Name ______________________ Class ______________ Date ________

4.1 Identifying and Graphing Sequences

Resource Locker

Essential Question: What is a sequence and how are sequences and functions related?

Explore Understanding Sequences

A go-kart racing track charges \$5 for a go-kart license and \$2 for each lap. If you list the charges for 1 lap, 2 laps, 3 laps, and so on, in order, the list forms a sequence of numbers:

$$7, 9, 11, 13, \ldots$$

A **sequence** is a list of numbers in a specific order. Each element in a sequence is called a **term**. In a sequence, each term has a position number. In the sequence 7, 9, 11, 13, . . . , the second term is 9, so its position number is 2.

(A) The total cost (term) of riding a go-kart for different numbers of laps (position) is shown below. Complete the table.

Position number, n	1	2	3		5		*Domain*
Term of the sequence, $f(n)$	7	9		13		17	*Range*

(B) You can use the term and position number of a sequence to write a function. Using function notation, $f(2) = 9$ indicates that the second term is 9. Use the table to complete the following statements.

$f(1) = \square$ $\quad f(3) = \square$ $\quad f(6) = \square$ $\quad f(\square) = 13$ $\quad f(\square) = 15$

(C) Identify the domain of the function $f(n)$. ______________________

(D) Identify the range of the function $f(n)$. ______________________

Reflect

1. **Discussion** What does $f(4) = 13$ mean in the context of the go-kart problem?

____

2. **Discussion** Explain how to find the missing values in the table.

____

3. **Communicate Mathematical Ideas** Explain why the relationship between the position numbers and the corresponding terms of a sequence can be considered a function.

____

Explain 1 Generating Sequences Using an Explicit Rule

An **explicit rule** for a sequence defines the nth term as a function of n for any whole number n greater than 0. Explicit rules can be used to find any specific term in a sequence without finding any of the previous terms.

Example 1 **Write the first 4 terms of the sequence defined by the explicit rule.**

(A) $f(n) = n^2 + 2$

Make a table and substitute values for $n = 1, 2, 3, 4$ to find the first 4 terms.

The first 4 terms of the sequence defined by the explicit rule $f(n) = n^2 + 2$ are 3, 6, 11, and 18.

n	$f(n) = n^2 + 2$	$f(n)$
1	$f(1) = 1^2 + 2 = 3$	3
2	$f(2) = 2^2 + 2 = 6$	6
3	$f(3) = 3^2 + 2 = 11$	11
4	$f(4) = 4^2 + 2 = 18$	18

(B) $f(n) = 3n^2 + 1$

Make a table and substitute values for $n =$ ______.

The first 4 terms are ______.

n	$f(n) = 3n^2 + 1$	$f(n)$
1	$f(\square) = 3(\square)^2 + 1 = \square$	$\square$
2	$f(\square) = 3(\square)^2 + 1 = \square$	$\square$
3	$f(\square) = 3(\square)^2 + 1 = \square$	$\square$
4	$f(\square) = 3(\square)^2 + 1 = \square$	$\square$

Reflect

4. Communicate Mathematical Ideas Explain how to find the 20th term of the sequence defined by the explicit rule $f(n) = n^2 + 2$.

5. Justify Reasoning The number 125 is a term of the sequence defined by the explicit rule $f(n) = 3n + 2$. Which term in the sequence is 125? Justify your answer.

Your Turn

6. Write the first 4 terms of the sequence defined by the explicit rule. $f(n) = n^2 - 5$

7. Find the 15th term of the sequence defined by the explicit rule. $f(n) = 4n - 3$.

Explain 2 Generating Sequences Using a Recursive Rule

A **recursive rule** for a sequence defines the nth term by relating it to one or more previous terms.

The following is an example of a recursive rule:

$$f(1) = 4, f(n) = f(n-1) + 10 \text{ for each whole number } n \text{ greater than } 1$$

This rule means that after the first term of the sequence, every term $f(n)$ is the sum of the pervious term $f(n-1)$ and 10.

Example 2 **Write the first 4 terms of the sequence defined by the recursive rule.**

(A) $f(1) = 2, f(n) = f(n-1) + 3$ for each whole number n greater than 1

For the first 4 terms, the domain of the function is 1, 2, 3, and 4.

The first term of the sequence is 2.

n	$f(n) = f(n-1) + 3$	$f(n)$
1	$f(1) = 2$	2
2	$f(2) = f(1) + 3 = 2 + 3 = 5$	5
3	$f(3) = f(2) + 3 = 5 + 3 = 8$	8
4	$f(4) = f(3) + 3 = 8 + 3 = 11$	11

The first 4 terms are 2, 5, 8, and 11.

(B) $f(1) = 4, f(n) = f(n-1) + 5$ for each whole number n greater than 1

For the first 4 terms, the domain of the function is ______________

The first term of the sequence is $\boxed{}$.

n	$f(n) = f(n-1) + 5$	$f(n)$
1	$f(1) = \boxed{}$	$\boxed{}$
2	$f(2) = f\left(\boxed{}\right) + 5 = \boxed{} + 5 = \boxed{}$	$\boxed{}$
3	$f(3) = f\left(\boxed{}\right) + 5 = \boxed{} + 5 = \boxed{}$	$\boxed{}$
4	$f(4) = f\left(\boxed{}\right) + 5 = \boxed{} + 5 = \boxed{}$	$\boxed{}$

The first 4 terms are ______________ .

Reflect

8. Describe how to find the 12^{th} term of the sequence in Example 2A.

9. Suppose you want to find the 40^{th} term of a sequence. Would you rather use a recursive rule or an explicit rule? Explain your reasoning.

Your Turn

Write the first 5 terms of the sequence.

10. $f(1) = 35$ and $f(n) = f(n - 1) - 2$ for each whole number n greater than 1.

11. $f(1) = 45$ and $f(n) = f(n - 1) - 4$ for each whole number n greater than 1.

Explain 3 Constructing and Graphing Sequences

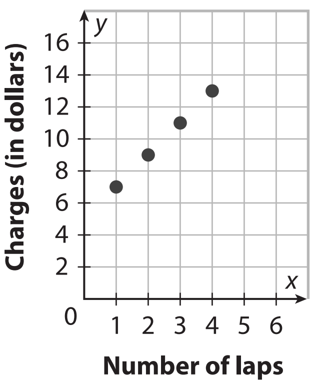
You can graph a sequence on a coordinate plane by plotting the points $(n, f(n))$ indicated in a table that you use to generate the terms.

Example 3 **Construct and graph the sequence described.**

Ⓐ The go-kart racing charges are \$5 for a go-kart license and \$2 for each lap. Use the explicit rule $f(n) = 2n + 5$.

Complete the table to represent the cost for the first 4 laps.

n	$f(n) = 2n + 5$	$f(n)$
1	$f(1) = 2(1) + 5 = 2 + 5 = 7$	7
2	$f(2) = 2(2) + 5 = 4 + 5 = 9$	9
3	$f(3) = 2(3) + 5 = 6 + 5 = 11$	11
4	$f(4) = 2(4) + 5 = 8 + 5 = 13$	13



The ordered pairs are (1, 7), (2, 9), (3, 11), (4, 13).

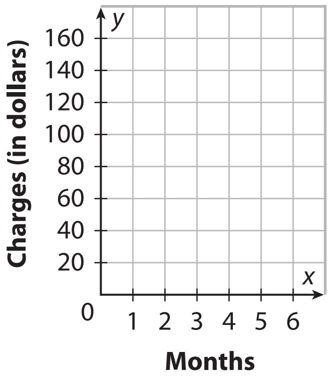
Graph the sequence using the ordered pairs.

Notice that the graph is a set of points that are not connected.

Ⓑ A movie rental club charges \$20 a month plus a \$5 membership fee. Use the explicit rule $f(n) = 20n + 5$.

Complete the table to represent the charges paid for 6 months.

n	$f(n) = \square n + \square$	$f(n)$
1	$f(\square) = \square(\square) + \square = \square$	$\square$
2	$f(\square) = \square(\square) + \square = \square$	$\square$
3	$f(\square) = \square(\square) + \square = \square$	$\square$
4	$f(\square) = \square(\square) + \square = \square$	$\square$
5	$f(\square) = \square(\square) + \square = \square$	$\square$
6	$f(\square) = \square(\square) + \square = \square$	$\square$



The ordered pairs are ______________________________.

Graph the sequence using the ordered pairs.

Notice that the graph is a set of points that are not connected.

Reflect

12. Explain why the points in the graphs in Example 3 are not connected.

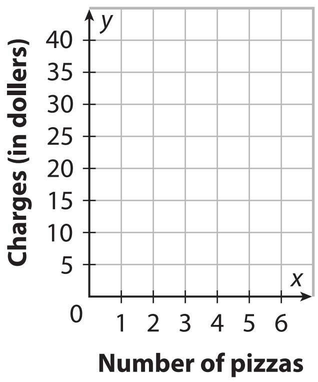
____

____

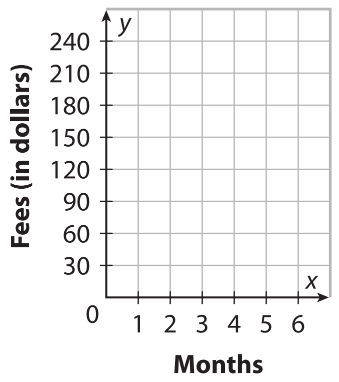
Your Turn

Construct and graph the sequence described.

13. A pizza place is having a special. If you order a large pizza for a regular price \$17, you can order any number of additional pizzas for \$8.50 each. Use the recursive rule $f(1) = 17$ and $f(n) = f(n - 1) + 8.5$ for each whole number n greater than 1.



14. A gym charges \$100 as the membership fee and \$20 monthly fee. Use the explicit rule $f(n) = 20n + 100$ to construct and graph the sequence.



Elaborate

15. What is the difference between an explicit rule and a recursive rule?

16. Describe how to use an explicit rule to find the position number of a given term in a sequence.

17. Explain why the graph of a sequence is a set of points that are not connected.

18. **Essential Question Check-In** Why can the rule for a sequence be considered a function?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Complete the table, and state the domain and range for the sequence it represents. Assume that the sequence continues without end.

1.

n	1	2	3		5	
f(*n*)	15	30		60		90

2.

n	1		3	4		6
f(*n*)	6	8	10		14	

Write the first 4 terms of the sequence defined by the given rule.

3. $f(1) = 65{,}536,\ f(n) = \sqrt{f(n-1)}$

4. $f(n) = n^3 - 1$

5. $f(1) = 7, f(n) = -4 \cdot f(n-1) + 15$

6. $f(n) = 2n^2 + 4$

7. $f(1) = 3,\ f(n) = [f(n-1)]^2$

8. $f(n) = (2n-1)^2$

Find the 10th term of the sequence defined by the given rule.

9. $f(1) = 2, f(n) = f(n - 1) + 7$

10. $f(n) = \sqrt{n + 2}$

11. $f(1) = 30, f(n) = 2 \cdot f(n - 1) - 50$

12. $f(n) = \frac{1}{2}(n - 1) + 3$

The explicit rule for a sequence and one of the specific terms is given. Find the position of the given term.

13. $f(n) = 1.25n + 6.25$; 25

14. $f(n) = -3(n - 1)$; -51

15. $f(n) = (2n - 2) + 2$; 52

The recursive rule for a sequence and one of the specific terms is given. Find the position of the given term.

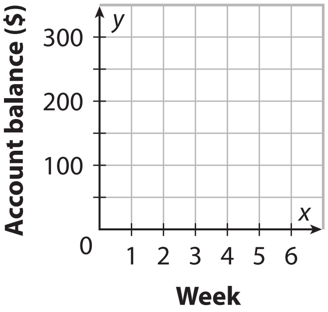
16. $f(1) = 8\frac{1}{2}$; $f(n) = f(n - 1) - \frac{1}{2}$; $5\frac{1}{2}$

17. $f(1) = 99$, $f(n) = f(n - 1) + 4$; 119

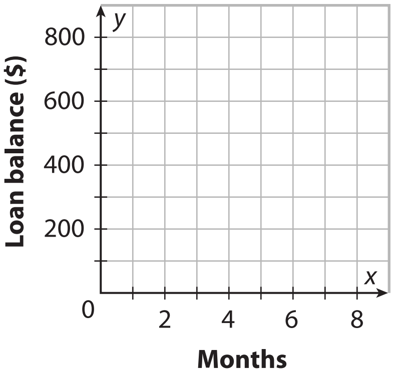
18. $f(1) = 33.3, f(n) = f(n - 1) + 0.2$; 34.9

Graph the sequence that represents the situation on a coordinate plane.

19. Jessica had $150 in her savings account after her first week of work. She then started adding $35 each week to her account for the next 5 weeks. The savings account balance can be represented by a sequence.



20. Carrie borrowed $840 from a friend to pay for a car repair. Carrie promises to repay her friend in 8 equal monthly payments. The remaining amount Carrie has to repay can be represented by a sequence.



H.O.T. Focus on Higher Order Thinking

21. A park charges $12 for one round of miniature golf and a reduced fee for each additional round played. If Tom paid $47 for 6 rounds of miniature golf, what is the reduced fee for each additional round played?

22. **Analyze Relationships** Construct a recursive rule to describe the sequence: 2, 4, 6, 8,…

23. Explain the Error To find the 5th term of a sequence where $f(1) = 4$ and $f(n) = 2 \cdot f(n-1) + 1$ for each whole number greater than 1, Shane calculates $(4 \cdot 2 \cdot 2 \cdot 2 \cdot 2) + 1 = 65$. Is this correct? Justify your answer.

24. Critical Thinking Write a recursive rule for a sequence where every term is the same.

Lesson Performance Task

A museum charges $10 per person for admission and $2 for each of 8 special exhibits.

a. Use function notation to write an equation to represent the cost for attending n events.

b. Make a table to represent the total cost of admission plus 1, 2, and 3 special exhibits.

c. What would $f(0) = 10$ represent?

d. What would the total cost be for going to all 8 special exhibits?

e. Determine an explicit rule for the total cost if the first special exhibit were free.

Name ____________________ Class __________ Date ______

4.2 Constructing Arithmetic Sequences

Essential Question: What is an arithmetic sequence?

Resource Locker

Explore Exploring Arithmetic Sequences

You can order tickets for the local theater online. There is a fee of \$2 per order. Matinee tickets cost \$10 each. The total cost, in dollars, of ordering n matinee tickets online can be found by using $C(n) = 10n + 2$. The table shows the cost of 1, 2, 3, and 4 tickets.

Ⓐ Complete the table of values for $C(n) = 10n + 2$.

Tickets	1	2	3	4
Total Cost (\$)				

What is the domain of the sequence? __________

Ⓑ What is the range of the sequence? __________

Ⓒ What is the first term of the sequence? __________

Ⓓ Find the difference between each two consecutive terms in the sequence:

$22 - 12 =$ __________ $32 - 22 =$ __________ $42 - 32 =$ __________

Reflect

1. **Discussion** Suppose you extended the table for up to 15 tickets. Would you expect the difference between consecutive terms to be the same? Explain your reasoning.

____

____

____

2. **Communicate Mathematical Ideas** Explain how the domain is limited in this situation.

____

____

____

Explain 1 Constructing Rules for Arithmetic Sequences

In an **arithmetic sequence,** the difference between consecutive terms is always equal. This difference, written as d, is called the **common difference.**

An arithmetic sequence can be described in two ways, explicitly and recursively. As you saw earlier, in an **explicit** rule for a sequence, the nth term of the arithmetic sequence is defined as a function of n. In a **recursive** rule for a sequence, the first term of the sequence is given and the nth term is defined by relating it to the previous term. An arithmetic sequence can be defined using either a recursive rule or an explicit rule.

Example 1 **Write a recursive rule and an explicit rule for the sequence described by each table.**

(A) The table shows the monthly balance in a savings account with regular monthly deposits. The savings account begins with \$2000, and \$500 is deposited each month.

Time (months)	n	1	2	3	4	5
Balance	$f(n)$	2000	2500	3000	3500	4000

Write a recursive rule.

$f(1) = 2000$, and the common difference d is 500.

The recursive rule is $f(1) = 2000$, $f(n) = f(n-1) + 500$ for $n \geq 2$.

Write an explicit rule.

n	$f(n)$	$f(1) + d \cdot x = f(n)$
1	2000	$2000 + 500(0) = 2000$
2	2500	$2000 + 500(1) = 2500$
3	3000	$2000 + 500(2) = 3000$

Since d is always multiplied by a number equal to $(n-1)$, you can generalize the result from the table. The explicit rule is $f(n) = 2000 + 500(n-1)$.

(B) The table shows the monthly balance in a savings account with regular monthly deposits.

Time (months)	n	1	2	3	4	5
Balance	$f(n)$	5000	6000	7000	8000	9000

Write a recursive rule.

$f(1) =$ ____________ and the common difference d is ____________.

The recursive rule is $f(1) =$ ____________, $f(n) = f(n-1) +$ ____________ for $n \geq 2$.

Write an explicit rule.

n	$f(n)$	$f(1) + d \cdot x = f(n)$
1	5000	$5000 + 1000(0) = 5000$
2		
3		

Since d is always multiplied by a number equal to ____________, you can generalize the result from the table. $f(n) =$ ____________ $+$ ____________.

Reflect

3. **Critique Reasoning** Jerome says that the sequence 1, 8, 27, 64, 125, ... is not an arithmetic sequence. Is that correct? Explain.

4. An arithmetic sequence has a common difference of 3. If you know that the third term of the sequence is 15, how can you find the fourth term? ________________

YourTurn

5. The table shows the number of plates left at a buffet after n hours. Write a recursive rule and an explicit rule for the arithmetic sequence represented by the table.

Time (hours)	n	1	2	3	4	5
Number of plates	$f(n)$	155	141	127	113	99

Explain 2 Using a General Form to Construct Rules for Arithmetic Sequences

Arithmetic sequences can be described by a set of general rules. Values can be substituted into these rules to find a recursive and explicit rule for a given sequence.

General Recursive Rule	General Explicit Rule
Given $f(1)$, $f(n) = f(n-1) + d$ for $n \geq 2$	$f(n) = f(1) + d(n-1)$

Example 2 **Write a general recursive and general explicit rule for each arithmetic sequence.**

(A) 100, 88, 76, 64, ...

$f(1) = 100$, common difference $= 88 - 100 = -12$

The recursive rule is $f(1) = 100$, $f(n) = f(n-1) - 12$ for $n \geq 2$.

The explicit rule is $f(n) = 100 - 12(n-1)$.

(B) 0, 8, 16, 24, 32,...

$f(1) =$ ________, common difference $=$ ________ $-$ ________ $=$ ________.

The recursive rule is $f(1) =$ ________, $f(n) = f(n-1) +$ ________ for $n \geq 2$.

The explicit rule is $f(n) =$ ________ $+$ ________ $(n-1)$.

Reflect

6. What is the recursive rule for the sequence $f(n) = 2 + (-3)(n - 1)$? How do you know?

YourTurn

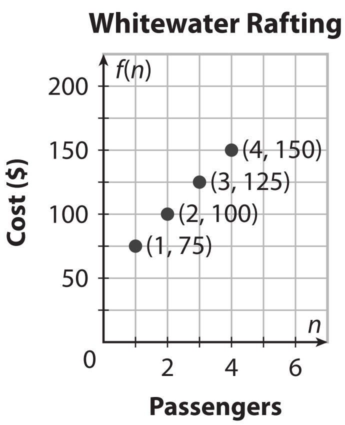
7. Write a recursive rule and an explicit rule for the arithmetic sequence 6, 16, 26, 36, . . .

Explain 3 Relating Arithmetic Sequences and Functions

The explicit rule for an arithmetic sequence can be expressed as a function. You can use the graph of the function to write an explicit rule.

Example 3 **Write an explicit rule in function notation for each arithmetic sequence.**

Ⓐ The cost of a whitewater rafting trip depends on the number of passengers. The base fee is \$50, and the cost per passenger is \$25. The graph shows the sequence.



Step 1 Represent the sequence in a table.

Number of passengers n	1	2	3	4
Cost (\$) $f(n)$	75	100	125	150

Step 2 Find the common difference.

$f(2) - f(1) = 100 - 75 = 25$

$f(3) - f(2) = 125 - 100 = 25$

$f(4) - f(3) = 150 - 125 = 25$

The common difference d is 25.

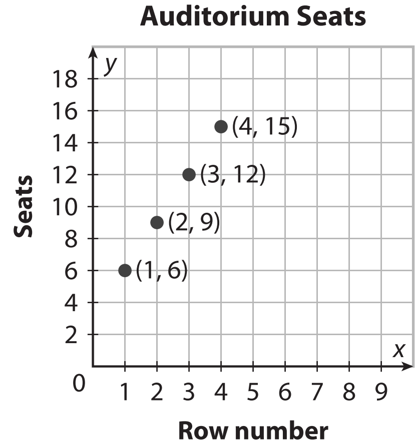
Step 3 Write an explicit rule for the sequence.

Substitute 75 for $f(1)$ and 25 for d.

$f(n) = f(1) + d(n - 1)$

$f(n) = 75 + 25(n - 1)$

(B) The number of seats per row in an auditorium depends on which row it is. The first row has 6 seats, the second row has 9 seats, the third row has 12 seats, and so on. The graph shows the sequence.



Step 1 Represent the sequence in a table.

Row number n	1	2	3	4
Number of seats $f(n)$				

Step 2 Find the common difference.

$f(2) - f(1) = \boxed{} - \boxed{} = \boxed{}$

$f(3) - f(2) = \boxed{} - \boxed{} = \boxed{}$

$f(4) - f(3) = \boxed{} - \boxed{} = \boxed{}$

The common difference is $d =$ ____________.

Step 3 Write an explicit rule for the sequence.

$f(n) = f(1) + d(n - 1)$

Substitute ____________ for $f(1)$ and ____________ for d.

$f(n) = \boxed{} + \boxed{}(n - 1)$

Reflect

8. **Analyze Relationships** Compare the graph of the function $f(x) = 3 + 5(x - 1)$ and the graph of the sequence $f(n) = 3 + 5(n - 1)$.

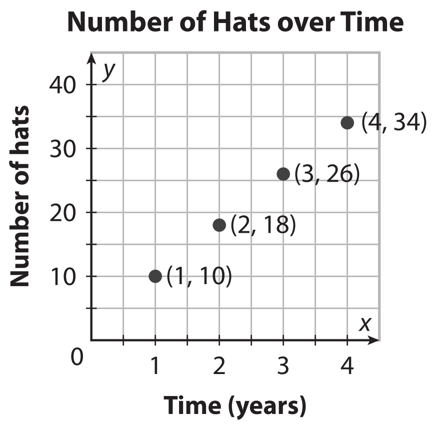
____

____

____

YourTurn

9. Jerry collects hats. The total number of hats in Jerry's collection depends on how many years he has been collecting hats. After the first year, Jerry had 10 hats. Each year he has added the same number of hats to his collection. The graph shows the sequence. Write an explicit rule in function notation for the arithmetic sequence.



Elaborate

10. What information do you need to write a recursive rule for an arithmetic sequence that you do not need to write an explicit rule?

11. Suppose you want to be able to determine the ninetieth term in an arithmetic sequence and you have both an explicit and a recursive rule. Which rule would you use? Explain.

12. **Essential Question Check-In** The explicit equation for an arithmetic sequence and a linear equation have a similar form. How is the value of m in the linear equation $y = mx + b$ similar to the value of d in the explicit equation $f(n) = f(1) + d(n - 1)$?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. Farah pays a \$25 signup fee to join a car sharing service and a \$7 monthly charge. The total cost of using the car sharing service for n months can be found using $C(n) = 25 + 7n$. The table shows the cost of the service for 1, 2, 3, and 4 months.

a. Complete the table for $C(n) = 25 + 7n$

Months	n	1	2	3	4
Cost (\$)	$f(n)$				

b. What are the domain and range of the sequence?

c. What is the common difference d?

Tell whether each sequence is an arithmetic sequence.

2.

a. 6, 7, 8, 9, 10,…

b. 5, 10, 20, 35, 55,…

c. 0, –1, 1, –2, 2,…

d. 1, 16, 81, 625, 1296

e. –2, –4, –6, –8, –10, …

3. **Chemistry** A chemist heats up several unknown substances to determine their boiling point. Use the table to determine whether the sequence is arithmetic. If it is arithmetic, write an explicit rule and a recursive rule for the sequence. If not, explain why it is not arithmetic.

Substance	1	2	3	4	5
Boiling Point (°F)	100	135	149	165	188

Write a recursive rule and an explicit rule for the arithmetic sequence described by each table.

4.

Month	n	1	2	3	4	5
Account balance ($)	$f(n)$	35	32	29	26	23

5.

Tickets	n	1	2	3	4	5
Total cost (S)	$f(n)$	58	65	72	79	86

6.

Month	n	1	2	3	4	5
Total deposits ($)	$f(n)$	84	100	116	132	148

7.

Delivery number	n	1	2	3	4	5
Weight of truck (lb)	$f(n)$	4567	3456	2345	1234	123

8.

Week	n	1	2	3	4	5
Account owed ($)	$f(n)$	125	100	75	50	25

9.

Skaters	n	1	2	3	4	5
Charge for lesson ($)	$f(n)$	60	80	100	120	140

Write a recursive rule and an explicit rule for each arithmetic sequence.

10. 95, 90, 85, 80, 75, . . .

11. 63, 70, 77, 84, 91, . . .

12. 86, 101, 116, 131, 146, . . .

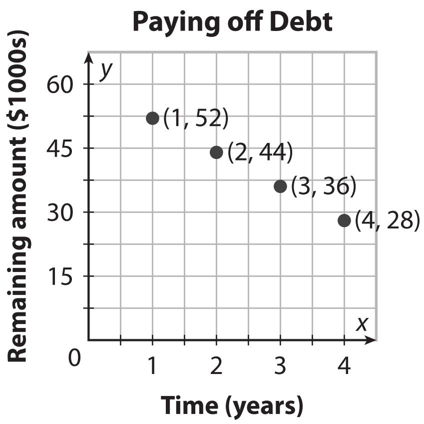
13. 112, 110, 108, 106, 104, . . .

14. 5, 9, 13, 17, 21, . . .

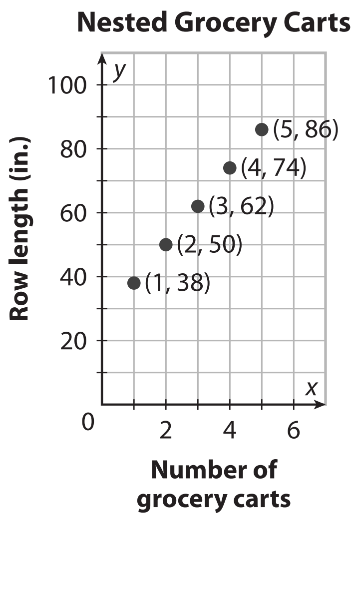
15. 67, 37, 7, −23, −53, . . .

Write an explicit rule in function notation for each arithmetic sequence.

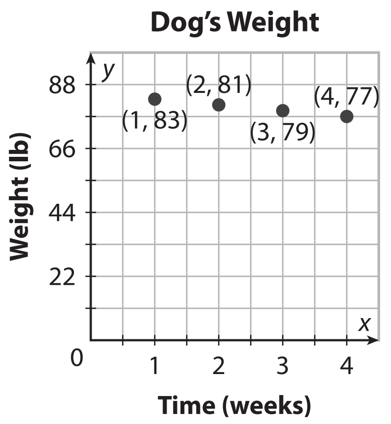
16. A student loan needs to be paid off beginning the first year after graduation. Beginning at Year 1, there is $52,000 remaining to be paid. The graduate makes regular payments of $8,000 each year. The graph shows the sequence.



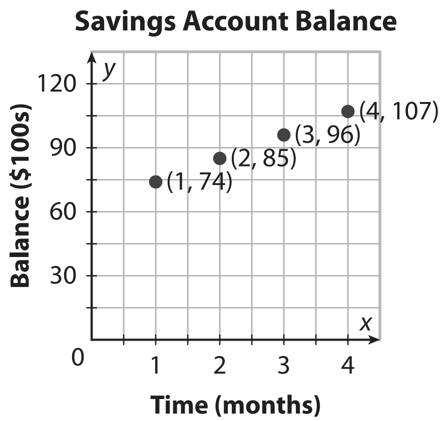
17. A grocery cart is 38 inches long. When the grocery carts are put away in a nested row, the length of the row depends on how many carts are nested together. Each cart added to the row adds 12 inches to the row length. The graph shows the sequence.



18. A dog food for overweight dogs claims that a dog weighing 85 pounds will lose about 2 pounds per week for the first 4 weeks when following the recommended feeding guidelines. The graph shows the sequence.



19. A savings account is opened with $6300. Monthly deposits of $1100 are made. The graph shows the sequence.



20. **Biology** The wolf population in a local wildlife area is currently 12. Due to a new conservation effort, conservationists hope the wolf population will increase by 2 animals each year for the next 50 years. Assume that the plan will be successful. Write an explicit rule for the population sequence. Use the rule to predict the number of animals in the wildlife area in the fiftieth year.

21. How are the terms in the sequence in the table related? Is the sequence an arithmetic sequence? Explain.

n	1	2	3	4	5
$f(n)$	3	9	27	81	243

H.O.T. Focus on Higher Order Thinking

22. **Explain the Error** The cost of a hamburger is \$2.50. Each additional hamburger costs \$2.00. Sully wrote this explicit rule to explain the sequence of costs: $f(n) = 2 + 2.5(n - 1)$. Using this rule, he found the cost of 12 hamburgers to be \$29.50. Is this number correct? If not, identify Sully's error.

23. **Critical Thinking** Lucia knows the fourth term in a sequence is 55 and the ninth term in the same sequence is 90. Explain how she can find the common difference for the sequence. Then use the common difference to find the second term of the sequence.

24. **Represent Real-World Problems** Write and solve a real-world problem involving a situation that can be represented by the sequence $f(n) = 15 + 2(n - 1)$.

Lesson Performance Task

For Carl's birthday, his grandparents gave him a \$50 gift card to a local movie theater. The theater charges \$6 admission for each movie. How can Carl use an arithmetic sequence to determine the value left on his card after each movie he sees?

a. Write an explicit rule for the arithmetic sequence and use it to determine how much value is left on the card after Carl has seen 4 movies.

b. How much is left on the card after Carl has seen the maximum number of movies?

Name ______________________ Class ______________ Date ________

4.3 Modeling with Arithmetic Sequences

Essential Question: How can you solve real-world situations using arithmetic sequences?

Resource Locker

Explore Interpreting Models of Arithmetic Sequences

You can model real-world situations and solve problems using models of arithmetic sequences. For example, suppose watermelons cost \$6.50 each at the local market. The total cost, in dollars, of n watermelons can be found using $c(n) = 6.5n$.

(A) Complete the table of values for 1, 2, 3, and 4 watermelons.

Watermelons n	1	2	3	4
Total cost (\$) $c(n)$				

(B) What is the common difference?

(C) What does n represent in this context?

(D) What are the dependent and independent variables in this context?

(E) Find $c(7)$. What does this value represent?

Reflect

1. **Discussion** What domain values make sense for $c(n) = 6.5n$ in this situation?

Explain 1 Modeling Arithmetic Sequences From a Table

Given a table of data values from a real-world situation involving an arithmetic sequence, you can construct a function model and use it to solve problems.

Example 1 **Construct an explicit rule in function notation for the arithmetic sequence represented in the table. Then interpret the meaning of a specific term of the sequence in the given context.**

(A) Suppose the table shows the cost, in dollars, of postage per ounce of a letter.

Number of ounces	n	1	2	3	4
Cost ($) of postage	$f(n)$	0.35	0.55	0.75	0.95

Determine the value of $f(9)$, and tell what it represents in this situation.

Find the common difference, d. $d = 0.55 - 0.35 = 0.20$

Substitute 0.35 for $f(1)$ and 0.20 for d.

$f(n) = f(1) + d(n - 1)$

$f(n) = 0.35 + 0.20(n - 1)$

$f(9) = 0.35 + 0.20(8) = 1.95$

So, the cost of postage for a 9-ounce letter is $1.95.

(B) The table shows the cumulative total interest paid, in dollars, on a loan after each month.

Number of months	n	1	2	3	4
Cumulative total ($)	$f(n)$	160	230	300	370

Determine the value of $f(20)$ and tell what it represents in this situation.

Find the common difference, d. $d = \boxed{} - 160 = \boxed{}$

Substitute $\boxed{}$ for $f(1)$ and $\boxed{}$ for d.

$f(n) = f(1) + d(n - 1)$

$f(n) = \boxed{} + \boxed{}(n - 1)$

Find $f(20)$ and interpret the value in context.

$f(n) = f(1) + d(n - 1)$

$f\left(\boxed{}\right) = \boxed{} + \boxed{}\left(\boxed{}\right) = \boxed{}$

So, the cumulative total ________ paid after ________ months is ________.

Your Turn

Construct an explicit rule in function notation for the arithmetic sequence represented in the table. Then interpret the meaning of a specific term of the sequence in the given context.

2. The table shows $f(n)$, the distance, in miles, from the store after Mila has traveled for n minutes.

Time (min)	n	1	2	3	4
Distance (mi)	$f(n)$	20	32	44	56

Determine the value of $f(10)$ and tell what it represents in this situation.

3. The table below shows the total cost, in dollars, of purchasing n battery packs.

Number of battery packs	n	1	2	3	4
Total cost ($)	$f(n)$	4.90	8.90	12.90	16.90

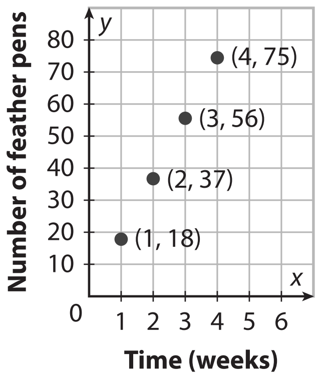
Determine the value of $f(18)$ and tell what it represents in this situation.

Explain 2 Modeling Arithmetic Sequences From a Graph

Given a graph of a real-world situation involving an arithmetic sequence, you can construct a function model and use it to solve problems.

Example 2 **Construct an explicit rule in function notation for the arithmetic sequence represented in the graph, and use it to solve the problem.**

(A) D'Andre collects feather pens. The graph shows the number of feather pens D'Andre has collected over time, in weeks. According to this pattern, how many feather pens will D'Andre have collected in 12 weeks?



Represent the sequence in a table.

n	1	2	3	4
$f(n)$	18	37	56	75

Find the common difference.

$d = 37 - 8 = 19$

Use the general explicit rule for an arithmetic sequence to write the rule in function notation. Substitute 18 for $f(1)$and 19 for d.

$f(n) = f(1) + d(n - 1)$

$f(n) = 18 + 19(n - 1)$

To determine the number of feather pens D'Andre will have collected after 12 weeks,

find $f(12)$.

$f(n) = 18 + 19(n - 1)$

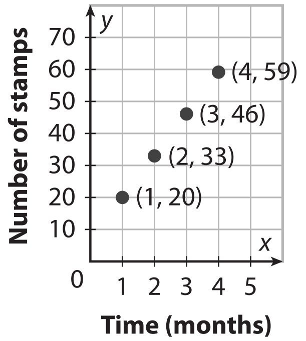
$f(12) = 18 + 19(11)$

$f(12) = 18 + 209$

$f(12) = 227$

So, if this pattern continues, D'Andre will have collected 227 feather pens in 12 weeks.

(B) Eric collects stamps. The graph shows the number of stamps that Eric has collected over time, in months. According to this pattern, how many stamps will Eric have collected in 10 months?



Represent the sequence in a table.

n	1	2	3	4
f(n)				

Find the common difference.

$d = \square - 20 = \square$

Use the general explicit rule for an arithmetic sequence to write the rule in function notation.

Substitute $\square$ for $f(1)$ and $\square$ for d.

$f(n) = f(1) + d(n - 1)$

$f(n) = \square + \square(n - 1)$

To determine the number of stamps Eric will have collected in 10 months, find $f(\square)$.

$f(n) = f(1) + d(n - 1)$

$f(\square) = \square + \square(\square) = \square$

So, if this pattern continues, Eric will have collected ____________ in ____________ months.

Reflect

4. How do you know which variable is the independent variable and which variable is the dependent variable in a real-world situation involving an arithmetic sequence?

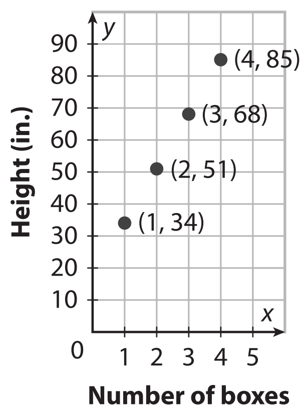
____

____

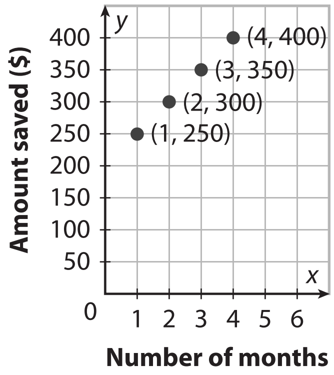
Your Turn

Construct an explicit rule in function notation for the arithmetic sequence represented in the graph, and use it to solve the problem.

5. The graph shows the height, in inches, of a stack of boxes on a table as the number of boxes in the stack increases. Find the height of the stack with 7 boxes.



6. Quynh begins to save the same amount each month to save for a future shopping trip. The graph shows total amount she has saved after each month, n. What will be the total amount Quynh has saved after 12 months?



Explain 3 Modeling Arithmetic Sequences From a Description

Given a description of a real-world situation involving an arithmetic sequence, you can construct a function model and use it to solve problems.

Example 3 **Construct an explicit rule in function notation for the arithmetic sequence represented, and use it to solve the problem. Justify and evaluate your answer.**

The odometer on a car reads 34,240 on Day 1. Every day the car is driven 57 miles. If this pattern continues, what will the odometer read on Day 15?

Analyze Information

- The odometer on the car reads ________ miles on Day 1.
- Every day the car is driven ________ miles.

$f(1) =$ ________ and

$d =$ ________

Formulate a Plan

Write an explicit rule in function notation for the arithmetic sequence, and use it to find ________, the odometer reading on Day 15.

Solve

$$f(n) = f(1) + d(n - 1)$$

$$f(n) = \boxed{} + \boxed{}(n - 1)$$

$$f\left(\boxed{}\right) = \boxed{} + \boxed{}\left(\boxed{}\right)$$

$$f\left(\boxed{}\right) = \boxed{}$$

On the Day 15, the odometer will show ________ miles.

Justify and Evaluate

Using an arithmetic sequence model ________ reasonable because the number of miles on the odometer increases by the same amount each day.

By rounding and estimation:

$34{,}200 + 60(14) = \boxed{} + \boxed{} = \boxed{}$ miles

So ________ miles is a reasonable answer.

Your Turn

Construct an explicit rule in function notation for the arithmetic sequence represented, and use it to solve the problem. Justify and evaluate your answer.

7. Ruby signed up for a frequent-flier program. She receives 3400 frequent-flier miles for the first round-trip she takes and 1200 frequent-flier miles for all additional round-trips. How many frequent-flier miles will Ruby have after 5 round-trips?

8. A gym charges each member $100 for the first month, which includes a membership fee, and $50 per month for each month after that. How much money will a person spend on their gym membership for 6 months?

Elaborate

9. What domain values usually make sense for an arithmetic sequence model that represents a real-world situation?

10. When given a graph of an arithmetic sequence that represents a real-world situation, how can you determine the first term and the common difference in order to write a model for the sequence?

11. What are some ways to justify your answer when creating an arithmetic sequence model for a real-world situation and using it to solve a problem?

12. **Essential Question Check-In** How can you construct a model for a real-world situation that involves an arithmetic sequence?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. A T-shirt at a department store costs \$7.50. The total cost, in dollars, of a T-shirts is given by the function $C(a) = 7.5a$.

a. Complete the table of values for 4 T-shirts.

T-shirts	1	2	3	4
Cost (\$)				

b. Determine the common difference.

c. What does the variable a represent? What are the reasonable domain values for a?

2. A car dealership sells 5 cars per day. The total number of cars C sold over time in days is given by the function $C(t) = 5t$.

a. Complete the table of values for the first 4 days of sales.

Time (days)	1	2	3	4
Number of Cars				

b. Determine the common difference.

c. What do the variables represent? What are the reasonable domain and range values for this situation?

3. A telemarketer makes 82 calls per day. The total number of calls made over time, in days, is given by the function $C(t) = 82t$.

a. Complete the table of values for 4 days of calls.

Time (days)	1	2	3	4
Number of Calls				

b. Determine the common difference.

c. What do the variables represent? What are the reasonable domain and range values for this situation?

Construct an explicit rule in function notation for the arithmetic sequence represented in the table. Then determine the value of the given term, and explain what it means.

4. Darnell starts saving the same amount from each week's paycheck. The table shows the total balance $f(n)$ of his savings account over time in weeks.

Time (weeks) *n*	1	2	3	4
Savings Account Balance($) *f*(*n*)	$250	$380	$510	$640

Determine the value of $f(9)$, and explain what it represents in this situation.

5. Juan is traveling to visit universities. He notices mile markers along the road. He records the mile markers every 10 minutes. His father is driving at a constant speed. Complete the table.

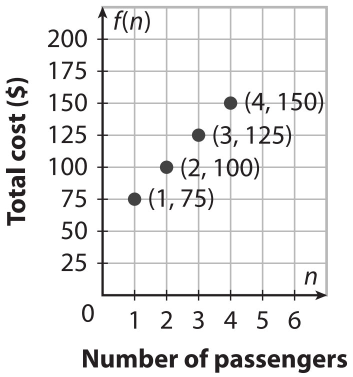
a.

Time Interval	Mile Marker
1	520
2	509
3	498
4	
5	
6	

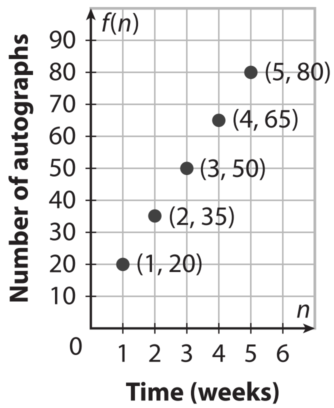
b. Find $f(10)$, and tell what it represents in this situation.

Construct an explicit rule in function notation for the arithmetic sequence represented in the graph. Then determine the value of the given term, and explain what it means.

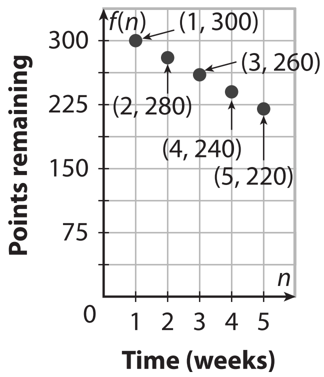
6. The graph shows total cost of a whitewater rafting trip and the corresponding number of passengers on the trip. Find $f(8)$, and explain what it represents.



7. Ed collects autographs. The graph shows the total number of autographs that Ed has collected over time, in weeks. Find $f(12)$, and explain what it represents.

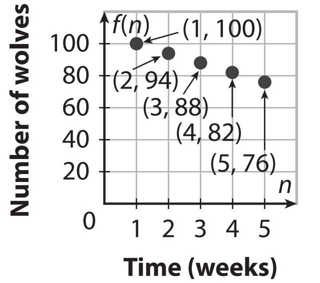


8. **Finance** Bob purchased a bus pass card with 320 points. Each week costs 20 points for unlimited bus rides. The graph shows the points remaining on the card over time in weeks. Determine the value of $f(10)$, and explain what it represents.



9. **Biology** The local wolf population is declining. The graph shows the local wolf population over time, in weeks.

Find $f(9)$, and explain what it represents.



Construct an explicit rule in function notation for the arithmetic sequence. Then determine the value of the given term, and explain what it means.

10. **Economics** To package and ship an item, it costs \$5.75 for the first pound and \$0.75 for each additional pound. Find the 12th term, and explain what it represents.

11. A new bag of cat food weighs 18 pounds. At the end of each day, 0.5 pound of food is removed to feed the cats. Find the 30th term, and explain what it represents.

12. Carrie borrows \$960 interest-free to pay for a car repair. She will repay \$120 monthly until the loan is paid off. How many months will it take Carrie to pay off the loan? Explain.

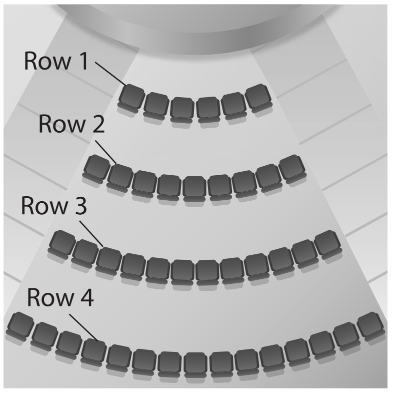
13. The rates for a go-kart course are shown.

Number of Laps n	1	2	3	4
Total cost (\$) $f(n)$	7	9	11	13

a. What is the total cost for 15 laps?

b. Suppose that after paying for 9 laps, the 10th lap is free. Will the sequence still be arithmetic? Explain.

14. **Multi-Part** Seats in a concert hall are arranged in the pattern shown.



a. The numbers of seats in the rows form an arithmetic sequence. Write a rule for the arithmetic sequence.

b. How many seats are in Row 15?

c. Each ticket costs \$40. If every seat in the first 10 rows is filled, what is the total revenue from those seats?

d. An extra chair is added to each row. Write the new rule for the arithmetic sequence and find the new total revenue from the first 10 rows.

H.O.T. Focus on Higher Order Thinking

15. Explain the Error The table shows the number of people who attend an amusement park over time, in days.

Time (days) n	1	2	3	4
Number of people $f(n)$	75	100	125	150

Sam writes an explicit rule for this arithmetic sequence: $f(n) = 25 + 75(n - 1)$

He then claims that according to this pattern, 325 people will attend the amusement park on Day 5. Explain the error that Sam made.

16. Communicate Mathematical Ideas Explain why it may be harder to find the nth value of an arithmetic sequence from a graph if the points are not labeled.

17. Make a prediction Verona is training for a marathon. The first part of her training schedule is given in the table.

Session n	1	2	3	4	5	6
Distance (mi) $f(n)$	3.5	5	6.5	8	9.5	11

a. Is this training schedule an arithmetic sequence? Explain. If it is, write an explicit rule for the sequence.

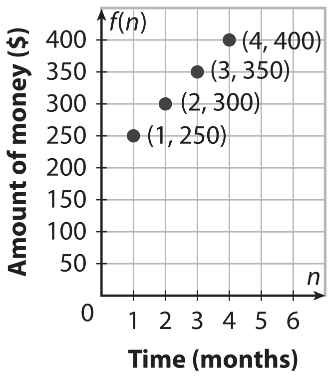
b. If Verona continues this pattern, during which training session will she run 26 miles?

18. If Verona's training schedule starts on a Monday and she runs every third day, on which day will she run 26 miles?

19. Multiple Representations Determine whether the following graph, table, and verbal description all represent the same arithmetic sequence.

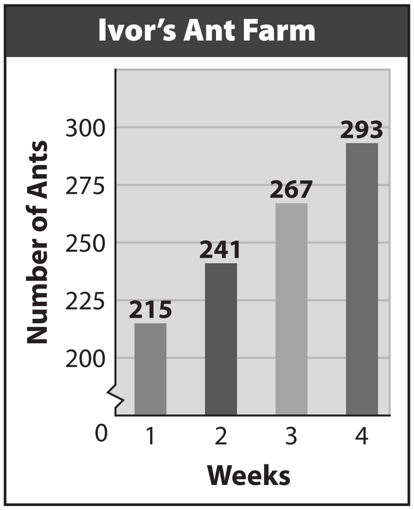
Time (months) *n*	1	2	3	4
Amount of money ($) *f* (*n*)	250	300	350	400

A person deposits $250 dollars into a bank account. Each month, he adds $25 dollars to the account, and no other transactions occur in the account.



Lesson Performance Task

The graph shows the population of Ivor's ant colony over the first four weeks. Assume the ant population will continue to grow at the same rate.



a. Write an explicit rule in function notation.

b. If Ivor's ants have a mass of 1.5 grams each, what will be the total mass of all of his ants in 13 weeks?

c. When the colony reaches 1385 ants, Ivor's ant farm will not be big enough for all of them. In how many weeks will the ant population be too large?