Name __________ Class __________ Date __________

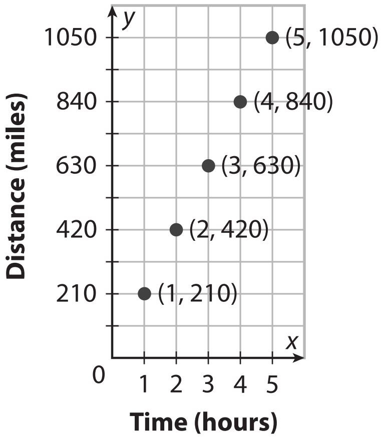
5.1 Understanding Linear Functions

Essential Question: What is a linear function?

Resource Locker

Explore 1 Recognizing Linear Functions

A race car can travel up to 210 mph. If the car could travel continuously at this speed, $y = 210x$ gives the number of miles y that the car would travel in x hours. Solutions are shown in the graph below.



The graph of the car's speed is a function because every x-value is paired with exactly one y-value. Because the graph is a non-vertical straight line, it is also a **linear function.**

Ⓐ Fill in the table using the data points from the graph above.

x	y

Ⓑ Using the table, check that x has a constant change between consecutive terms.

Ⓒ Now check that y has a constant change between consecutive terms.

Ⓓ Using the answers from before, what change in x corresponds to a change in y?

Ⓔ All linear functions behave similarly to the one in this example. Based on this information, a generalization can be made that a ____________ change in x will correspond to a ____________ change in y.

Reflect

1. Discussion Will a non-linear function have a constant change in x that corresponds to a constant change in y?

____

____

2. $y = x^2$ represents a typical non-linear function. Using the table of values, check whether a constant change in x corresponds to a constant change in y.

____

____

____

x	$y = x^2$
1	1
2	4
3	9
4	16
5	25

Explore 2 Proving Linear Functions Grow by Equal Differences Over Equal Intervals

Linear functions change by a constant amount (change by equal differences) over equal intervals. Now you will explore the proofs of these statements. $x_2 - x_1$ and $x_4 - x_3$ represent two intervals in the x-values of a linear function.

It is also important to know that any linear function can be written in the form $f(x) = mx + b$, where m and b are constants.

Complete the proof that linear functions grow by equal differences over equal intervals.

Given: $x_2 - x_1 = x_4 - x_3$

f is a linear function of the form $f(x) = mx + b$.

Prove: $f(x_2) - f(x_1) = f(x_4) - f(x_3)$

Proof:

Statement	Reason
1. $x_2 - x_1 = x_4 - x_3$	Given.
2. $m(x_2 - x_1) = \boxed{}(x_4 - x_3)$	Mult. Property of Equality
3. $mx_2 - \boxed{} = mx_4 - \boxed{}$	________
4. $mx_2 + b - mx_1 - b = mx_4 + \boxed{} - mx_3 - \boxed{}$	________
5. $mx_2 + b - (mx_1 + b) = mx_4 + b - \boxed{}$	________
6. $f(x_2) - f(x_1) = \boxed{}$	Definition of $f(x)$

Reflect

3. Discussion Consider the function $y = x^3$. Use two equal intervals to determine if the function is linear. The table for $y = x^3$ is shown.

x	$y = x^3$
1	1
2	8
3	27
4	64
5	125

4. In the given of the proof it states that: f is a linear function of the form $f(x) = mx + b$. What is the name of the form for this linear function?

Explain 1 Graphing Linear Functions Given in Standard Form

Any linear function can be represented by a linear equation. A **linear equation** is any equation that can be written in the **standard form** expressed below.

Standard Form of a Linear Equation

$Ax + By = C$ where A, B, and C are real numbers and A and B are not both 0.

Any ordered pair that makes the linear equation true is a **solution of a linear equation in two variables**. The graph of a linear equation represents all the solutions of the equation.

Example 1 **Determine whether the equation is linear. If so, graph the function.**

(A) $5x + y = 10$

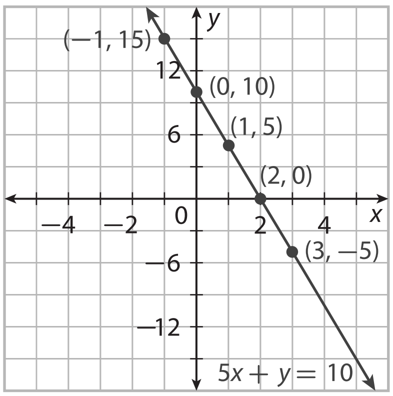
The equation is linear because it is in the standard form of a linear equation:
$A = 5$, $B = 1$, and $C = 10$.

To graph the function, first solve the equation for y.

$$\begin{aligned} 5x + y &= 10 \\ y &= 10 - 5x \end{aligned}$$

Make a table and plot the points. Then connect the points.

x	−1	0	1	2	3
y	15	10	5	0	−5



Note that because the domain and range of functions of a non-horizontal line are all real numbers, the graph is continuous.

(B) $-4x + y = 11$

The equation is linear because it is in the ________ form of a linear equation:

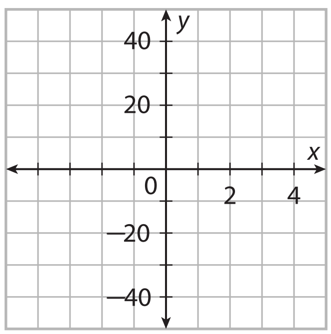
$A =$ ______, $B =$ ______, and $C =$ ______.

To graph the function, first solve the equation for ______.

$$\begin{aligned} -4x + y &= 11 \\ y &= 11 + \underline{\hspace{2cm}}x \end{aligned}$$

Make a table and plot the points. Then connect the points.

x	−4	−2	0	2	4
y					



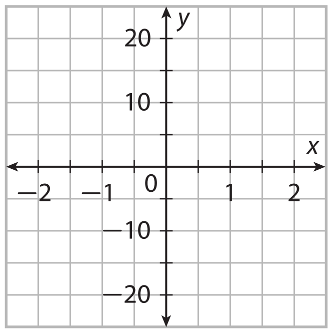
Reflect

5. Write an equation that is linear but is not in standard form.

6. If $A = 0$ in an equation written in standard form, how does the graph look?

Your Turn

7. Determine whether $6x + y = 12$ is linear. If so, graph the function.



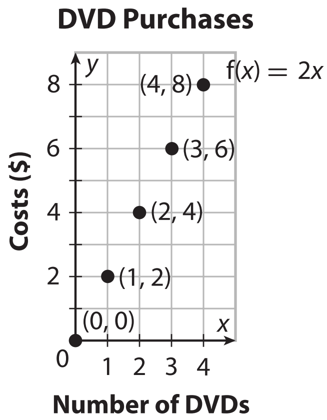
Explain 2 Modeling with Linear Functions

A **discrete function** is a function whose graph has unconnected points, while a **continuous function** is a function whose graph is an unbroken line or curve with no gaps or breaks. For example, a function representing the sale of individual apples is a discrete function because no fractional part of an apple will be represented in a table or a graph. A function representing the sale of apples by the pound is a continuous function because any fractional part of a pound of apples will be represented in a table or graph.

Example 2 **Graph each function and give its domain and range.**

(A) Sal opens a new video store and pays the film studios \$2.00 for each DVD he buys from them. The amount Sal pays is given by $f(x) = 2x$, where x is the number of DVDs purchased.

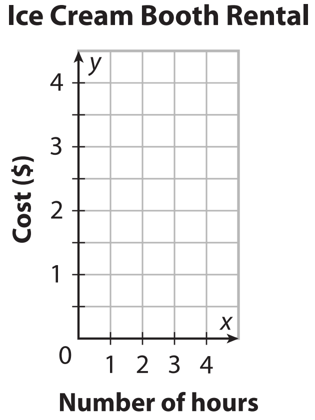
x	$f(x) = 2x$
0	$f(0) = 2(0) = 0$
1	$f(1) = 2(1) = 2$
2	$f(2) = 2(2) = 4$
3	$f(3) = 2(3) = 6$
4	$f(4) = 2(4) = 8$



This is a discrete function. Since the number of DVDs must be a whole number, the domain is $\{0, 1, 2, 3, \ldots\}$ and the range is $\{0, 2, 4, 6, 8 \ \ldots\}$.

(B) Elsa rents a booth in her grandfather's mall to open an ice cream stand. She pays \$1 to her grandfather for each hour of operation. The amount Elsa pays each hour is given by $f(x) = x$, where x is the number of hours her booth is open.

x	$f(x) = x$
0	$f(0) =$
1	$f(1) =$
2	$f(2) =$
3	$f(3) =$
4	$f(4) =$



This is a ____________ function. The domain is ______________________________

and the range is ______________________________.

Reflect

8. Why are the points on the graph in Example 2B connected?

____

____

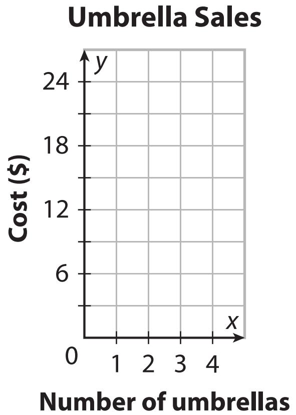
9. **Discussion** How is the graph of the function in Example 2A related to the graph of an arithmetic sequence?

____

____

Your Turn

10. Kristoff rents a kiosk in the mall to open an umbrella stand. He pays \$6 to the mall owner for each umbrella he sells. The amount Kristoff pays is given by $f(x) = 6x$, where x is the number of umbrellas sold. Graph the function and give its domain and range.



Elaborate

11. What is a solution of a linear equation in two variables?

12. What type of function has a graph with a series of unconnected points?

13. **Essential Question Check-In** What is the standard form for a linear equation?

Evaluate: Homework and Practice

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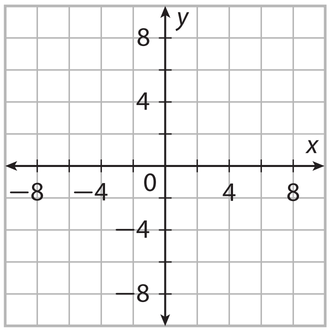
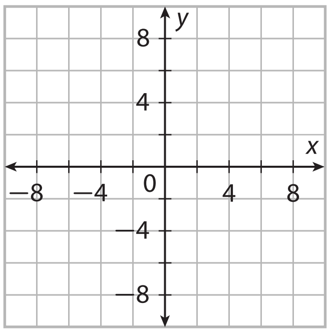
- Online Homework
- Hints and Help
- Extra Practice

Determine if the equation is linear. If so, graph the function.

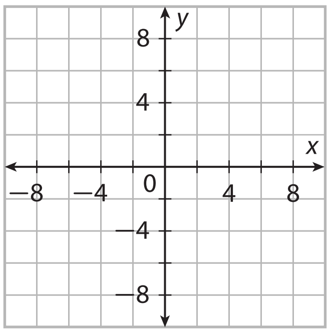
1. $2x + y = 4$



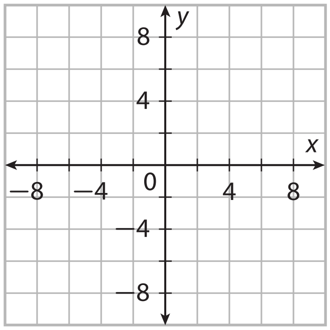
2. $2x^2 + y = 6$



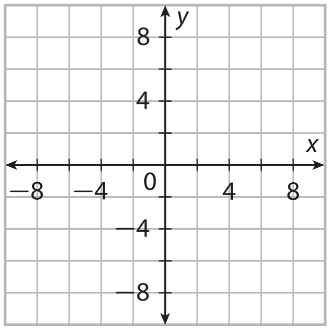
3. $\frac{2}{x} + \frac{y}{4} = \frac{3}{2}$



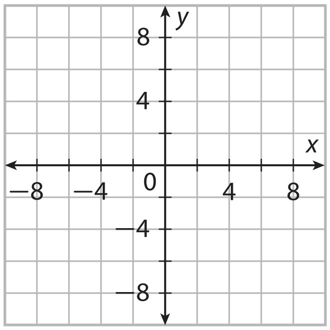
4. $3x + 4y = 8$



5. $x + y^2 = 1$



6. $x + y = 1$



State whether each function is discrete or continuous.

7. The number of basketballs manufactured per day

8. $x = \frac{y}{4}$, where x is the number of hours and y is the miles walked

9. The number of bulls eyes scored for each hour of practice

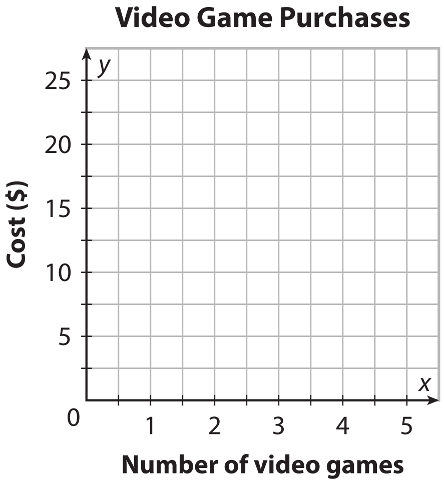
10. $y = 4^4x$, where x is the time and y is gallons of water

11. $y = 35x^1$, where x is distance and y is height

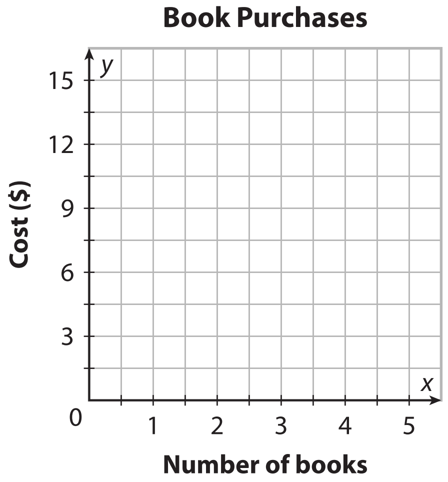
12. The amount of boxes shipped per shift

Graph each function and give its domain and range.

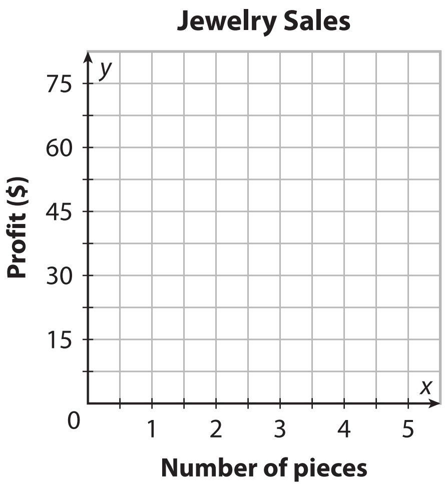
13. Hans opens a new video game store and pays the gaming companies \$5.00 for each video game he buys from them. The amount Hans pays is given by $f(x) = 5x$, where x is the number of video games purchased.



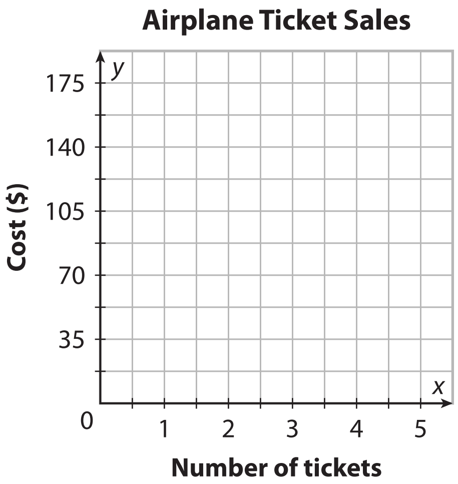
14. Peter opens a new bookstore and pays the book publisher \$3.00 for each book he buys from them. The amount Peter pays is given by $f(x) = 3x$, where x is the number of books purchased.



15. Steve opens a jewelry shop and makes \$15.00 profit for each piece of jewelry sold. The amount Steve makes is given by $f(x) = 15x$, where x is the number of pieces of jewelry sold.



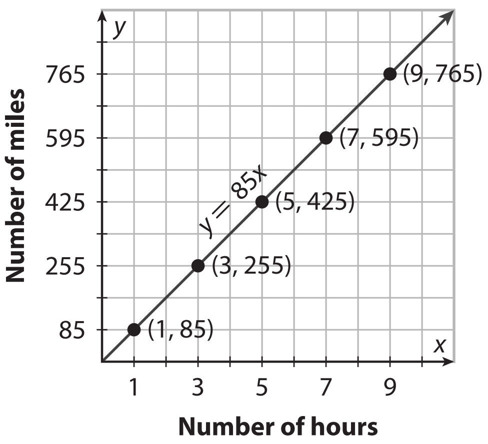
16. Anna owns an airline and pays the airport \$35.00 for each ticket sold. The amount Anna pays is given by $f(x) = 35x$, where x is the number of tickets sold.



17. A hot air balloon can travel up to 85 mph. If the balloon travels continuously at this speed, $y = 85x$ gives the number of miles y that the hot air balloon would travel in x hours.

Fill in the table using the data points from the graph. Determine whether x and y have constant change between consecutive terms and whether they are in a linear function.

x					
y					



18. State whether each function is in standard form.

a. $3x + y = 8$

b. $x - y = 15z$

c. $x^2 + y = 11$

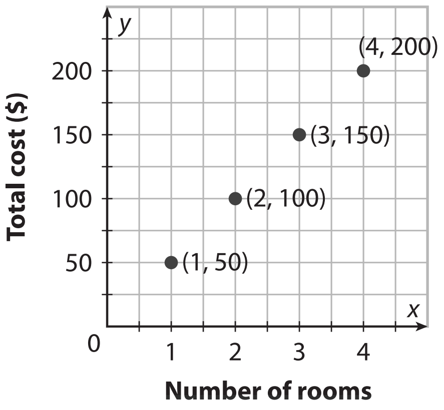
d. $3xy + y^2 = 4$

e. $x + 4y = 12$

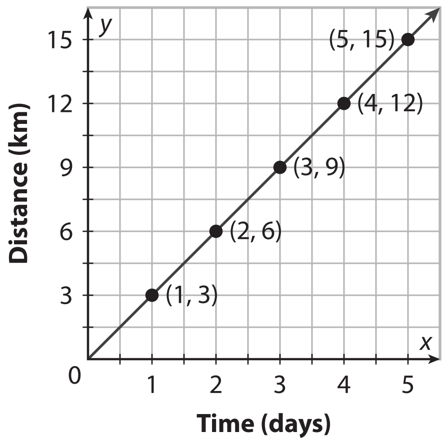
f. $5x + 24y = 544$

19. **Physics** A physicist working in a large laboratory has found that light particles traveling in a particle accelerator increase velocity in a manner that can be described by the linear function $-4x + 3y = 15$, where x is time and y is velocity in kilometers per hour. Use this function to determine when a certain particle will reach 30 km/hr.

20. Travel The graph shows the costs of a hotel for one night for a group traveling. The total cost depends on the number of hotel rooms the group needs. Does the plot follow a linear function? Is the graph discrete or continuous?



21. Biology The migration pattern of a species of tree frog to different swamp areas over the course of a year can be described using the graph below. Fill in the table and express whether this pattern follows a linear function. If the migration pattern is a linear function, express what constant change in y corresponds to a constant change in x.



x	y
1	
2	
3	
4	
5	

H.O.T. Focus on Higher Order Thinking

22. Representing Real-World Problems Write a real-world problem that is a discrete non-linear function.

23. Explain the Error A student used the following table of values and stated that the function described by the table was a linear function. Explain the student's error.

x	−1	0	2	3	4
y	−5	0	5	10	15

24. Communicate Mathematical Ideas Explain how graphs of the same function can look different.

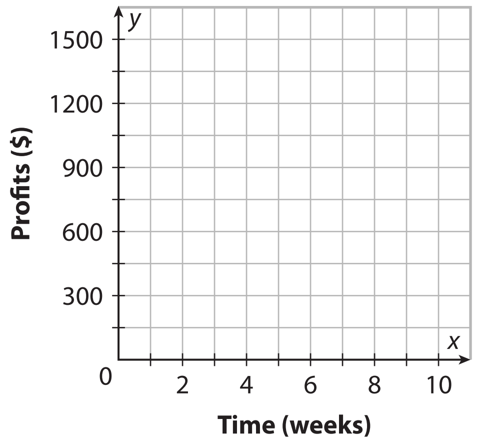
Lesson Performance Task

Jordan has started a new dog-walking service. His total profits over the first 4 weeks are expressed in this table.

Time (weeks)	Profits($)
1	150
2	300
3	450
4	600

a. Show that his business results can be described by a linear function.

b. Graph this function and use the graph to predict his business profit 9 weeks after he opens.



c. Explain why it is or is not a good idea to project his profits so far into the future. Give examples to support your answer.

Name ______________________ Class ____________ Date ________

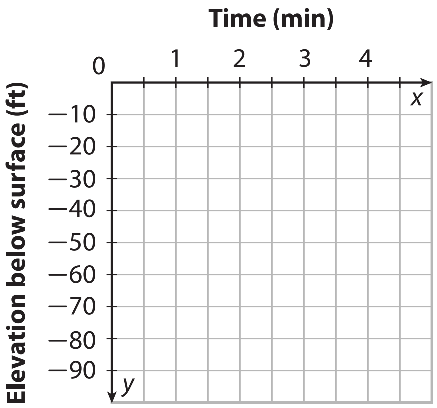
5.2 Using Intercepts

Essential Question: How can you identify and use intercepts in linear relationships?

Resource Locker

Explore Identifying Intercepts

Miners are exploring 90 feet underground. The miners ascend in an elevator at a constant rate over a period of 3 minutes until they reach the surface. In the coordinate grid, the horizontal axis represents the time in minutes from when the miners start ascending, and the vertical axis represents the miners' elevation relative to the surface in feet.



(A) What point represents the miners' elevation at the beginning of the ascent?

_________ Plot this point.

(B) What point represents the miners' elevation at the end of the ascent?

_________ Plot this point.

(C) Connect the points with a line segment.

(D) What is the point where the graph crosses the *y*-axis? _________ the *x*-axis? _________

Reflect

1. Discussion The point where the graph intersects the *y*-axis represents the beginning of the miners' ascent. Will the point where a graph intersects the *y*-axis always be the lowest point on a linear graph? Explain.

____

____

____

Explain 1 Determining Intercepts of Linear Equations

The graph in the Explore intersected the axes at (0, –90) and (3, 0).

The **y-intercept** of a graph is the y-coordinate of the point where the graph intersects the y-axis. The x-coordinate of this point is always 0. The y-intercept of the graph in the Explore is –90.

The **x-intercept** of a graph is the x-coordinate of the point where the graph intersects the x-axis. The y-coordinate of this point is always 0. The x-intercept of the graph in the Explore is 3.

Example 1 **Find the x- and y-intercepts.**

(A) $3x - 2y = 6$

To find the x-intercept, replace y with 0 and solve for x.

$$3x - 2(0) = 6$$
$$3x = 6$$
$$x = 2$$

The x-intercept is 2.

To find the y-intercept, replace x with 0 and solve for y.

$$3(0) - 2y = 6$$
$$-2y = 6$$
$$y = -3$$

The y-intercept is -3.

(B) $-5x + 6y = 60$

To find the x-intercept, replace y with ______ and solve for x.

$$-5x + 6\left(\boxed{}\right) = 60$$
$$-5x = 60$$
$$x = \boxed{}$$

The x-intercept is ______.

To find the y-intercept, replace ______ with 0 and solve for ______.

$$-5\left(\boxed{}\right) + 6y = 60$$
$$6y = 60$$
$$y = \boxed{}$$

The y-intercept is ______.

Reflect

2. If the point (5, 0) is on a graph, is (5, 0) the y-intercept of the graph? Explain.

______

Your Turn

Find the x- and y-intercepts.

3. $8x + 7y = 28$

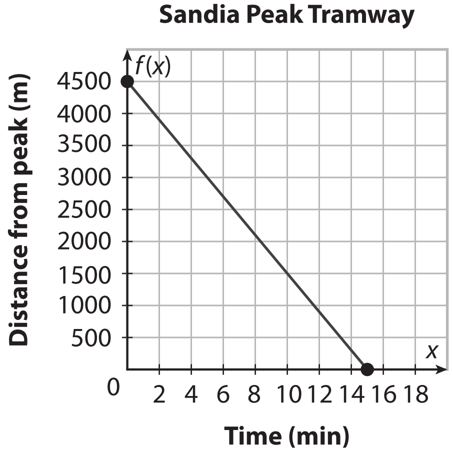
4. $-6x - 8y = 24$

Explain 2 Interpreting Intercepts of Linear Equations

You can use intercepts to interpret a situation that is modeled by a linear function.

Example 2 **Find and interpret the x- and y-intercepts for each situation.**

Ⓐ The Sandia Peak Tramway in Albuquerque, New Mexico, travels a distance of about 4500 meters to the top of Sandia Peak. Its speed is 300 meters per minute. The function $f(x) = 4500 - 300x$ gives the tram's distance in meters from the top of the peak after x minutes.



To find the x-intercept, replace $f(x)$ with 0 and solve for x.

$f(x) = 4500 - 300x$

$0 = 4500 - 300x$

$x = 15$

It takes 15 minutes to reach the peak.

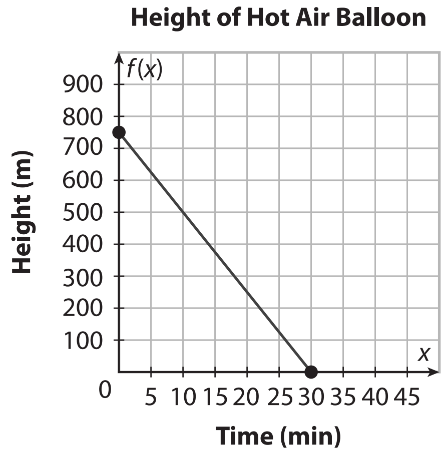
To find the y-intercept, replace x with 0 and find $f(0)$.

$f(x) = 4500 - 300x$

$f(0) = 4500 - 300(0) = 4500$

The distance from the peak when it starts is 4500 m.

Ⓑ A hot air balloon is 750 meters above the ground and begins to descend at a constant rate of 25 meters per minute. The function $f(x) = 750 - 25x$ represents the height of the hot air balloon after x minutes.



To find the x-intercept, replace $f(x)$ with 0 and solve x.

$f(x) = 750 - 25x$

$\boxed{} = 750 - 25x$

$x = \boxed{}$

It takes ____________ to reach the ground.

To find the y-intercept, replace x with 0 and find $f(0)$.

$f(x) = 750 - 25x$

$f(0) = 750 - 25\left(\boxed{}\right) = 750$

The height above ground when it starts is ____________.

Reflect

5. **Critique Reasoning** A classmate says that the graph shows the path of the tram. Do you agree?

____

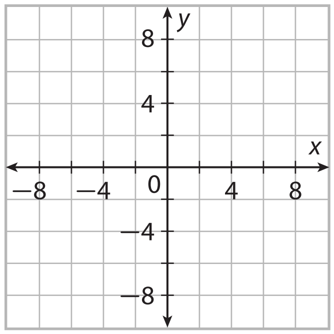
Your Turn

6. The temperature in an experiment is increased at a constant rate over a period of time until the temperature reaches 0 °C. The equation $y = \frac{5}{2}x - 70$ gives the temperature y in degrees Celsius x hours after the experiment begins. Find and interpret the x- and y-intercepts.

Explain 3 Graphing Linear Equations Using Intercepts

You can use the x- and y-intercepts to graph a linear equation.

Example 3 **Use intercepts to graph the line described by each equation.**



(A) $\frac{1}{2}y = 3 - \frac{3}{4}x$

Write the equation in standard form. $\frac{3}{4}x + \frac{1}{2}y = 3$

Find the intercepts.

x-intercept:

$$\frac{3}{4}x + \frac{1}{2}(0) = 3$$
$$\frac{3}{4}x = 3$$
$$x = 4$$

y-intercept:

$$\frac{3}{4}(0) + \frac{1}{2}y = 3$$
$$\frac{1}{2}y = 3$$
$$y = 6$$

Graph the line by plotting the points (4, 0) and (0, 6) and drawing a line through them.

(B) $18y = 12x + 108$

Write the equation in standard form. $\boxed{} = 108$

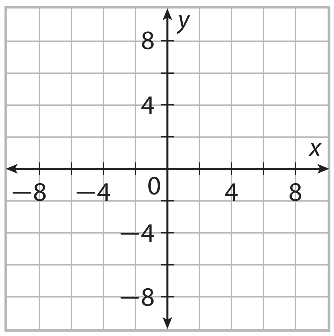
Find the intercepts.

x-intercept:

$$-12x + 18\left(\boxed{}\right) = 108$$
$$-12x = 108$$
$$x = \boxed{}$$

y-intercept:

$$-12\boxed{} + 18y = 108$$
$$18y = 108$$
$$y = \boxed{}$$

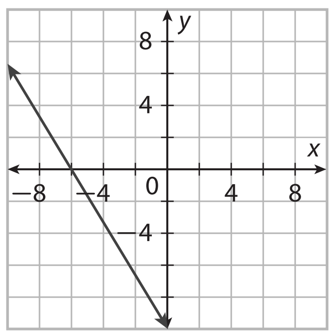


Graph the line by plotting the points ________________ and ________________

and draw ________________ through them.

Your Turn

7. Use intercepts to graph $3y = -5x - 30$.



Elaborate

8. A line intersects the y-axis at the point (a, b). Is $a = 0$? Is $b = 0$? Explain.

9. What does a negative y-intercept mean for a real-world application?

10. **Essential Question Check-in** How can you find the x-intercept of the graph of a linear equation using the equation? How is using the graph of a linear equation to find the intercepts like using the equation?

Evaluate: Homework and Practice

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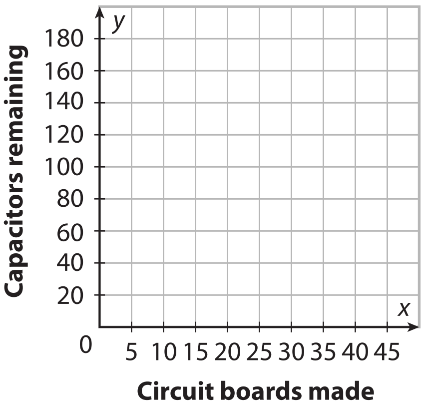
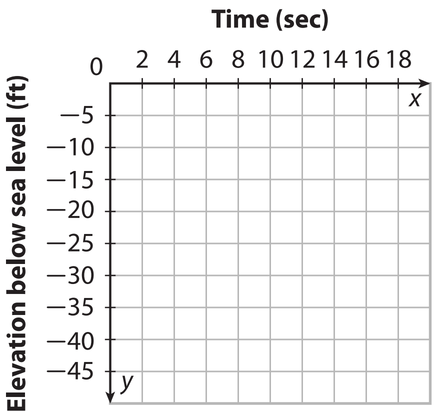
- Online Homework
- Hints and Help
- Extra Practice

Identify and interpret the intercepts for each situation, plot the points on the graph, and connect the points with a line segment.

1. An electronics manufacturer has 140 capacitors, and the same number of capacitors is needed for each circuit board made. The manufacturer uses the capacitors to make 35 circuit boards.



2. A dolphin is 42 feet underwater and ascends at a constant rate for 14 seconds until it reaches the surface.



Find the x- and y-intercepts.

3. $2x - 3y = -6$

4. $-4x - 5y = 40$

5. $8x + 4y = -56$

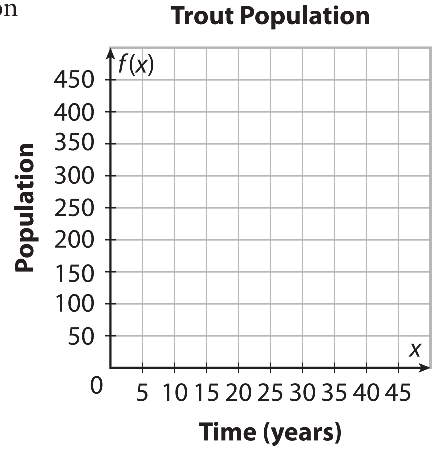
6. $-9x + 6y = 72$

7. $\frac{3}{5}x + \frac{1}{2}y = 30$

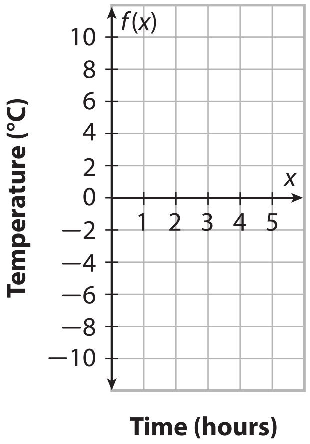
8. $-\frac{3}{4}x + \frac{5}{6}y = 15$

Interpret the intercepts for each situation. Use the intercepts to graph the function.

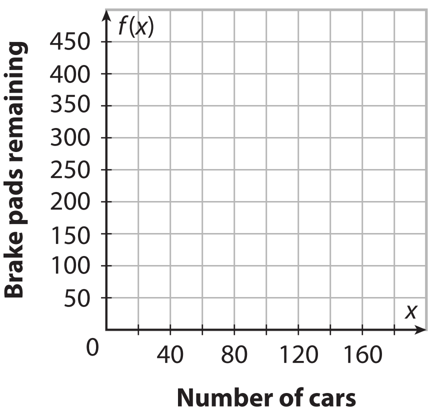
9. **Biology** A lake was stocked with 350 trout. Each year, the population decreases by 14. The population of trout in the lake after x years is represented by the function $f(x) = 350 - 14x$.



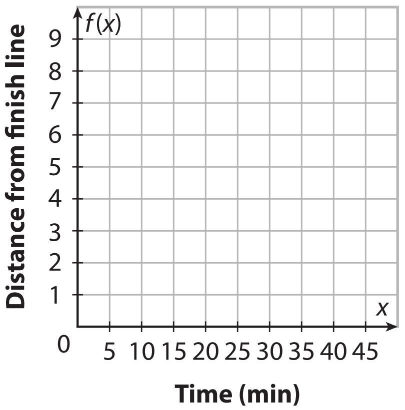
10. The air temperature is –6 °C at sunrise and rises 3 °C every hour for several hours. The air temperature after x hours is represented by the function $f(x) = 3x - 6$.



11. The number of brake pads needed for a car is 4, and a manufacturing plant has 480 brake pads. The number of brake pads remaining after brake pads have been installed on x cars is $f(x) = 480 - 4x$.

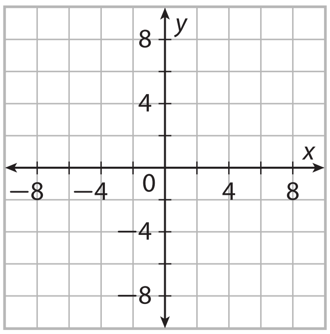


12. Connor is running a 10-kilometer cross country race. He runs 1 kilometer every 4 minutes. Connor's distance from the finish line after x minutes is represented by the function $f(x) = 10 - \frac{1}{4}x$.

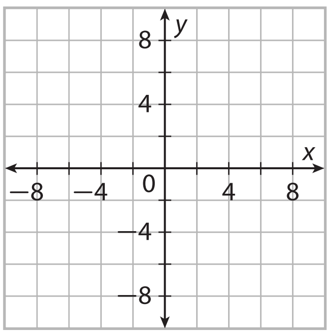


Use intercepts to graph the line described by each equation.

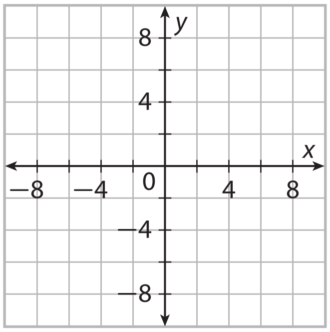
13. $-6y = -4x + 24$



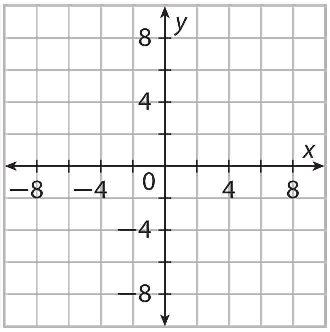
14. $9y = 3x + 18$



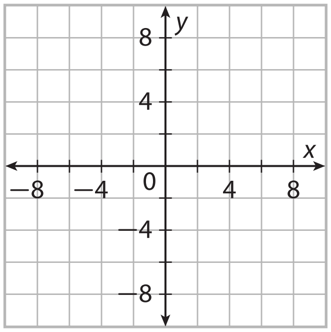
15. $y = \frac{1}{5}x + 2$



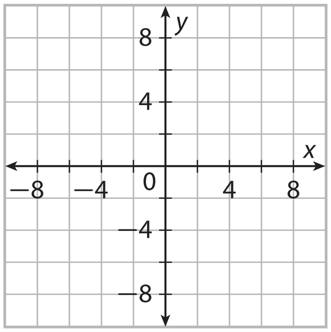
16. $-3y = 7x - 21$



17. $\frac{3}{2}x = -4y - 12$



18. $\frac{2}{3}y = 2 - \frac{1}{2}x$



19. Kim owes her friend \$245 and plans to pay \$35 per week. Write an equation of the function that shows the amount Kim owes after x weeks. Then find and interpret the intercepts of the function.

20. **Explain the Error** Arlo incorrectly found the x-intercept of $9x + 12y = 144$. His work is shown.

$$9x + 12y = 144$$
$$9(0) + 12y = 144$$
$$12y = 144$$
$$y = 12 \text{ The } x\text{-intercept is 12.}$$

Explain Arlo's error.

21. Determine whether each point could represent an x-intercept, y-intercept, both, or neither.

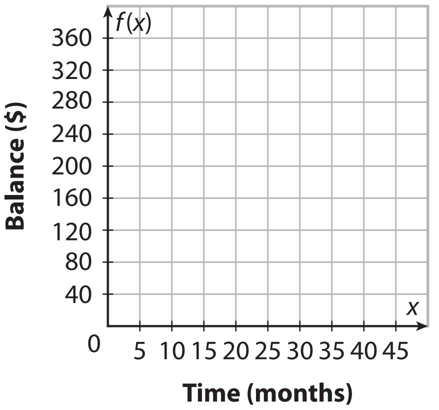
A. (0, 5) **B.** (0, 0) **C.**(−7, 0)

D. (3, −4) **E.** (19, 0)

22. A bank employee notices an abandoned checking account with a balance of \$360. The bank charges an \$8 monthly fee for the account.

a. Write and graph the equation that gives the balance $f(x)$ in dollars as a function of the number of months, x.

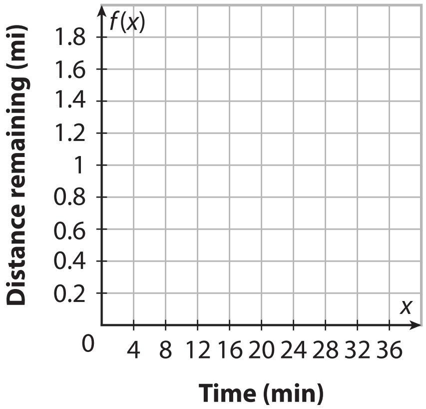
b. Find and interpret the x- and y-intercepts.



23. Kathryn is walking on a treadmill at a constant pace for 30 minutes. She has programmed the treadmill for a 2-mile walk. The display counts backward to show the distance remaining.

a. Write and graph the equation that gives the distance $f(x)$ left in miles as a function of the number x of minutes she has been walking.

b. Find and interpret the x- and y-intercepts.

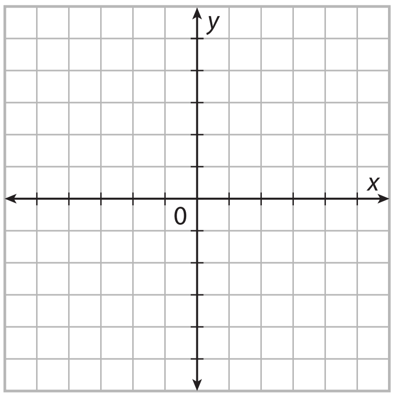


H.O.T. Focus on Higher Order Thinking

24. Represent Real-World Problems Write a real-world problem that could be modeled by a linear function whose x-intercept is 6 and whose y-intercept is 60.

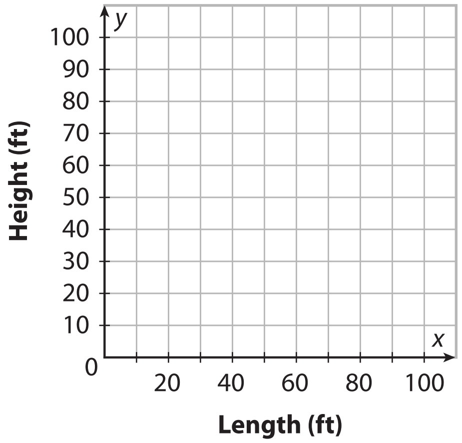
25. Draw Conclusions For any linear equation $Ax + By = C$, what are the intercepts in terms of A, B, and C?

26. Multiple Representations Find the intercepts of $3x + 40y = 1200$. Explain how to use the intercepts to determine appropriate scales for the graph and then create a graph.



Lesson Performance Task

A sail on a boat is in the shape of a right triangle. If the sail is superimposed on a coordinate plane, the point where the horizontal and vertical sides meet is (0, 0) and the sail is above and to the right of (0, 0). The equation of the line that represents the sail's hypotenuse in feet is $10x + 3y = 240$.



a. Find and interpret the intercepts of the line and use them to graph the line. Then use the triangle formed by the x-axis, y-axis, and the line described by the above equation to find the area of the sail.

b. Now find the area of a sail whose hypotenuse is described by the equation $Ax + By = C$, where A, B, and C are all positive.

Name ______________________ Class ______________ Date ________

5.3 Interpreting Rate of Change and Slope

Essential question: How can you relate rate of change and slope in linear relationships?

Resource Locker

Explore Determining Rates of Change

For a function defined in terms of x and y, the **rate of change** over a part of the domain of the function is a ratio that compares the change in y to the change in x in that part of the domain.

rate of change $= \dfrac{\text{change in } y}{\text{change in } x}$

The table shows the year and the cost of sending 1-ounce letter in cents.

Years after 2000 (x)	3	4	6	8	13
Cost (cents)	37	37	39	42	46

Find the rate of change, $\dfrac{\text{change in postage}}{\text{change in year}}$, for each time period using the table.

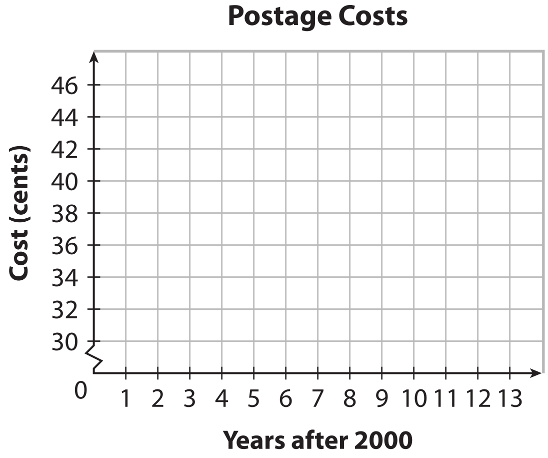
(A) From 2003 to 2004: $\dfrac{\square - \square}{4-3} = \square$ cent(s) per year

(B) From 2004 to 2006: $\dfrac{\square - \square}{6-4} = \square = \square$ cent(s) per year

(C) From 2006 to 2008: $\dfrac{\square - \square}{8-6} = \square = \square$ cent(s) per year

(D) From 2008 to 2013: $\dfrac{\square - \square}{13-8} = \square = \square$ cent(s) per year

Ⓔ Plot the points represented in the table. Connect the points with line segments to make a statistical line graph.



Find the rate of change for each time period using the graph.

Ⓕ Label the vertical increase (*rise*) and the horizontal increase (*run*) between points (4, 37) and (6, 39). Then find the rate of change, $\frac{\text{rise}}{\text{run}}$.

$\frac{\text{rise}}{\text{run}} = \frac{\square}{\square} = \square$ cent(s) per year

Ⓖ Label the vertical increase (*rise*) and the horizontal increase (*run*) between points (6, 39) and (8, 42). Then find the rate of change, $\frac{\text{rise}}{\text{run}}$.

$\frac{\text{rise}}{\text{run}} = \frac{\square}{\square} = \square$ cent(s) per year

Ⓗ Label the vertical increase (*rise*) and the horizontal increase (*run*) between points (8, 42) and (13, 46). Then find the rate of change, $\frac{\text{rise}}{\text{run}}$.

$\frac{\text{rise}}{\text{run}} = \frac{\square}{\square} = \square$ cent(s) per year

Reflect

1. **Discussion** Between which two years is the rate of change $\frac{\text{change in postage}}{\text{change in years}}$ the greatest?

2. **Discussion** Compare the line segment between 2006 and 2008 with the line segment between 2008 and 2013. Which is steeper? Which represents a greater rate of change?

3. **Discuss** How do you think the steepness of the line segment between two points is related to the rate of change it represents?

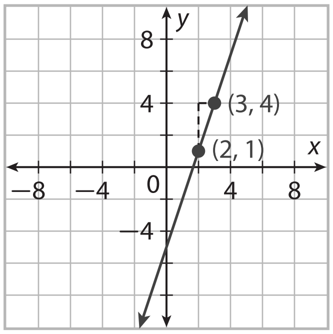
Explain 1 Determining the Slope of a Line

The rate of change for a linear function can be calculated using the rise and run of the graph of the function. The **rise** is the difference in the y-values of two points on a line. The **run** is the difference in the x-values of two points on a line.

The **slope** of a line is the ratio of rise to run for any two points on the line.

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{difference in } y\text{-values}}{\text{difference in } x\text{-values}}$$

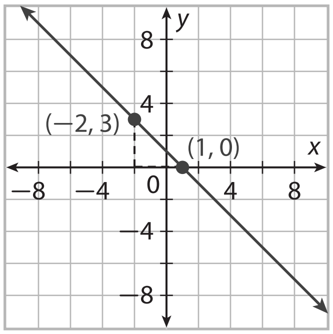
Example 1 **Determine the slope of each line.**



(A) Use $(3, 4)$ as the first point. Subtract y-values to find the change in y, or rise. Then subtract x-values to find the change in x, or run.

$$\text{slope} = \frac{4-1}{3-2} = \frac{3}{1} = 3.$$

Slope of the line is 3.



(B) Use $(-2, \boxed{})$ as the first point. Subtract y-values to find the change in y, or rise. Then subtract x-values to find the change in x, or run.

$$\text{slope} = \frac{\boxed{} - \boxed{}}{\boxed{} - \boxed{}} = \frac{\boxed{}}{\boxed{}} = \boxed{}.$$

The slope of the line is $\boxed{}$.

Reflect

4. Find the rise of a horizontal line. What is the slope of a horizontal line?

---

5. Find the run of a vertical line. What is the slope of a vertical line?

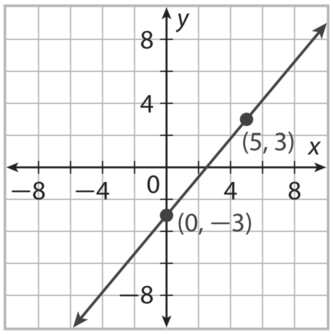
---

6. **Discussion** If you have a graph of a line, how can you determine whether the slope is positive, negative, zero, or undefined without using points on the line?

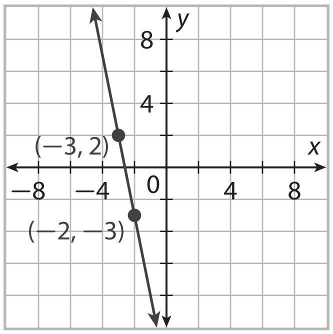
Your Turn

Find the slope of each line.

7.



8.



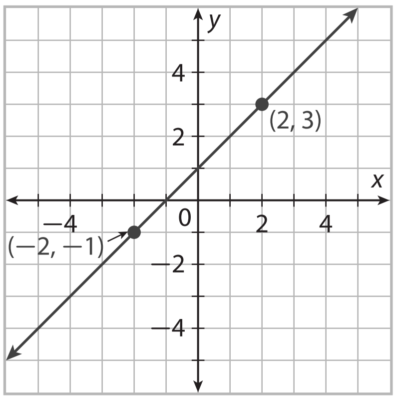
Explain 2 Determining Slope Using the Slope Formula

The **slope formula** for the slope of a line is the ratio of the difference in y-values to the difference in x-values between any two points on the line.

Slope Formula

If (x_1, x_2) and (y_1, y_2) are any two points on a line, the slope of the line is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Example 2 **Find the slope of each line passing through the given points using the slope formula. Describe the slope as positive, negative, zero, or undefined.**



(A) The graph shows the linear relationship.

$$y_2 - y_1 = 3 - (-1) = 3 + 1 = 4$$

$$x_2 - x_1 = 2 - (-2) = 2 + 2 = 4$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4}{4} = 1$$

The slope is positive. The line rises from left to right.

(B)

x	3	3	3	3
y	2	4	6	8

Let $(\square, 4)$ be (x_1, y_1) and $(\square, 8)$ be (x_2, y_2).

$$y_2 - y_1 = 8 - \square = \square$$

$$x_2 - x_1 = \square - \square = \square$$

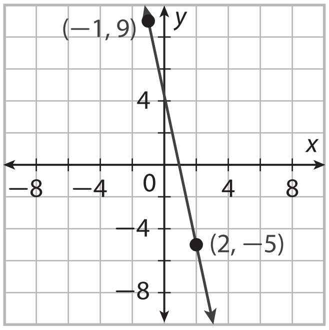
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\square}{\square}$$

The slope is ____________ and the line is __________.

Your Turn

Find the slope of each line passing through the given points using the slope formula. Describe the slope as positive, negative, zero, or undefined.

9. The graph shows the linear relationship.



10.

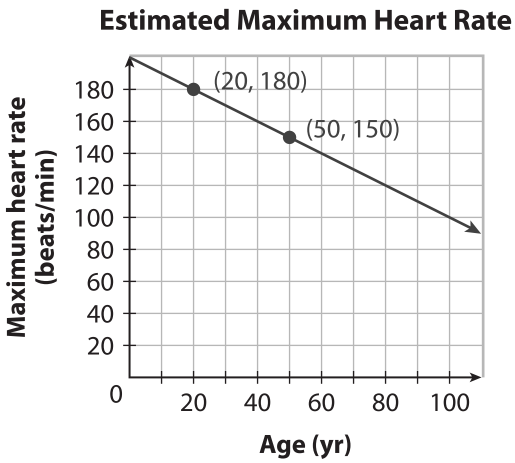
x	1	2	3	4
y	5	5	5	5

Explain 3 Interpreting Slope

Given a real-world situation, you can find the slope and then interpret the slope in terms of the context of the situation.

Example 3 **Find and interpret the slope for each real-world situation.**

(A) The graph shows the relationship between a person's age and his or her estimated maximum heart rate.



Use the two points that are labeled on the graph.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{180 - 150}{50} = \frac{30}{-30} = -1$$

Interpret the slope.

The slope being −1 means that for every year a person's age increases, his or her maximum heart rate decreases by 1 beat per minute.

(B) The height of a plant y in centimeters after x days is a linear relationship. The points (30, 15) and (40, 25) are on the line.

Use the two points that are given.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\square - 15}{\square - \square} = \frac{\square}{\square} = \square$$

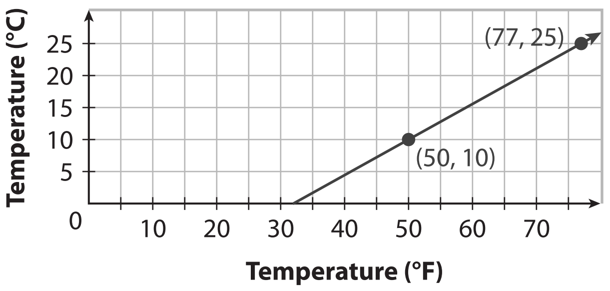
Interpret the slope.

The slope being ____________ means ____.

Your Turn

Find and interpret the slope.

11. The graph shows the relationship between the temperature expressed in °F and the temperature expressed in °C.



12. The number of cubic feet of water y in a reservoir x hours after the water starts flowing into the reservoir is a linear function. The points (40, 3000) and (60, 4000) are on the line of the function.

Elaborate

13. How can you relate the rate of change and slope in the linear relationships?

14. How is the slope formula related to the definition of slope?

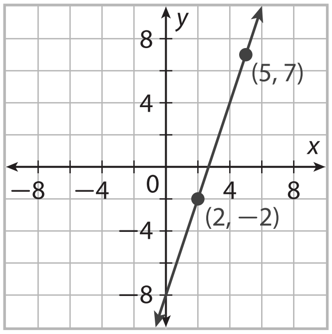
15. How can you interpret slope in a real-world situation?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

Determine the slope of each line.

1.



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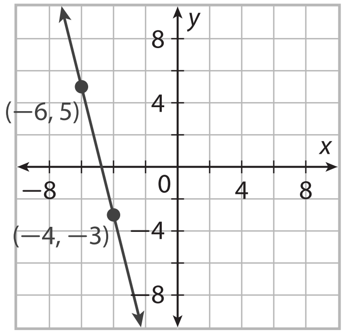

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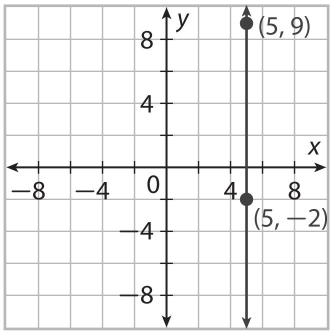

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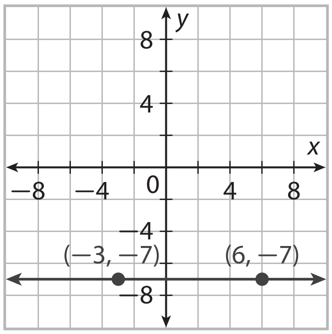

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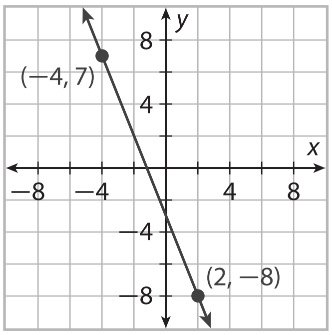

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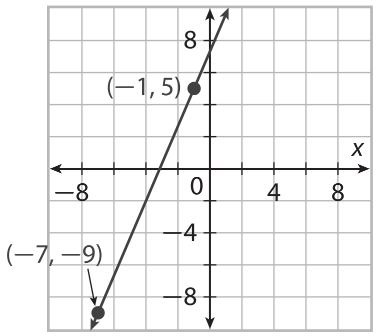


Find the slope of each line passing through the given points using the slope formula. Describe the slope as positive, negative, zero, or undefined.

7. $(5, 3)$ and $(10, 8)$

8. $(-5, 14)$ and $(-1, 2)$

9. $(-5, 6)$ and $(8, 6)$

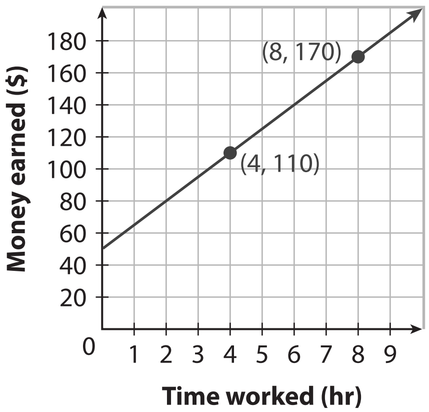
10. $(-4, -17)$ and $(-4, -3)$

11. $(12, -7)$ and $(2, -2)$

12. $(-3, -10)$ and $(-1, -1)$

Find and interpret the slope for each real-world situation.

13.



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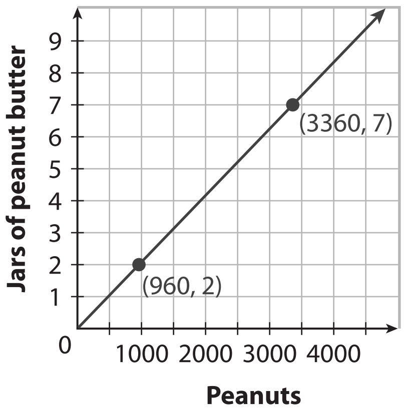

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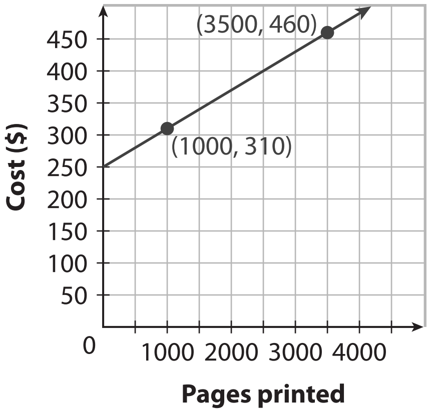

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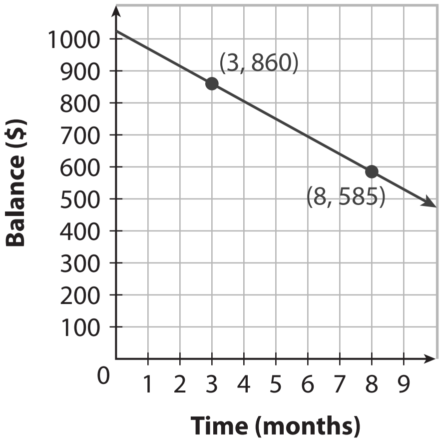
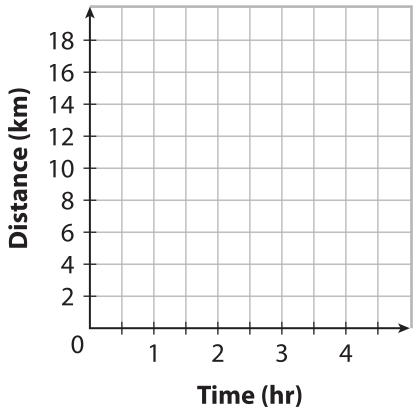


17. **a.** The table shows the distance that a group of hikers has traveled from the start of the trail.

Time (hr)	0.5	1	2	3
Distance (km)	3	5	7	13

Use the table to plot the 4 points on the graph and join the points using line segments.



b. Find the slope for each of the three line segments.

c. Which line segment has the greatest slope? Does this line segment appear to be the steepest on the graph?

18. Determine whether each set of points is on a line that has a positive slope, negative slope, zero slope, or undefined slope. Select the correct answer for each part.

a. $(5, 0)$ and $(8, 4)$ ☐ positive ☐ negative ☐ zero ☐ undefined

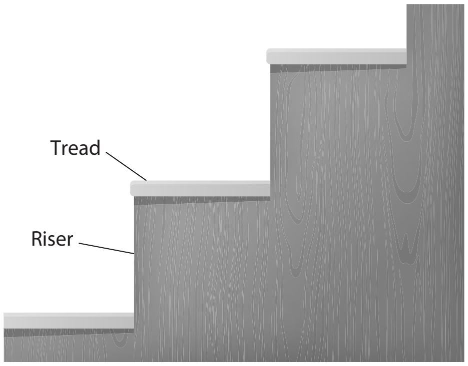
b. $(-6, 1)$ and $(-6, 9)$ ☐ positive ☐ negative ☐ zero ☐ undefined

c. $(2, 6)$ and $(11, -3)$ ☐ positive ☐ negative ☐ zero ☐ undefined

d. $(3, 4)$ and $(-2, 12)$ ☐ positive ☐ negative ☐ zero ☐ undefined

e. $(-3, 5)$ and $(7, 5)$ ☐ positive ☐ negative ☐ zero ☐ undefined

19. What is the slope of the segment shown for a staircase with 10-inch treads and 7.75-inch risers? As you walk up (or down) the stairs, your vertical distance from the floor is a linear function of your horizontal distance from the point on the floor where you started. Is the function discrete or continuous? Explain.



20. The Mount Washington Cog Railway in New Hampshire is one of the steepest cog railways in the world. A section of the railway has a slope of approximately 0.37. In this section, a vertical change of 1 unit corresponds to a horizontal change of what length? Round your answer to the nearest hundredth.

21. a. Biology The table shows how the number of cricket chirps per minute changes with the air temperature.

Temperature (°F)	40	50	60	70	80	90
Chirps per minute	0	40	80	120	160	200

Find the rates of change.

b. Is the graph of the data a line? If so, what is the slope? If not, explain why not.

H.O.T. Focus on Higher Order Thinking

22. Explain the Error A student is asked to find the slope of a line containing the points (4, 3) and (−2, 15) and finds the slope as shown. Explain the error.

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4-(-2)}{3-15} = \frac{6}{-12} = -\frac{1}{2}$$

23. Critical Thinking In this lesson, you learned that the slope of a line is constant. Does this mean that all lines with the same slope are the same line? Explain.

24. a. Represent Real-World Problems A ladder is leaned against a building. The bottom of the ladder is 11 feet from the building. The top of the ladder is 19 feet above the ground. What is the slope of the ladder?

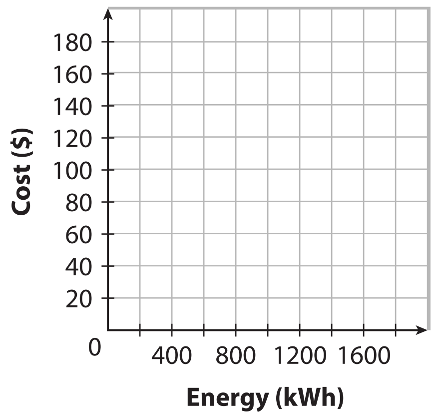
b. What does the slope of the ladder mean in the real world?

c. If the ladder were set closer to the building, would it be harder or easier to climb? Explain in terms of the slope of the ladder.

25. a. The table shows the cost, in dollars, charged by an electric company for various amounts of energy in kilowatt-hours.

Graph the data and show the rates of change.

Energy (kWh)	0	200	400	600	1000	2000
Cost ($)	8	8	34	60	112	157



b. Compares the rates of change for each interval. Are they all the same? Explain.

c. What do the rates of change represent?

d. Describe in words the electric company's billing plan.

Lesson Performance Task

A city has three Internet service providers (ISP), each of which charges a usage fee when a subscriber goes over 100 megabytes (MB) per billing cycle. The table below relates the amount of data a subscriber uses with the cost for each ISP.

ISP	100 MB	200 MB	400 MB
A	\$54	\$74	\$94
B	\$42	\$57	\$87
C	\$60	\$72	\$96

Use the table to find the rate of change for each interval of each ISP, and use the rates of change to determine whether the usage fee is constant for each ISP. Interpret the meaning of the rates of change for each ISP. Then determine and explain which ISP would be the least expensive and which ISP would be the most expensive for a subscriber that uses a high amount of data.