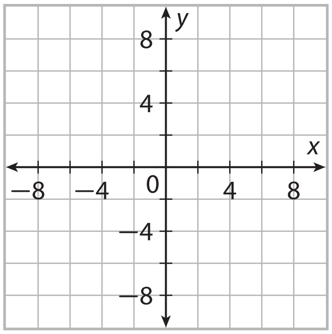
19. $-4x + 2y = 10$



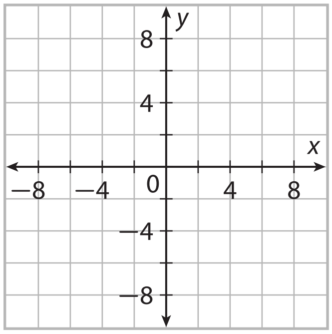
20. $3x - 6y = -12$



21. $-5x - 2y = 8$

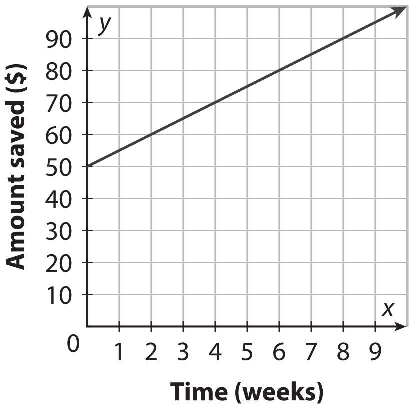


22. $3x + 4y = -12$



23. **Sports** A figure skating school offers introductory lessons at \$25 per session. There is also a registration fee of \$30. Write a linear equation in slope-intercept form that represents the situation. You want to take at least 6 lessons. Can you pay for those lessons using a \$200 gift certificate? If so, how much money, if any, will be left on the gift certificate? If not, explain why not.

24. **Represent Real World Problems** Lorena and Benita are saving money. They began on the same day. Lorena started with \$40. Each week she adds \$8. The graph describes Benita's savings plan. Which girl will have more money in 6 weeks? How much more will she have? Explain your reasoning.



H.O.T. Focus on Higher Order Thinking

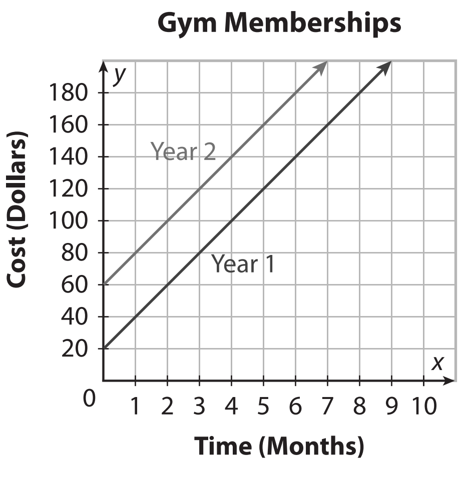
25. Analyze Relationships Julio and Jake start their reading assignments the same day. Jake is reading a 168-page book at a rate of 24 pages per day. Julio's book is 180 pages long and his reading rate is $1\frac{1}{4}$ times Jake's rate. After 5 days, who will have more pages left to read? How many more? Explain your reasoning.

26. Explain the Error John has $2 in his bank account when he gets a job. He begins making $107 dollars a day. A student found that the equation that represents this situation is $y = 2x + 107$. What is wrong with the student's equation? Describe and correct the student's error.

27. Justify Reasoning Is it possible to write the equation of every line in slope-intercept form? Explain your reasoning.

Lesson Performance Task

The graph shows the cost of a gym membership in each of two years. What are the values that represent the sign-up fee and the monthly membership fee? How did the values change between the years?



a. Write an equation in slope-intercept form for each of the two lines in the graph.

b. What are the values that represent the sign-up fee and the membership cost? How did the values change between the years?

Name ____________________ Class ____________ Date ________

6.2 Point-Slope Form

Essential Question: **How can you represent a linear function in a way that reveals its slope and a point on its graph?**

Resource Locker

Explore Deriving Point-Slope Form

Suppose you know the slope of a line and the coordinates of one point on the line. How can you write an equation of the line?

(A) A line has a slope m of 4, and the point $(2, 1)$ is on the line. Let (x, y) be any other point on the line. Substitute the information you have in the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\square = \frac{y - \square}{x - \square}$$

(B) Use the Multiplication Property of Equality to get rid of the fraction.

$$\square\left(x - \square\right) = \left(\frac{y - \square}{x - \square}\right)\left(x - \square\right)$$

(C) Simplify.

$$\square\left(x - \square\right) = \left(y - \square\right)$$

Reflect

1. **Discussion** The equation that you derived is written in a form called *point-slope form*. The equation $y = 2x + 1$ is in slope-intercept form. How can you rewrite it in point-slope form?

____

____

____

Explain 1 Creating Linear Equations Given Slope and a Point

Point-Slope Form
The line with slope m that contains the point (x_1, y_1) can be described by the equation $y - y_1 = m(x - x_1)$.

Example 1 **Write an equation in point-slope form for each line.**

(A) Slope is 3.5, and $(-3, 2)$ is on the line.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 2 = 3.5\left(x - (-3)\right)$	Substitute.
$y - 2 = 3.5(x + 3)$	Simplify.

(B) Slope is 0, and $(-2, -1)$ is on the line.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - \square = \square\left(x - \square\right)$	Substitute.
$y + \square = \square$	Simplify.

Reflect

2. **Communicate Mathematical Ideas** Suppose that you are given that the slope of a line is 0. What is the only additional information you need to write an equation of the line? Explain.

Your Turn

Write an equation in point-slope form for each line.

3. Slope is 6, and (1, 2) is on the line.

4. Slope is $\frac{1}{3}$, and $(-3, 1)$ is on the line.

Explain 2 Creating Linear Models Given Slope and a Point

You can write an equation in point-slope form to describe a real-world linear situation. Then you can use that equation to solve a problem.

Example 2 **Solve the problem using an equation in point-slope form.**

(A) Paul wants to place an ad in a newspaper. The newspaper charges \$10 for the first 2 lines of text and \$3 for each additional line of text. Paul's ad is 8 lines long. How much will the ad cost?

Let x represent the number of lines of text. Let y represent the cost in dollars of the ad. Because 2 lines of text cost \$10, the point $(2, 10)$ is on the line. The rate of change in the cost is \$3 per line, so the slope is 3.

Write an equation in point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 10 = 3(x - 2)$	Substitute 3 for m, 2 for x_1, and 10 for y_1.

To find the cost of 8 lines, substitute 8 for x and solve for y.

$y - 10 = 3(8 - 2)$	Substitute
$y - 10 = 18$	Simplify.
$y = 28$	

The cost of 8 lines is \$28.

(B) Paul would like to shop for the best price to place the ad. A different newspaper has a base cost of \$15 for 3 lines and \$2 for every extra line. How much will an 8-line ad cost in this paper?

$y - y_1 = m(x - x_1)$	Point-slope form
$y - \boxed{} = 2\left(x - \boxed{}\right)$	Substitute.
$y - \boxed{} = 2\left(\boxed{} - \boxed{}\right)$	Substitute for x.
$y - \boxed{} = \boxed{}$	Simplify the right side.
$y = \boxed{}$	Solve for y.

The cost of 8 lines is \$$\boxed{}$.

Reflect

5. **Analyze Relationships** Suppose that you find that the cost of an ad with 8 lines in another publication is \$18. How is the ordered pair (8, 18) related to the equation that represents the situation? How is it related to the graph of the equation?

____

____

Your Turn

6. Daisy purchases a gym membership. She pays a signup fee and a monthly fee of \$11. After 4 months, she has paid a total of \$59. Use a linear equation in point-slope form to find the signup fee.

Explain 3 Creating Linear Equations Given Two Points

You can use two points on a line to create an equation of the line in point-slope form. There is more than one such equation.

Example 3 **Write an equation in point-slope form for each line.**

(A) $(2, 1)$ and $(3, 4)$ are on the line.

Let $(2, 1) = (x_1, y_1)$ and let $(3, 4) = (x_2, y_2)$.

Find the slope of the line by substituting the given values in the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{3 - 2} = 3$$

You can choose either point and substitute the coordinates in the point-slope form.

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 1 = 3(x - 2)$	Substitute 3 for m, 2 for x_1, and 1 for y_1.

Or:

$y - y_1 = m(x - x_1)$	Point-slope form
$y - 4 = 3(x - 3)$	Substitute 3 for m, 3 for x_1, and 4 for y_1.

B $(1, 3)$ and $(2, 3)$ are on the line.

Let $(1, 3) = (x_1, y_1)$ and let $(2, 3) = (x_2, y_2)$.

Find the slope of the line by substituting the given values in the slope formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{\square - \square}{\square - \square}$$

$$= \square$$

Choose either point and substitute the coordinates in the point-slope form.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - \square = \square(x - \square)$ Substitute 0 for m, 1 for x_1, and 3 for y_1.

Or:

$y - y_2 = m(x - x_2)$ Point-slope form

$y - \square = \square(x - \square)$ Substitute 0 for m, 2 for x_2, and 3 for y_2.

Reflect

7. Given two points on a line, Martin and Minh each found the slope of the line. Then Martin used (x_1, y_1) and Minh used (x_2, y_2) to write the equation in point-slope form. Each student's equation was correct. Explain how they can show both equations are correct.

Your Turn

Write an equation in point-slope form for each line.

8. $(2, 4)$ and $(3, 1)$ are on the line.

9. $(0, 1)$ and $(1, 1)$ are on the line.

Explain 4 Creating a Linear Model Given Two Points

In a real-world linear situation, you may have information that represents two points on the line. You can write an equation in point-slope form that represents the situation and use that equation to solve a problem.

Example 4 **Solve the problem using an equation in point-slope form.**

An animal shelter asks all volunteers to take a training session and then to volunteer for one shift each week. Each shift is the same number of hours. The table shows the numbers of hours Joan and her friend Miguel worked over several weeks. Another friend, Lili, plans to volunteer for 24 weeks over the next year. How many hours will Lili volunteer?

Volunteer	Weeks worked	Hours worked
Joan	6	15
Miguel	10	23

Analyze Information

Identify the important information.

- Joan worked for ________ weeks for a total of ________ hours.
- Miguel worked for ________ weeks for a total of ________ hours.
- Lili will work for ________ weeks.

Formulate a Plan

To create the equation, identify the two ordered pairs represented by the situation. Find the ________ of the line that contains the two points. Write the equation in point-slope form. Substitute the ________________ that Lili works for x to find y, the ____________________________.

Let x represent the number of weeks worked and y represent the number of hours worked.

The points __________ and __________ are on the line. Substitute the coordinates in the slope formula to find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\square - \square}{\square - \square}$$

$$m = \square$$

Next choose one of the points and find an equation of the line in point-slope form.

$y - y_1 = m(x - x_1)$ Point-slope form

$y - \square = \square\left(x - \square\right)$ Substitute $\square$ for m, $\square$ for x_1, and $\square$ for y_1.

Or:

$y - y_2 = m(x - y_2)$ Point-slope form

$y - \square = \square\left(x - \square\right)$ Substitute $\square$ for m, $\square$ for x_2, and $\square$ for y_2.

Finally, substitute ______ in the equation to find y.

$y - \square = \square\left(x - \square\right)$ Substitute $\square$ for m, $\square$ for x_1, and $\square$ for y_1.

$y - \square = \square\left(\square - \square\right)$ Substitute $\square$ for x.

$y - \square = \square\left(\square\right)$ Simplify.

$y = \square$ Simplify.

Or:

$y - \square = \square\left(x - \square\right)$ Substitute $\square$ for m, $\square$ for x_2, and $\square$ for y_2.

$y - \square = \square\left(\square - \square\right)$ Substitute $\square$ for x.

$y - \square = \square\left(\square\right)$ Simplify.

$y = \square$ Simplify.

Lili will work a total of ________ hours.

Justify and Evaluate

The ordered pair $\left(\square, \square\right)$ is a solution of both equations obtained using the given information.

$y - \square = \square\left(x - \square\right)$	Substitute 2 for m, 6 for x_1, and 15 for y_1.
$\square - \square = \square\left(\square - \square\right)$	Substitute $\square$ for x and $\square$ for y.
$\square = \square$	Simplify.

Or:

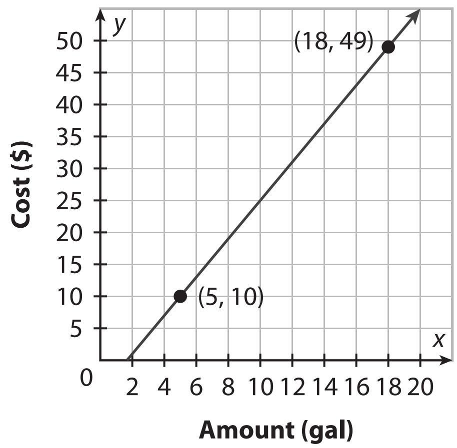
$y - \square = \square\left(x - \square\right)$	Substitute 2 for m, 10 for x_2, and 23 for y_2.
$\square - \square = \square\left(\square - \square\right)$	Substitute $\square$ for x and $\square$ for y.
$\square = \square$	Simplify.

The answer makes sense because the rate of change in the number of hours is the slope, ______. Because Lili will work ______ more weeks than Miguel, she will work $23 + 2$ ______ hours, or ______ hours.

Your Turn

Solve the problem using an equation in point-slope form.

10. A gas station has a customer loyalty program. The graph shows the amount y dollars that two members paid for x gallons of gas. Use an equation in point-slope to find the amount a member would pay for 22 gallons of gas.



11. A roller skating rink offers a special rate for birthday parties. On the same day, a party for 10 skaters cost $107 and a party for 15 skaters cost $137. How much would a party for 12 skaters cost?

Elaborate

12. Can you write an equation in point-slope form that passes through any two given points in a coordinate plane?

13. Compare and contrast the slope-intercept form of a linear equation and the point-slope form.

14. **Essential Question Check-In** Given a linear graph, how can you write an equation in point-slope form of the line?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. Is the equation $y + 1 = 7(x + 2)$ in point-slope form? Justify your answer.

Write an equation in point-slope form for each line.

2. Slope is 1 and $(-2, -1)$ is on the line.

3. Slope is -2, and $(1, 1)$ is on the line.

4. Slope is 0, and $(1, 2)$ is on the line.

5. Slope is $\frac{1}{4}$, and $(1, 2)$ is on the line.

6. $(1, 6)$ and $(2, 3)$ are on the line.

7. $(-1, 1)$ and $(1, -1)$ are on the line.

8. $(7, 7)$ and $(-3, 7)$ are on the line.

9. $(0, 3)$ and $(2, 4)$ are on the line.

Solve the problem using an equation in point-slope form.

10. An oil tank is being filled at a constant rate. The depth of the oil is a function of the number of minutes the tank has been filling, as shown in the table. Find the depth of the oil one-half hour after filling begins.

Time (min)	Depth (ft)
0	3
10	5
15	6

11. James is participating in a 5-mile walk to raise money for a charity. He has received \$200 in fixed pledges and raises \$20 extra for every mile he walks. Use a point-slope equation to find the amount he will raise if he completes the walk.

12. Keisha is reading a 325-page book at a rate of 25 pages per day. Use a point-slope equation to determine whether she will finish reading the book in 10 days.

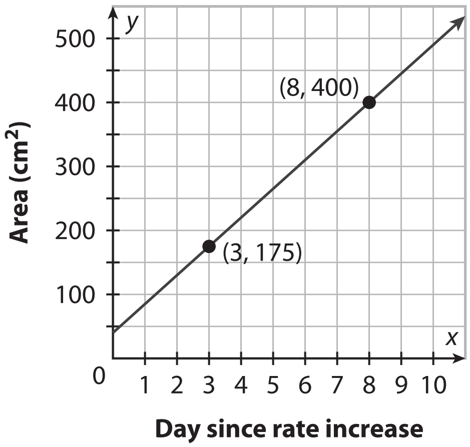
13. Lizzy is tiling a kitchen floor for the first time. She had a tough time at first and placed only 5 tiles the first day. She started to go faster, and by the end of day 4, she had placed 35 tiles. She worked at a steady rate after the first day. Use an equation in point-slope form to determine how many days Lizzy took to place all of the 100 tiles needed to finish the floor.

14. The amount of fresh water left in the tanks of a nineteenth-century clipper ship is a linear function of the time since the ship left port, as shown in the table. Write an equation in point-slope form that represents the function. Then find the amount of water that will be left in the ship's tanks 50 days after leaving port.

Time (days)	Amount (gal)
1	3555
8	3240
15	2925

15. At higher altitudes, water boils at lower temperatures. This relationship between altitude and boiling point is linear. At an altitude of 1000 feet, water boils at 210 °F. At an altitude of 3000 feet, water boils at 206 °F. Use an equation in point-slope form to find the boiling point of water at an altitude of 6000 feet.

16. In art class,Tico is copying a detail from a painting. He paints slowly for the first few days, but manages to increase his rate after that. The graph shows his progress after he increased his rate. How many square centimeters of his painting will he finish in 5 days after the increase in rate?



17. A hot air balloon in flight begins to ascend at a steady rate of 120 feet per minute. After 1.5 minutes, the balloon is at an altitude of 2150 feet. After 3 minutes, it is at an altitude of 2330 feet. Use an equation in point-slope form to determine whether the balloon will reach an altitude of 2500 feet in 4 minutes.

18. A candle burned at a steady rate. After 32 minutes, the candle was 11.2 inches tall. Eighteen minutes later, it was 10.75 inches tall. Use an equation in point-slope form to determine the height of the candle after 2 hours.

19. **Volume** A rectangular swimming pool has a volume capacity of 2160 cubic feet. Water is being added to the pool at a rate of about 20 cubic feet per minute. Determine about how long it will take to fill the pool completely if there were already about 1200 gallons of water in the pool. Use the fact that 1 cubic foot of space holds about 7.5 gallons of water.

20. **Multi-Step** Marisa is walking from her home to her friend Sanjay's home. When she is 12 blocks away from Sanjay's home, she looks at her watch. She looks again when she is 8 blocks away from Sanjay's home and finds that 6 minutes have passed.

a. What do you need to assume in order to treat this as a linear situation?

b. Identify the variables for the linear situation and identify two points on the line. Explain the meaning of the points in the context of the problem.

c. Find the slope of the line and describe what it means in the context of the problem.

d. Write an equation in point-slope form for the situation and use it to find the number of minutes Marisa takes to reach Sanjay's home. Show your work.

21. Match each equation with the pair of points used to create the equation.

a. $y - 10 = 1(x + 2)$ ________________ $(0, 0), (-1, 1)$

b. $y - 0 = 1(x - 0)$ ________________ $(1, 1), (-1, -1)$

c. $y - 3 = -1(x + 3)$ ________________ $(-2, 10), (0, 12)$

d. $y - 3 = 0(x - 2)$ ________________ $(1, 3), (-3.5, 3)$

H.O.T. Focus on Higher Order Thinking

22. Explain the Error Carlota wrote the equation $y + 1 = 2(x - 3)$ for the line passing through the points $(-1, 3)$ and $(2, 9)$. Explain and correct her error.

23. Communicate Mathematical Ideas Explain why it is possible for a line to have no equation in point-slope form or to have infinitely many, but it is not possible that there is only one.

24. Persevere in Problem Solving If you know that $A \neq 0$ and $B \neq 0$, how can you write an equation in point-slope form of the equation $Ax + By = C$?

Lesson Performance Task

Alberto is snow boarding down a mountain with a constant slope. The slope he is on has an overall length of 1560 feet. The top of the slope has a height of 4600 feet, and the slope has a vertical drop of 600 feet. It takes him 24 seconds to reach the bottom of the slope.

a. If we assume that Alberto's speed down the slope is constant, what is his height above the bottom of the slope at 10 seconds into the run?

b. Alberto says that he must have been going 50 miles per hour down the slope. Do you agree? Why or why not?

Name ____________________ Class ____________ Date ________

6.3 Standard Form

Essential Question: How can you write a linear equation in standard form given properties of the line including its slope and points on the line?

Resource Locker

Explore Comparing Forms of Linear Equations

You have seen that the standard form of a linear equation is $Ax + By = C$ where A and B are not both zero. For instance, the equation $2x + 3y = -12$ is in standard form. You can write equivalent equations in slope-intercept form and point-slope form. For instance, the equation in slope-intercept form and an equation in point-slope are shown.

Standard form: $2x + 3y = -12$

Slope-intercept form: $y = -\frac{2}{3}x - 4$

Point-slope from: $y + 8 = -\frac{2}{3}(x - 6)$

Use the three forms of the equation shown to complete each step.

(A) Circle true or false.

You can read the slope from the equation.

Standard form	True	False
Slope-intercept form	True	False
Point-slope form	True	False

(B) The slope is ____________.

(C) Circle true or false.

You can read the y-intercept from the equation.

Standard form	True	False
Slope-intercept form	True	False
Point-slope form	True	False

(D) Identify the y-intercept ____________

Reflect

1. Explain how you can find both intercepts using the standard form.

____

2. How can you find a point on the line using the slope-intercept form?

____

Explain 1 Creating Linear Equations in Standard Form Given Slope and a Point

Given the slope of a line and a point on the line, you can write an equation of the line in standard form.

Example 1 **Write an equation in standard form for each line.**

(A) Slope is 2 and $(-2, 2)$ is on the line.

Method 1: Use the point-slope form. Substitute the given slope and the coordinates of the given point.

$y - 2 = 2\left(x - (-2)\right)$	Point-slope form	Rewrite in standard form.
$y - 2 = 2x + 4$	Simplify.	$y - 2 = 2x + 4$
		$-2x + y = 6$

Method 2: Use the slope-intercept form. Substitute the given slope and the coordinates of the given point. Solve for b.

$y = mx + b$	Slope-intercept form	The slope-intercept form is $y = 2x + 6$.
$2 = 2(-2) + b$	Substitute for x.	Rewrite in standard form as in Method 1: $-2x + y = 6$
$6 = b$	Simplify.	

(B) Slope is 5, and $(-2, 4)$ is on the line.

Method 1: Use the point-slope form. Substitute the given slope and the coordinates of the given point.

$y - \boxed{} = \boxed{}\left(x - \left(\boxed{}\right)\right)$ Point-slope form

$y - \boxed{} = \boxed{}x + \boxed{}$ Simplify.

Rewrite in standard form.

$y - \boxed{} = \boxed{}x + \boxed{}$

$\boxed{}x + y = \boxed{}$

Method 2: Use the slope-intercept form and substitute the slope and point into the equation to solve for b.

$\boxed{} = \boxed{} \cdot \boxed{} + b$	This gives the equation $y = \boxed{}x + \boxed{}$.
$\boxed{} = \boxed{} + b$	Rewrite in standard form as in Method 1:
$\boxed{} = b$	$\boxed{}x + y = \boxed{}$

Reflect

3. **Discussion** Which method do you prefer? Why?

Your Turn

Write an equation in standard form for each line.

4. Slope is -2, and $(-7, -10)$ is on the line.

5. Slope is 4, and $(-3, 0)$ is on the line.

Explain 2 Creating Linear Equations in Standard Form Given Two Points

You can use two points on a line to create an equation of the line in standard form.

Example 2 **Write an equation in standard form for each line.**

A $(-2, -1)$ and $(0, 4)$ are on the line.

Find the slope using the given points.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - (-1)}{0 - (-2)} = \frac{5}{2}$$

Substitute the slope and the coordinates of either of the given points in the point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - 4 = \frac{5}{2}(x - 0)$$

$$y - 4 = \frac{5}{2}x$$

Rewrite in standard form.

$$2y - 8 = 5x$$

$$5x - 2y = -8$$

B $(5, 2)$ and $(3, -6)$ are on the line.

Find the slope.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\boxed{} - \boxed{}}{\boxed{} - \boxed{}} = \frac{\boxed{}}{\boxed{}} = \boxed{}$$

Substitute the slope and the coordinates of either of the given points in the point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - \boxed{} = \boxed{}\left(x - \boxed{}\right)$$

$$y - \boxed{} = \boxed{}x - \boxed{}$$

Or:

$$y - \boxed{} = \boxed{}\left(x - \boxed{}\right)$$

$$y + \boxed{} = \boxed{}x - \boxed{}$$

Rewrite in standard form.

Reflect

6. Why does it not matter which of the two given points you use in the point-slope form?

Your Turn

Write an equation in standard form for each line.

7. $(4, -7)$ and $(2, -3)$ are on the line.

8. $(1, 5)$ and $(-10, -6)$ are on the line.

Explain 3 Creating Linear Models in Standard Form

Equations in standard form can be used to model real-world linear situations.

Example 3 **Write an equation in standard form to model the linear situation.**

(A) A tank is filling up with water at a rate of 3 gallons per minute. The tank already had 3 gallons in it before it started being filled.

Let x represent the time in minutes since the filling began and y represent the amount of water in gallons. Since 3 gallons were in the tank before filling started, the point $(0, 3)$ is on the line.

The rate is 3 gallons per minute, so $m = 3$.

Substitute the slope and the coordinates of the point in the point-slope form and rewrite in standard form.

$y - 3 = 3(x - 0)$

$y - 3 = 3x$

$3x - y = -3$

(B) A hot tub filled with 440 gallons of water is being drained. After 1.5 hours, the amount of water had decreased to 320 gallons.

The initial amount of water in the hot tub was 440 gallons, so $(0, 440)$ is on the line.

After 1.5 hours, the amount of water had decreased to 320 gallons, so $(1.5, 320)$ is on the line.

Use the given information to find the slope.

$$m = \frac{\boxed{} - \boxed{}}{\boxed{} - \boxed{}} = \frac{\boxed{}}{\boxed{}} = \boxed{}$$

Substitute the slope and the coordinates of one of the points in the point-slope form, and rewrite in standard form.

$y - \boxed{} = \boxed{}\left(x - \boxed{}\right)$

$y - \boxed{} = \boxed{}x$

$\boxed{}$

Reflect

9. Your school sells adult and student tickets to a school play. Adult tickets cost $15 and student tickets cost $4. The total value of all the tickets sold is $7000. How could you write an equation in standard form to describe the linear situation?

Your Turn

Write an equation in standard form to model the linear situation.

10. A tank is being filled with gasoline at a rate of 4.5 gallons per minute. The gas tank contained 1.5 gallons of gasoline before filling started.

11. A pool that is being drained contained 18,000 gallons of water. After 2 hours, 12,500 gallons of water remain.

Elaborate

12. Describe a method other than the one given in the example for writing an equation in standard form given two points on a line.

13. Why might you choose the standard form of an equation over another form?

14. **Essential Question Check-In** When writing a linear equation in standard form, what other forms might you need to use?

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

Identify the form of each equation.

1. **a.** $5x + 4y = 8$ **b.** $y - 3 = 8(x - 2)$

c. $y = 3x + 6$ **d.** $2x - 3y = -7$

Rewrite each equation in standard form.

2. $y = 6x - 4$

3. $y - 2 = -(x + 7)$

4. $y = \frac{4}{3}x - \frac{2}{3}$

5. $y - 4 = \frac{7}{3}(x - 3)$

Use the information given to write an equation in standard form.

6. Slope is 3, and (1, 4) is on the line.

7. Slope is -2, and (4, 3) is on the line.

8. Slope is -3, and (0, -4) is on the line.

9. Slope is 0, and (0, 5) is on the line.

10. Slope is $\frac{4}{7}$, and (1, 3) is on the line.

11. Slope $= -\frac{3}{2}$, and (2, 3) is on the line.

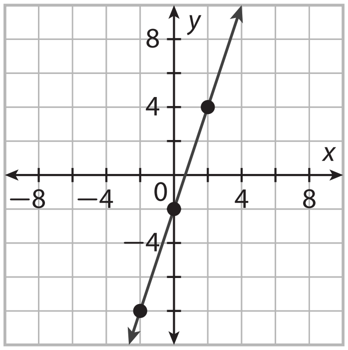
12. (-1, 1) and (0, 4) are on the line.

13. (6, 11) and (5, 9) are on the line.

14. (2, -5) and (-1, 1) are on the line.

15. (25, 34) and (35, 50) are on the line.

16. Use the information on the graph to write an equation in standard form.



Write an equation in standard form to model the linear situation.

17. A bathtub that holds 32 gallons of water contains 12 gallons of water. You begin filling it, and after 5 minutes, the tub is full.

18. A barrel of oil was filled at a constant rate of 7.5 gal/min. The barrel had 10 gallons before filling began.

19. **Represent Real-World Situations**
A restaurant needs to plan seating for a party of 150 people. Large tables seat 10 people and small tables seat 6. Let x represent the number of large tables and y represent the number of small tables. An expression like the total number of people you can seat using A large tables and B small tables is called a *linear combination*. For instance, 150 people could be seated using 12 large tables and 5 small tables. Use that expression to write an equation in standard form that models all the different combinations of tables the restaurant could use. Then identify at least one possible combination of tables other than $(12, 5)$.

20. Match each equation with an equivalent equation in standard form.

a. $y = \frac{2}{3}x + 3$ ______ $7x + 6y = 6$

b. $6 - y = -5x + 8$ ______ $5x - y = 2$

c. $y - 3 = 4(x - 3)$ ______ $2x - 3y = -9$

d. $-\frac{7}{6}x + 1 = y$ ______ $4x - y = 9$

21. **Explain the Error** Cody was given two points $\left(\frac{1}{2}, 4\right)$ and $\left(\frac{2}{3}, 1\right)$, on a line and asked to create a linear equation in standard form. Cody's work is shown. Identify any errors and correct them.

$$m = \frac{1-4}{\frac{2}{3}-\frac{1}{2}} = -\frac{3}{\frac{1}{6}} = -\frac{1}{2}$$

$$y - 1 = -\frac{1}{2}\left(x - \frac{2}{3}\right)$$

$$y - 1 = -\frac{1}{2}x + \frac{1}{3}$$

$$2y - 2 = -x + \frac{2}{3}$$

$$x + 2y = \frac{8}{3}$$

22. **Communicate Mathematical Ideas** In the equation $Ax + By = C$, A and B cannot both be zero. What if only A is zero? What if only B is zero? Explain.

Lesson Performance Task

An airplane takes off with a full tank of 40,000 gallons of fuel and flies at an average speed of 550 miles per hour. After 8 hours in flight, there are 14,000 gallons of fuel left. It will take another 3.5 hours for the plane to reach its destination. How much fuel will be left in the tank when the plane lands? What is the total distance of the flight?

Name ________________ Class __________ Date _______

6.4 Transforming Linear Functions

Essential Question: What are the ways in which you can transform the graph of a linear function?

Resource Locker

Explore 1 Building New Linear Functions by Translating

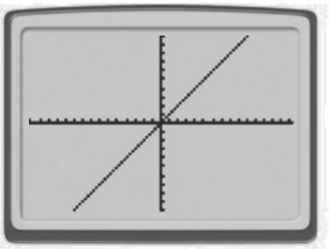
Investigate what happens to the graph of $f(x) = x + b$ when you change the value of b.

(A) Use a graphing calculator. Start with the standard viewing window, which you can obtain by pressing Zoom and selecting ZStandard. Because the distances between consecutive tick marks on the x-axis and on the y-axis are not equal, you can make them equal by pressing Zoom again and selecting ZSquare.

What interval on each axis does the viewing window now show? (Press Window to find out.)

____

(B) Graph the function $f(x) = x$ by pressing Y= and entering the function's rule next to $Y_1 =$. As shown, the graph of the function is a line that makes a 45° angle with each axis.



What are the slope and y-intercept of the graph of $f(x) = x$?

____

(C) Graph other functions of the form $f(x) = x + b$ by entering their rules next to $Y_2 =$, $Y_3 =$, and so on. Be sure to choose both positive and negative values of b. For instance, graph $f(x) = x + 2$ and $f(x) = x - 3$.

What do the graphs have in common? How are they different?

____

Reflect

1. **Discussion** A *vertical translation* moves all points on a figure the same distance either up or down. Use the idea of a vertical translation to describe what happens to the graph of $f(x) = x + b$ when you increase the value of b and decrease the value of b.

____

____

2. In this Explore, we replaced the linear function $f(x)$ by $f(x) + k$. Show how replacing $f(x)$ by $f(x + k)$ has exactly the same effect.

____

Explore 2 Building New Linear Functions by Stretching, Shrinking, or Reflecting

Investigate what happens to the graph of $f(x) = mx$ when you change the value of m.

(A) Use a graphing calculator. Press Y= and clear out all but the function $f(x) = x$ from the previous Explore Activity. Then graph other functions of the form $f(x) = mx$ by entering their rules next to $Y_2 =$, $Y_3 =$, and so on. Use only values of m that are greater than 1. For example, graph $f(x) = 2x$ and $f(x) = 6x$.

What do the graphs have in common? How are they different?

As the value of m increases from 1, does the graph become steeper or less steep?

(B) Again, press Y= and clear out all but the function $f(x) = x$. Then graph other functions of the form $f(x) = mx$ by entering their rules next to $Y_2 =$, $Y_3 =$, and so on. This time use only values of m that are less than 1 but greater than 0. For instance, graph $f(x) = 0.5x$ and $f(x) = 0.2x$.

As the value of m decreases from 1 to 0, does the graph become steeper or less steep?

(C) Again, press Y= and clear out all but the function $f(x) = x$. Then graph the function $f(x) = -x$ by entering its rule next to $Y_2 =$.

What are the slope and y-intercept of the graph of $f(x) = -x$?

How are the graphs of $f(x) = x$ and $f(x) = -x$ geometrically related?

(D) Again, press Y= and clear out all the functions. Graph $f(x) = -x$ by entering its rule next to $Y_1 =$. Then graph other functions of the form $f(x) = mx$ where $m < 0$ by entering their rules next to $Y_2 =$, $Y_3 =$, and so on. Be sure to choose values of m that are less than -1 as well as values of m between -1 and 0.

Describe what happens to the graph of $f(x) = mx$ as the value of m decreases from -1, and as it increases from -1 to 0.

Reflect

3. **Discussion** When $m > 1$, will the graph of $f(x) = mx$ be a *vertical stretch* or a *vertical shrink* of the graph of $f(x) = x$? When $0 < m < 1$, will the graph of $f(x) = mx$ be a *vertical stretch* or a *vertical shrink* of the graph of $f(x) = x$? Explain your answers.

Explore 3 Understanding Function Families

Investigate what happens to the graph of $f(x) = mx$ when you change the value of m.

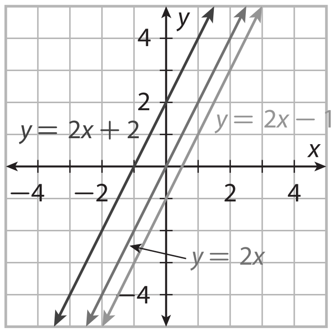
(A) A **family of functions** is a set of functions whose graphs have basic characteristics in common. What do all these variations on the original function $f(x) = x$ have in common?

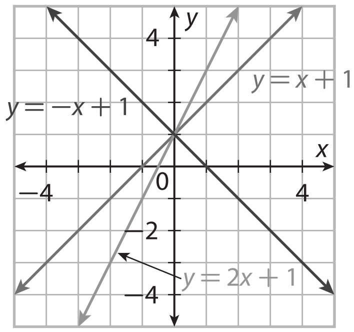
(B) The most basic function of a family of functions is called the **parent function**. What is the parent function of the family of functions explored in the first two Explore Activities?

(C) A **parameter** is one of the constants in a function or equation that determines which variation of the parent function one is considering. For functions of the form $f(x) = mx + b$, what are the two parameters?

Reflect

4. **Discussion** For the family of all linear functions, the parent function is $f(x) = x$, where the parameters are $m = 1$ and $b = 0$. Other examples of families of linear functions are shown below. The example on the left shows a family with the same parameter m and differing parameters b. The example on the right shows a family with the same parameter b and differing parameters m.





Describe the parameter that is left unchanged in the equations of the lines in the first graph.

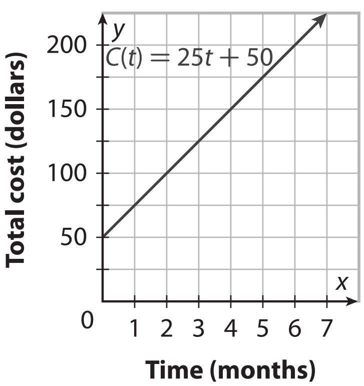
Explain 1 Interpreting Parameter Changes in Linear Models

Many real-world scenarios can be modeled by linear functions. Changes in a particular scenario can be analyzed by making changes in the corresponding parameter of the linear function.

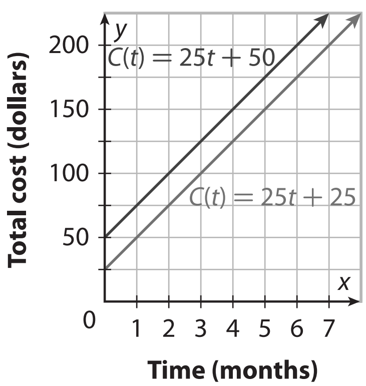
Example 1 **A gym charges a one-time new member fee of \$50 and then a monthly membership fee of \$25. The total cost C of being a member of the gym is given by the function $C(t) = 25t + 50$, where t is the time (in months) since joining the gym. For each situation described below, sketch a graph using the given graph of $C(t) = 25t + 50$ as a reference.**

(A) The gym decreases its one-time fee for new members.

What change did you make to the graph of $C(t) = 25t + 50$ to represent a lower one-time fee?



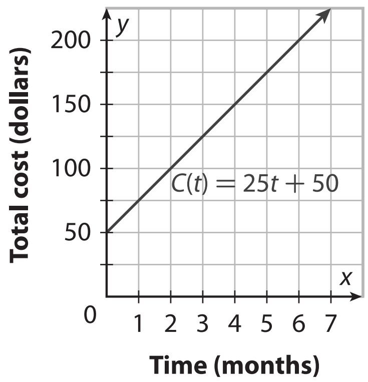
I decreased the y-intercept but the slope remained the same.



(B) The gym increases its monthly membership fee.

What change did you make to the graph of $C(t) = 25t + 50$ to represent an increased monthly fee?

I increased the slope but the y-intercept remained the same.



Reflect

5. Suppose the gym increases its one-time new member fee *and* decreases its monthly membership fee. Describe how you would alter the graph of $C(t) = 25t + 50$ to illustrate the new cost function.

Your Turn

Determine what will happen to each parent function when the described changes occur.

6. Once a year the gym offers a special in which the one-time fee for joining is waived for new members. What impact does this special offer have on the graph of the original function $C(t) = 25t + 50$?

7. Suppose the gym increases its one-time joining fee *and* decreases its monthly membership fee. Does this have any impact on the domain of the function? Does this have any impact on the range of the function? Explain your reasoning.

Elaborate

8. How do changes to m in the equation $f(x) = mx$ affect the graph of the equation?

9. How do changes to b in the equation $f(x) = x + b$ affect the graph of the equation?

10. Which parameter causes the steepness of the graph of the line to change for the family of linear functions of the form $f(x) = mx + b$?

11. **Essential Question Check-In** What are the different types of transformations?

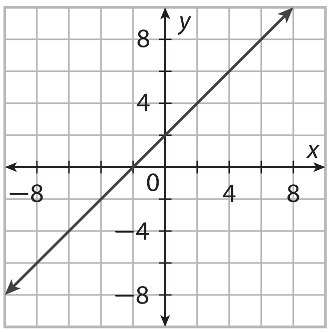
Evaluate: Homework and Practice

Personal Math Trainer

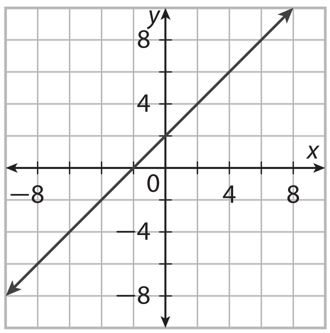
- Online Homework
- Hints and Help
- Extra Practice

In Exercises 1–4, the graph of $f(x) = x + 2$ is graphed.

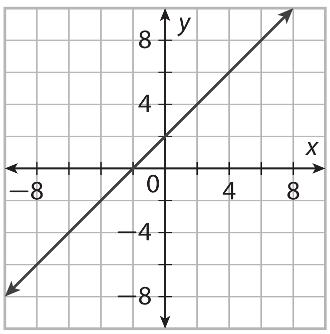
1. Graph two more functions in the same family for which the parameter being changed is the y-intercept, b.



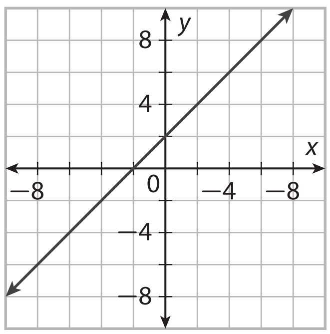
2. Graph two more functions in the same family for which the parameter being changed is the slope, m, and is greater than 1.



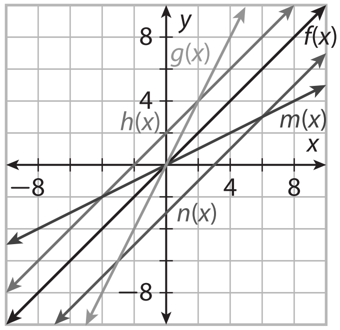
3. Graph two more functions in the same family for which the parameter being changed is the slope, m, and is between 0 and 1.



4. Graph two more functions in the same family for which the parameter being changed is the slope, m, and is less than 0.



5. The graph of the parent linear function $f(x) = x$ is shown in black on the coordinate grid. Write the function that represents this function with the indicated parameter changes.



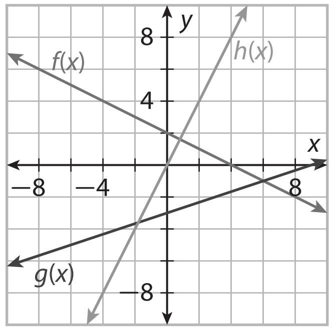
a. m increased, b unchanged ________

b. m decreased, b unchanged ________

c. m unchanged, b increased ________

d. m unchanged, b decreased, ________

6. For each linear function graphed on the coordinate grid, state the value of m and the value of b.



a. $f(x)$: $m =$ ______, $b =$ ______

b. $g(x)$: $m =$ ______, $b =$ ______

c. $h(x)$: $m =$ ______, $b =$ ______

Describe the transformation(s) on the graph of the parent function $f(x) = x$ that results in the graph of $g(x)$.

7. $g(x) = -x + 9$

8. $g(x) = 3x$

9. $g(x) = \frac{1}{4}x$

10. $g(x) = 7x - 8$

11. $g(x) = -\frac{3}{4}x + 5$

Use the parent function and the description of the transformation to write the new function.

12. Rotate the graph of $f(x) = -x + 2$ until it has the same steepness in the opposite direction.

13. Reflect the graph of $f(x) = x - 1$ across the y-axis, and then translate it 4 units down.

Determine how changes in parameters will affect a graph. Write the new function.

14. For large parties, a restaurant charges a reservation fee of \$25, plus \$15 per person. The total charge for a party of x people is $f(x) = 15x + 25$. How will the graph of this function change if the reservation fee is raised to \$50 and if the per-person charge is lowered to \$12?

15. The number of chaperones on a field trip must include 1 teacher for every 4 students, plus a total of 2 parents. The function describing the number of chaperones for a trip of x students is $f(x) = \frac{1}{4}x + 2$. How will the graph change if the number of parents is reduced to 0? If the number of teachers is raised to 1 for every 3 students?

16. A satellite dish company charges a one-time installation fee of \$75 and then a monthly usage charge of \$40. The total cost C of using that satellite service is given by the function $C(t) = 40t + 75$, where t is the time (in months) since starting the service. For the situation given below, describe the new function using the graph of $C(t) = 40t + 75$ as a reference.

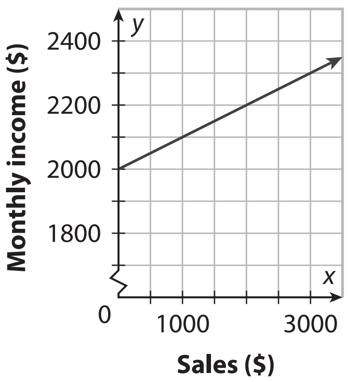
a. The satellite dish company reduces its one-time installation fee to \$60. What change would you make to the graph of $C(t) = 40t + 75$ to obtain the new graph?

b. The satellite dish company decreases its monthly fee to \$30. What change would you make to the graph of $C(t) = 40t + 75$ to obtain the new graph?

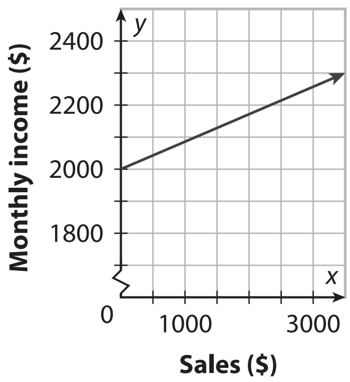
c. What is the new function with both changes?

17. A salesperson earns a base monthly salary of \$2000 plus a 10% commission on sales. The salesperson's monthly income I (in dollars) is given by the function $I(s) = 0.1s + 2000$, where s is the sales (in dollars) that the salesperson makes. Sketch a graph to illustrate each situation using the graph of $I(s) = 0.1s + 2000$ as a reference.

A. The salesperson's base salary is increased.

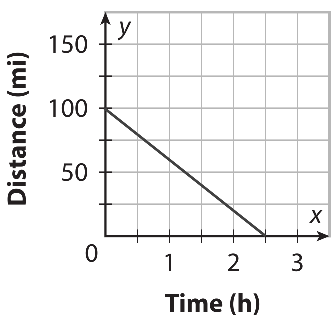


B. The salesperson's commission rate is decreased.

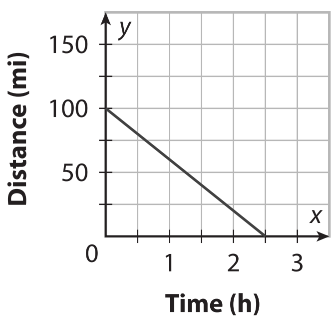


18. Mr. Resnick is driving at a speed of 40 miles per hour to visit relatives who live 100 miles away from his home. His distance d (in miles) from his destination is given by the function $d(t) = 100 - 40t$, where t is the time (in hours) since his trip began. Sketch a graph to illustrate each situation. The graphs shown already represent the function $d(t)$.

a. He increases his speed to get to the destination sooner. (Hint: His distance from the destination decreases faster.)



b. His starting distance from the destination is increased because a detour forces him to take a longer route.



c. Give an example of another linear function within the same family of functions as $d(t) = 100 - 40t$. Explain the meaning of each parameter in your example.

19. A book club charges a membership fee of \$20 and then \$12 for each book purchased.

a. Write a function to represent the cost y of membership in the club based on the number of books purchased x.

b. Write a second function to represent the cost of membership if the club raises its membership fee to \$30.

c. Describe the relationship between the functions from parts A and B.

20. Match each effect on a graph with the appropriate change in m. The *steepness* of a line refers to the absolute value of its slope. The greater the absolute value of the slope, the steeper the line. Complete the table to summarize, in terms of steepness, the effect of changing the value of m on the graph of $f(x) = mx$.

How the Value of m Changes	Effect on the Graph of $f(x) = mx$
A. Increase m when $m > 1$.	______ Graph becomes steeper.
B. Decrease m when $0 < m < 1$.	______ Graph becomes less steep.
C. Decrease m when $m < -1$.	______ Graph becomes steeper.
D. Increase m when $0 > m > -1$.	______ Graph becomes less steep.

H.O.T. Focus on Higher Order Thinking

21. Explain the Error A student is asked to explain what happens with each of the parameters for the following situation.

It costs a player \$20 up front to join a basketball league and then \$5 a week to play. If the cost to join the league is reduced to \$19 and the weekly fee increases to \$6 a week, what will happen to the function of the graph?

The student says that the graph will shift up because the value of b has increased and then the graph will become less steep because the value of m has decreased.

Explain what the student has done incorrectly.

22. Critique Reasoning Geoff says that changing the value of m while leaving b unchanged in $f(x) = mx + b$ has no impact on the intercepts of the graph. Marcus disagrees with this statement. Who is correct? Explain your reasoning.

23. Multiple Representations The graph of $y = x + 3$ is a vertical translation of the graph of $y = x + 1$, 2 units upward. Examine the intercepts of both lines and state another way that the geometric relationship between the two graphs can be described.

24. Critique Reasoning Stephanie says that the graphs of $y = 3x + 2$ and $y = 3x - 2$ are parallel. Isabella says that the graphs are perpendicular. Who is correct? Explain your reasoning.

25. Critical Thinking It has been shown that the graph of $g(x) = x + 3$ is the result of translating the graph of $f(x) = x$ three units up. However, this can also be thought of as a horizontal translation—that is, a translation left or right. Describe the horizontal translation of $f(x) = x$ to get the graph of $g(x) = x + 3$.

Lesson Performance Task

High-demand cars that are also in low supply tend to retain their value better than other cars. The data in the table is for a car that won a resale value award.

Year	1	3	5
Value (%)	84	64	44

a. Write a function to represent the change in the percentage of the car's value over time. Assume that the function is linear for the first 5 years.

b. According to the model, by what percent did the car's value drop the day it was purchased and driven off the lot?

c. Would the linear model be useful after 10 years? Explain why or why not.

d. Suppose months were used instead of years to write the function. How would the model change? What is the relationship of the new function to the original function?

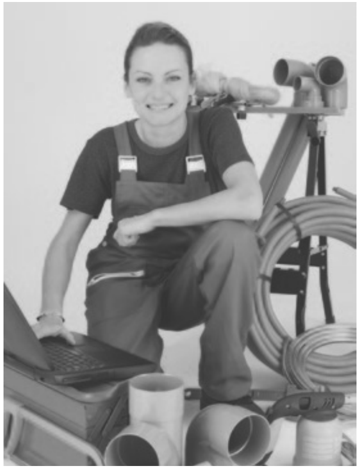
Name ______________________ Class ______________ Date ________

6.5 Comparing Properties of Linear Functions

Essential Question: How can you compare linear functions that are represented in different ways?

Resource Locker

Explore Comparing Properties of Linear Functions Given Algebra and a Description



Comparing linear relationships can involve comparing relationships that are expressed in different ways.

Dan's Plumbing and Kim's Plumbing have different ways of charging their customers. The function $D(t) = 35t$ represents the total amount in dollars that Dan's Plumbing charges for t hours of work. Kim's Plumbing charges \$35 per hour plus a \$40 flat-rate fee.

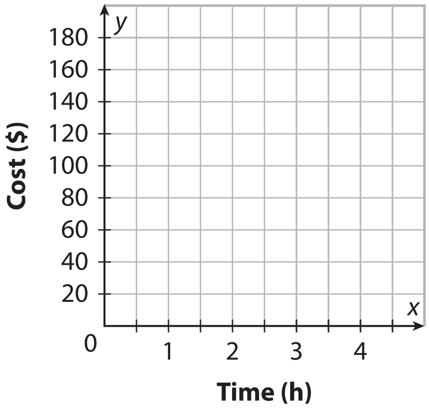
(A) Define a function $K(t)$ that represents the total amount Kim's Plumbing charges for t hours of work and then complete the tables.

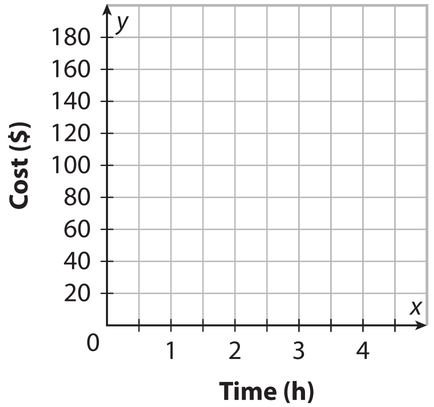
Cost for Dan's Plumbing		
t	$D(t) = 35t$	$(t, D(t))$
0		
1		
2		
3		

Cost for Kim's Plumbing		
t	$K(t) =$	$(t, K(t))$
0		
1		
2		
3		

(B) What domain and range values for the functions $D(t)$ and for $K(t)$ are reasonable in this context? Explain.

(C) Graph the two cost functions for the appropriate domain values.





(D) Compare the graphs. How are they alike? How are they different?

Reflect

1. **Discussion** What information could be found about the two functions without changing their representation?

Explain 1 Comparing Properties of Linear Functions Given Algebra and a Table

A table and a rule are two ways that a linear relationship may be expressed. Sometimes it may be helpful to convert one representation to the other when comparing two relationships. There are other times when comparisons are possible without converting either representation.

Example 1 **Compare the initial value and the range for each of the linear functions $f(x)$ and $g(x)$.**

(A) The domain of each function is the set of all real numbers x such that $5 \le x \le 8$. The table shows some ordered pairs for $f(x)$. The function $g(x)$ is defined by the rule $g(x) = 3x + 7$.

x	$f(x)$
5	20
6	24
7	28
8	32

The initial value is the output that is paired with the least input. The least input for $f(x)$ and $g(x)$ is 5.

The initial value of $f(x)$ is $f(5) = 20$.

The initial value of $g(x)$ is $g(5) = 3(5) + 7 = 22$.

Since $f(x)$ is a linear function and its domain is the set of all real numbers from 5 to 8, its range will be the set of all real numbers from $f(5)$ to $f(8)$. Since $f(5) = 20$ and $f(8) = 32$, the range of $f(x)$ is the set of all real numbers such that $20 \le f(x) \le 32$.

Since $g(x)$ is a linear function and its domain is the set of all real numbers from 5 to 8, its range will be the set of all real numbers from $g(5)$ to $g(8)$. Since $g(5) = 22$ and $g(8) = 3(8) + 7 = 31$, the range of $g(x)$ is the set of all real numbers such that $22 \le g(x) \le 31$.

(B) The domain of each function is the set of all real numbers x such that $6 \leq x \leq 10$. The table shows some ordered pairs for $f(x)$. The function $g(x)$ is defined by the rule $g(x) = 5x + 11$.

x	$f(x)$
6	36
7	42
8	48
9	54
10	60

The initial value is the output that is paired with the least input. The least input for $f(x)$ and $g(x)$ is ____.

The initial value of $f(x)$ is ____________________.

The initial value of $g(x)$ is ____________________.

Since $f(x)$ is a linear function, and its domain is the set of all real numbers from ____ to 10, its range will be the set of all real numbers from $f(\square)$ to $f(10)$. Since $f(\square) = \square$ and $f(10) = \square$, the range of $f(x)$ is the set of all real numbers such that $\square \leq f(x) \leq \square$.

Since $g(x)$ is a linear function and its domain is the set of all real numbers from ____ to 10, its range will be the set of all real numbers from $g(\square)$ to $g(10)$. Since $g(\square) = \square$ and $g(10) = 5(\square) + 11 = \square$, the range of $g(x)$ is the set of all real numbers such that $\square \leq g(x) \leq \square$.

Reflect

2. Discussion How can you use a table of values to find the rate of change for a linear function?

____

____

Your Turn

3. Find the rate of change for the linear function $f(x)$ that is shown in the table.

x	$f(x)$
3	22
4	29
5	36
6	43
7	50

4. The rule for $f(x)$ in Example 1B is $f(x) = 6x$. If the domains were extended to all real numbers, how would the slopes and y-intercepts of $f(x)$ and $g(x) = 5x + 11$ in Example 1B compare?

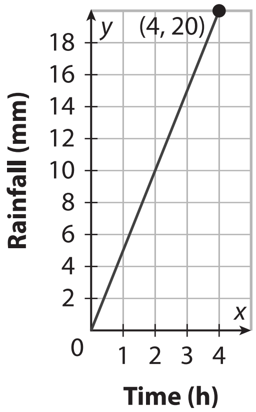
Explain 2 Comparing Properties of Linear Functions Given a Graph and a Description

Information about a linear relationship may have to be inferred from the context given in the problem.

Example 2 **Write a rule for each function, and then compare their domain, range, slope, and *y*-intercept.**

(A) A rainstorm in Austin lasted for 3.5 hours, during which time it rained a steady rate of 4.5 mm per hour. The function $A(t)$ represents the amount of rain that fell in t hours.

The graph shows the amount of rain that fell during the same rainstorm in Dallas, $D(t)$ (in millimeters), as a function of time t (in hours).



Write a rule for each function. $A(t) = 4.5t$ for $0 \leq t \leq 3.5$

The line representing $D(t)$ has endpoints at $(0, 0)$ and $(4, 20)$. The slope of $D(t)$ is $\frac{20 - 0}{4 - 0} = 5$. The y-intercept is 0, so substituting 5 for m and 0 for b in $y = mx + b$ produces the equation $y = 5x$. This can be represented by the function $D(t) = 5t$, for $0 \leq t \leq 4$.

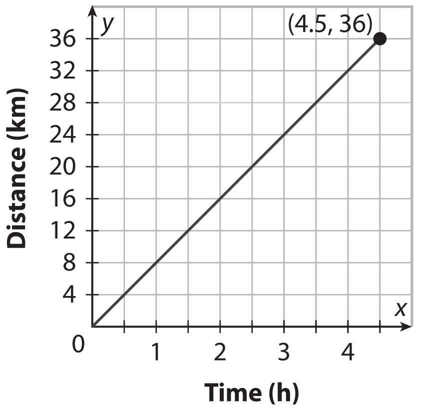
The domains of each function both begin at 0 but end for different values of t, because the lengths of time that it rained in Austin and Dallas were not the same.

The range for $A(t)$ is $0 \leq A(t) \leq 15.75$. The range for $D(t)$ is $0 \leq A(t) \leq 20$.

The slope for $D(t)$ is 5, which is greater than the slope for $A(t)$, which is 4.5.

The y-intercepts of both functions are 0.

B One group of hikers hiked at a steady rate of 6.5 kilometers per hour for 4 hours. The function $f(t)$ represents the distance this group of hikers hiked in t hours.



The graph shows the distance a second group of hikers hiked, $g(t)$ (in kilometers), as a function of t (in hours).

Write a rule for each function.

$f(t) = \boxed{}$ for $\boxed{} \le t \le \boxed{}$

The line representing $g(t)$ has endpoints at $(0, 0)$ and $\left(\boxed{}, \boxed{}\right)$. The slope of $g(t)$ is $\dfrac{\boxed{} - 0}{\boxed{} - 0} = \boxed{}$.

The y-intercept is ____, so substituting ____ for m and ____ for b in $y = mx + b$ produces the equation $y = \boxed{}$. This can be represented by the function $g(t) = \boxed{}$ for $\boxed{} \le t \le \boxed{}$.

The domains of each function both begin at __________ and end at __________ values of t.

The range for $f(t)$ is $\boxed{} \le f(t) \le \boxed{}$ and the range for $g(t)$ is $\boxed{} \le g(t) \le \boxed{}$.

The slope for ______ is greater than the slope for ______.

The y-intercepts are ________.

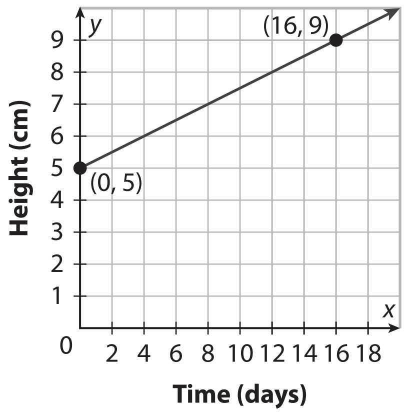
Reflect

5. What is the meaning of the y-intercepts for the functions $A(t)$ and $D(t)$ in Example 2A?

Your Turn

6. An experiment compares the heights of two plants over time. A plant was 5 cm tall at the beginning of the experiment and grew 0.3 centimeters each day. The function $f(t)$ represents the height of the plant (in centimeters) after t days. The graph shows the height of the second plant, $g(t)$ (in centimeters), as a function of time t (in days).

Find the rate of change $g(t)$ and compare it to the rate of change for $f(t)$.



Elaborate

7. When would representing a linear function by a graph be more helpful than by a table?

8. When would representing a linear function by a table be more helpful than by a graph?

9. **Essential Question-Check-In** How can you compare a linear function represented in a table to one represented as a graph?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Compare the initial value and the range for each of the linear functions $f(x)$ and $g(x)$.

1. The domain of each function is the set of all real numbers x such that $2 \leq x \leq 5$. The table shows some ordered pairs for $f(x)$. The function $g(x)$ is defined by the rule $g(x) = x + 6$.

x	$f(x)$
2	5
3	7
4	9
5	11

2. The domain of each function is the set of all real numbers x such that $8 \leq x \leq 12$. The table shows some ordered pairs for $f(x)$. The function $g(x)$ is defined by the rule $g(x) = 7x - 3$.

x	$f(x)$
8	34
9	38
10	42
11	46
12	50

3. The domain of each function is the set of all real numbers x such that $-4 \leq x \leq -1$. The function $f(x)$ is defined by the rule $f(x) = 2x + 9$. The table shows some ordered pairs for $g(x)$.

x	$g(x)$
−4	10
−3	9
−2	8
−1	7

4. The domain of each function is the set of all real numbers x such that $0 \leq x \leq 4$. The function $f(x)$ is defined by the rule $f(x) = -3x + 15$. The table shows some ordered pairs for $g(x)$.

x	$g(x)$
0	23
1	19
2	15
3	11
4	7

5. The domain of each function is the set of all real numbers x such that $10 \leq x \leq 13$. The table shows some ordered pairs for $f(x)$. The function $g(x)$ is defined by the rule $g(x) = \frac{1}{2}x + 12$.

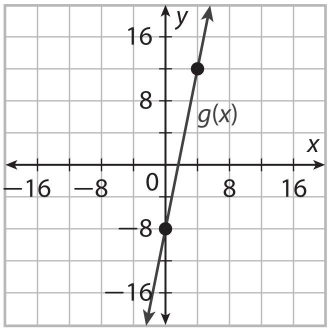
x	$f(x)$
10	22
11	$\frac{47}{2}$
12	25
13	$\frac{53}{2}$

6. The domain of each function is the set of all real numbers x such that $2 \leq x \leq 6$. The function $f(x)$ is defined by the rule $f(x) = -\frac{3}{4}x + 10$. The table shows some ordered pairs for $g(x)$.

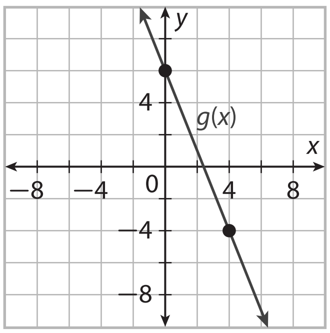
x	$g(x)$
2	14
3	$\frac{51}{4}$
4	$\frac{23}{2}$
5	$\frac{41}{4}$
6	9

Write a rule for each function *f* and *g*, and then compare their domains, ranges, slopes, and *y*-intercepts.

7. The function $f(x)$ has a slope of 6 and has a y-intercept of 20. The graph shows the function $g(x)$.



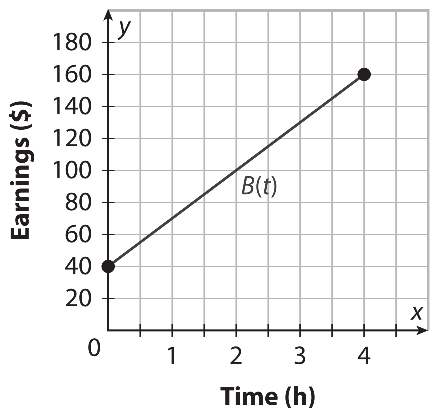
8. The function $f(x)$ has a slope of -3 and has a y-intercept of 5. The graph shows the function $g(x)$.



Write a rule for each function, and then compare their domains, ranges, slopes, and *y*-intercepts.

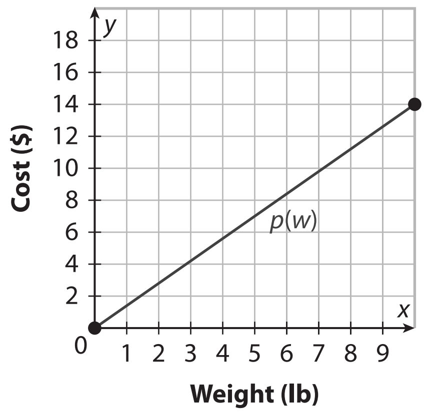
9. Jeff, an electrician, had a job that lasted 5.5 hours, during which time he earned \$32 per hour and charged a \$25 service fee. The function $J(t)$ represents the amount Jeff earns in t hours.

Brendan also works as an electrician. The graph of $B(t)$ shows the amount in dollars that Brendan earns as a function of time t in hours.



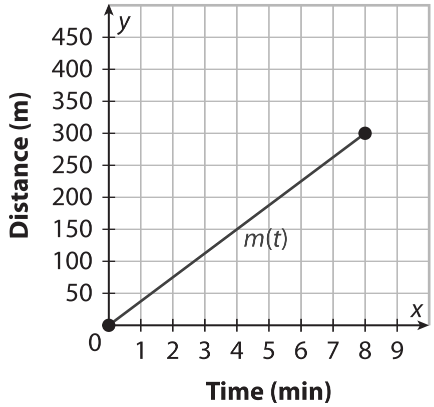
10. Apples can be bought at a farmer's market up to 10 pounds at a time, where each pound costs \$1.10. The function $a(w)$ represents the cost of buying w pounds of apples.

The graph of $p(w)$ shows the cost in dollars of buying w pounds of pears.



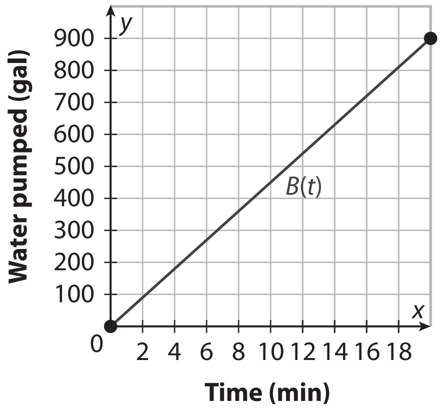
11. **Biology** A gecko travels for 6 minutes at a constant rate of 19 meters per minute. The function $g(t)$ represents the distance the gecko travels after t minutes.

The graph of $m(t)$ shows the distance in meters that a mouse travels after t minutes.



12. Cindy is buying a water pump. The box for Pump A claims that it can move 48 gallons per minute. The function $A(t)$ represents the amount of water (in gallons) Pump A can move after t minutes.

The graph of $B(t)$ shows the amount of water in gallons that Pump B can move after t minutes.



13. Erin is comparing two rental car companies for an upcoming trip. The function $A(d) = 0.20d$ represents the total amount in dollars of driving a car d miles from company A. Company B charges \$0.10 per mile and a \$10 fee.

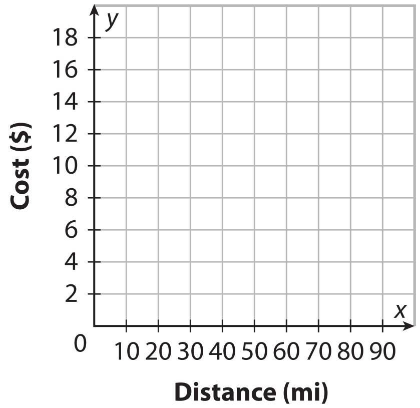
a. Define a function $B(d)$ that represents the total amount company B charges for driving d miles and then complete the tables.

Cost for Company A		
d	$A(d) = 0.20d$	$(d, A(d))$
0		
20		
40		

Cost for Company B		
d	$B(d) =$	$(d, B(d))$
0		
20		
40		

b. Graph and label the two cost functions for all appropriate domain values.

c. Compare the graphs. How are they alike? How are they different?



14. Snow is falling in two cities. The function $C(t) = 2t + 8$ represents the amount of snow on the ground, in centimeters, in Carlisle t hours after the snowstorm begins. There was 8 cm of snow on the ground in York when the storm began and the snow accumulates at 1.5 cm per hour.

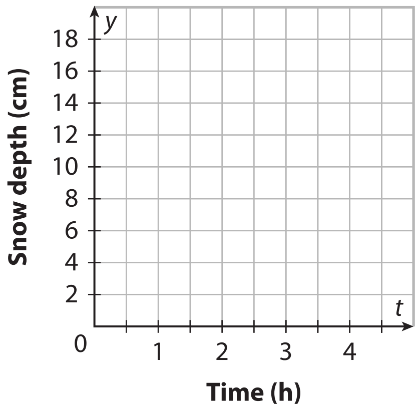
a. Define a function $Y(t)$ that represents the amount of snow on the ground after t hours in York and then complete the tables.

Carlisle		
t	$C(t) = 2t + 8$	$(t, C(t))$
0		
1		
2		

York		
t	$Y(t) =$	$(t, Y(t))$
0		
1		
2		

b. Graph and label the two cost functions for all appropriate domain values.

c. Compare the graphs. How are they alike? How are they different?



15. Gillian works from 20 to 30 hours per week during the summer. She earns \$12.50 per hour. Her friend Emily also has a job. Her pay for t hours each given is given by the function $e(t) = 13t$, where $15 \leq t \leq 25$.

a. Find the domain and range of each function.

b. Compare their hourly wages and the amount they earn per week.

16. The function $A(p)$ defined by the rule $A(p) = 0.13p + 15$ represents the cost in dollars of producing a custom textbook that has p pages for college A, where $0 < p \leq 500$. The table shows some ordered pairs for $B(p)$, where $B(p)$ represents the cost in dollars of producing a custom textbook that has p pages for college B, where $0 < p \leq 500$. For both colleges, only full pages may be printed.

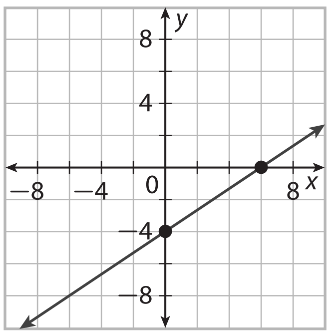
p	$B(p)$
0	24
50	30
100	36
150	42

Compare the domain, range, slope, and y-intercept of the functions. Interpret the comparisons in context.

17. Complete the table so that $f(x)$ is a linear function with a slope of 4 and a y-intercept of 7. Assume the domain includes all real numbers between the least and greatest values shown in the table. Compare $f(x)$ to $g(x) = 4x + 7$ if the range of $g(x)$ is $-1 \leq g(x) \leq 11$.

x	$f(x)$
−2	
−1	
0	
1	

18. Which functions have a rate of change that is greater than the one shown in the graph? Select all that apply.



a. $f(x) = \frac{1}{2}x - 5$

b. $g(x) = -x + 6$

c. $h(x) = \frac{3}{4}x - 9$

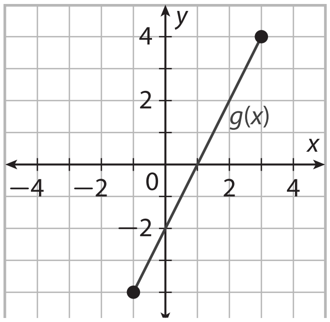
d. $j(x) = -\frac{1}{4}x + 8$

e. $k(x) = x$

19. Does the function $f(x) = 5x + 5$ with the domain $6 \leq x \leq 8$ have the same domain as function $g(x)$, whose only function values are shown in the table? Explain.

x	$g(x)$
6	35
7	40
8	45

20. The linear function $f(x)$ is defined by the table, and the linear function $g(x)$ is shown in the graph. Assume that the domain of $f(x)$ includes all real numbers between the least and greatest values shown in the table.

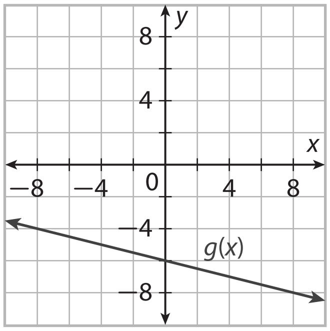


a. Find the domain and range of each function, and compare them.

x	$f(x)$
−1	−7
0	−4
1	−1
2	2
3	5

b. What is the slope of the line represented by each function? What is the y-intercept of each function?

21. The linear function $f(x)$ is defined by $f(x) = -\frac{1}{4}x + 6$ for all real numbers, and the linear function $g(x)$ is shown in the graph.



a. Find the domain and range of each function, and compare them.

b. What is the slope of the line represented by each function? What is the y-intercept of each function?

H.O.T. Focus on Higher Order Thinking

22. Communicate Mathematical Ideas Describe a linear function for which the least value in the range does not occur at the least value of the domain (a function for which the least value in the range is not the initial value.)

23. Draw Conclusions Two linear functions have the same slope, same x-intercept, and same y-intercept. Must these functions be identical? Explain your reasoning.

24. **Draw Conclusions** Let $f(x)$ be a line with slope -3 and y-intercept 0 with domain $\{0, 1, 2, 3\}$, and let $g(x) = \{(0, 0), (1, -1), (2, -4), (3, -9)\}$. Compare the two functions.

25. **Draw Conclusions** Let $f(x)$ be a line with slope 7 and y-intercept -17 with domain $0 \leq x \leq 5$, and let $g(x) = \{(0, -17), (1, -10), (2, -3), (3, 4), (4, 11), (5, 18)\}$. Compare the two functions.

Lesson Performance Task

Lindsay found a new job as an insurance salesperson. She has her choice of two different compensation plans. Plan F was described to her as a \$450 base weekly salary plus a 10% commission on the amount of sales she made that week. The function $f(x)$ represents the amount Lindsay earns in a week when making sales of x dollars with compensation plan F. Plan G was described to her with the graph shown. The function $g(x)$ represents the amount Lindsay earns in a week when making sales of x dollars with compensation plan G.

Write a rule for the functions $f(x)$ and $g(x)$; then identify and compare their domain, range, slope, and y-intercept. Compare the benefits and drawbacks of each compensation plan. Which compensation plan should Lindsay take? Justify your answer.