Name ____________________ Class ____________ Date ________

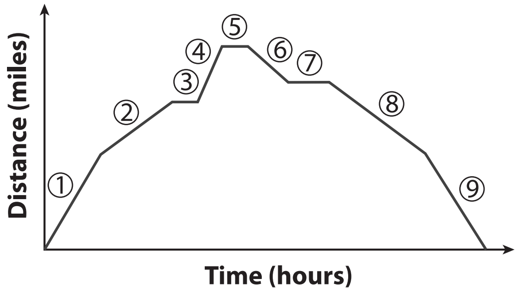
3.1 Graphing Relationships

Essential Question: How can you describe a relationship given a graph and sketch a graph given a description?

Resource Locker

Explore Interpreting Graphs

The distance a delivery van is from the warehouse varies throughout the day. The graph shows the distance from the warehouse for a day from 8:00 am to 5:00 pm.



(A) Segment 1 shows that the delivery van moved away from the warehouse. What does segment 2 show?

____

____

(B) Based on the time frame, what change in the distance from the warehouse is represented by segment 6?

____

____

(C) Which line segments show intervals where the distance did not change?

____

(D) What is a possible explanation for these segments?

____

Reflect

1. **Discussion** Explain how the slope of each segment of the graph is related to whether the delivery truck is not moving, is moving away from, or is moving toward the warehouse.

____

____

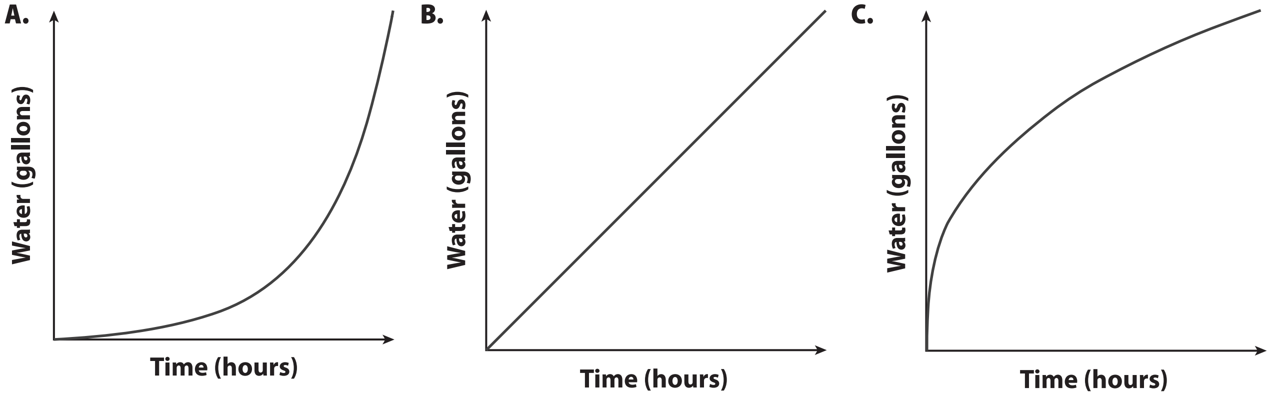
____

____

Explain 1 Relating Graphs to Situations

Graphs can often be drawn to represent real life situations. These graphs are not always easily derived from equations, but rather represent certain situations. For example, these graphs may include the amount of rain over a certain period of time, or the height of a bouncing ball over a certain period of time.

Example 1 **Three hoses fill three different water barrels. A green hose fills a water barrel at a constant rate. A black hose is slowly opened when filling the barrel. A blue hose is completely open at the beginning and then slowly closed. The three graphs of the situations are shown.**



Ⓐ Which graph best represents the amount of water in the barrel filled by the green hose?

Since the flow of the water is constant, the amount of water in the barrel should be a steady increase. Thus, graph B best represents the situation.

Ⓑ Describe the water level represented by each graph. Then determine which graph represents each situation.

Describe the water level for graph A.

____

Describe the water level for graph C.

____

Graph A represents the ____________ hose and graph C represents the ____________ hose.

Reflect

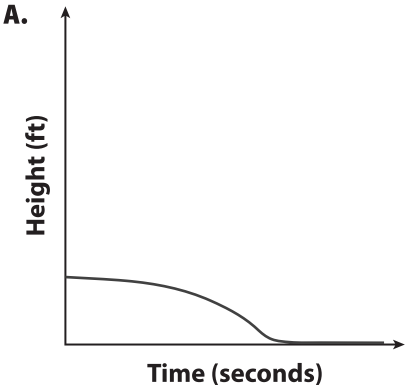
2. Could a graph of the amount of water in a water barrel slant downward from left to right? Explain.

____

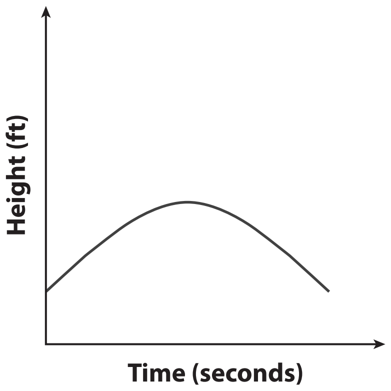
Your Turn

You and a friend are playing catch. You throw three different balls to your friend. You throw the first ball in an arc and your friend catches it. You throw the second ball in an arc, but this time the ball gets stuck in a tree. You throw the third ball directly at your friend, but it lands in front of your friend, and rolls the rest of the way on the ground. The three graphs of these situations are shown.

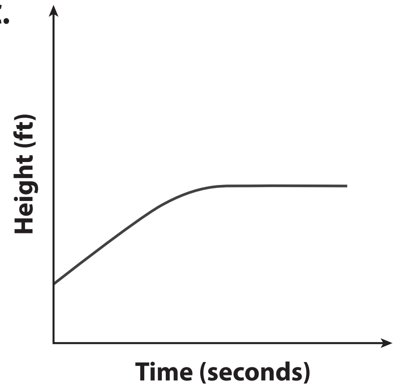
A.



B.



C.



3. Which graph represents the situation where the ball gets stuck in the tree?

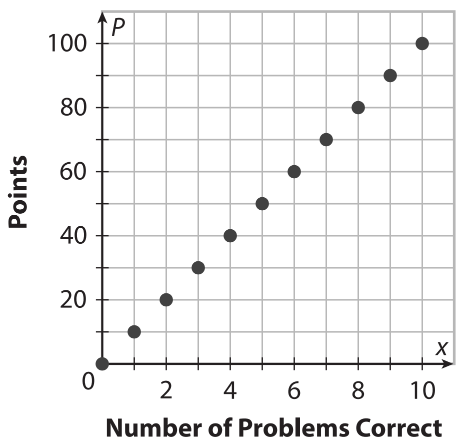
4. Describe the height of the ball represented by the other two graphs.

Explain 2 Sketching Graphs for Situations

Some graphs that represent real-world situations are drawn without any interruptions. In other words, they are *continuous graphs*. A **continuous graph** is a graph that is made up of connected lines or curves. Other types of graphs are not continuous. They are made up of distinct, unconnected points. These graphs are called **discrete graphs**.

Example 2 **Sketch a graph of the situation, tell whether the graph is continuous or discrete, and determine the domain and range.**

Ⓐ A student is taking a test. There are 10 problems on the test. For each problem the student answers correctly, the student received 10 points.

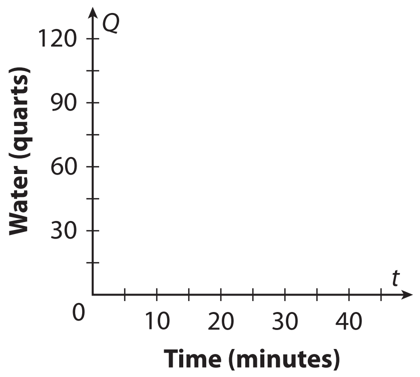


The graph is made up of multiple unconnected points, so the graph is discrete.

The student can get anywhere from 0 to 10 questions right, so the domain is the whole numbers from 0 to 10.

If the student gets 0 problems correct, the student gets 0 points. If the student gets 10 problems correct, the student gets 100 points. So the range is whole number multiples of 10 from 0 to 100.

(B) A bathtub is being filled with water. After 10 minutes, there are 75 quarts of water in the tub. Then someone accidentally pulls the drain plug while the water is still running, and the tub begins to empty. The tub loses 15 quarts in 5 minutes, and then someone plugs the drain and the tub fills for 6 more minutes, gaining another 45 quarts of water. After a 15-minute bath, the person gets out and pulls the drain plug. It takes 11 minutes for the tub to drain.



The graph is a ______________ graph.

The domain is ______________ .

The range is ______________ .

Your Turn

Sketch a graph of the situation, tell whether the graph is continuous or discrete, and determine the domain and range.

5. At the start of a snowstorm, it snowed two inches an hour for two hours, then slowed to one inch an hour for an additional hour before stopping. Three hours after the snow stopped, it began to melt at one-half an inch an hour for two hours.

____

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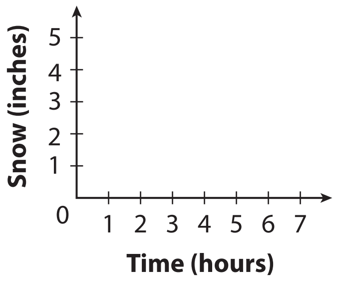
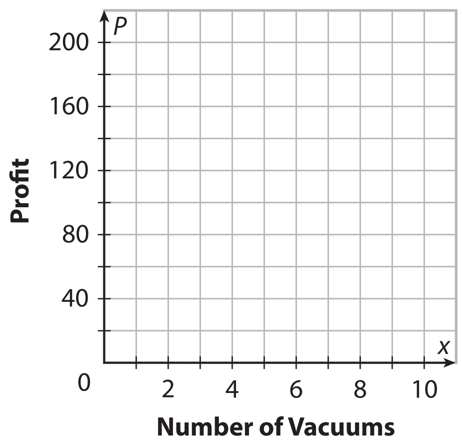


6. A local salesman is going door to door trying to sell vacuums. For every vacuum he sells, he makes $20. He can sell a maximum of 10 vacuums a day.

____

____

____



Elaborate

7. When interpreting graphs of real world situations, what can the slope of each part tell you about the situation?

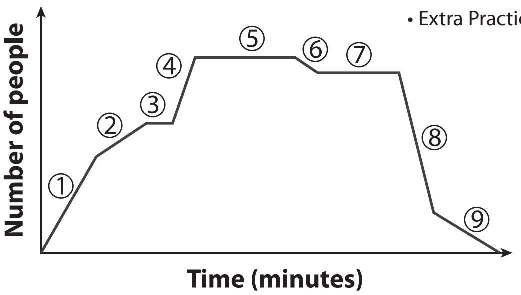
8. **Discussion** What is the best way to sketch the graph of a situation?

9. **Essential Question Check-In** How can you tell when to use a discrete graph as opposed to using a continuous graph? Give an example of each.

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

The graph shows the attendance at a hockey game, and the rate at which the fans enter and exit the arena.



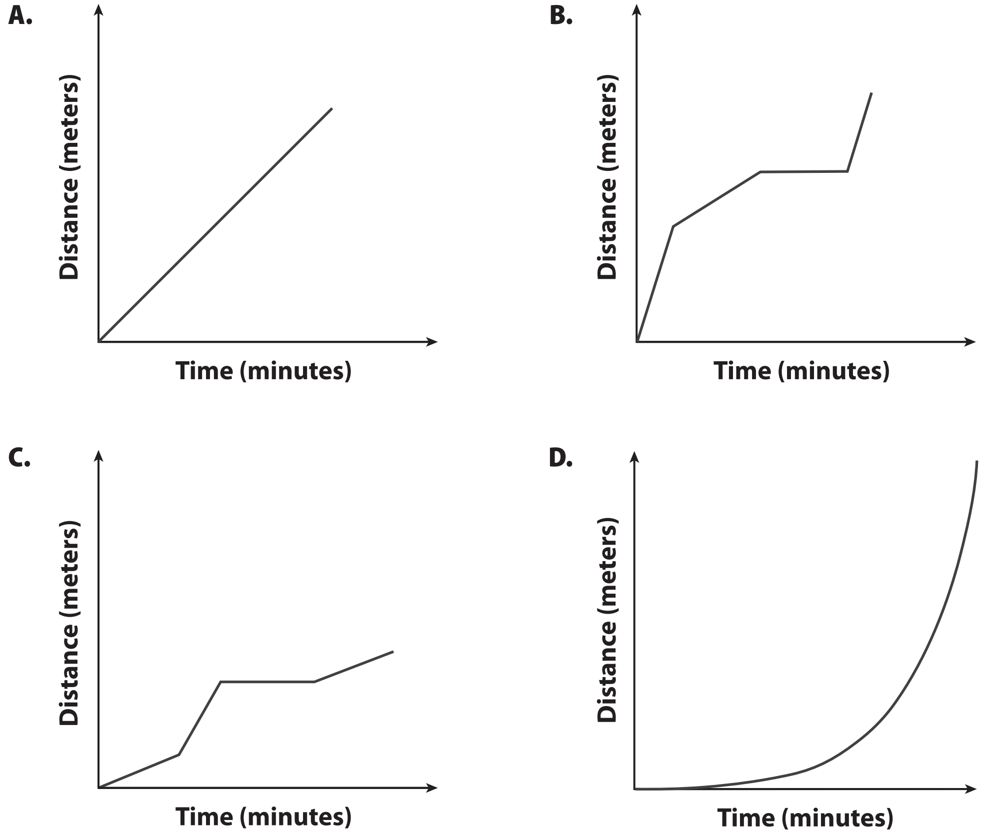
1. Compare segments 1 and 2. What do they represent?

2. What does segment 8 represent in terms of the game?

3. What is the significance of segments 5 and 7?

4. What does segment 6 mean?

Use Graphs A–D for Exercises 5–8. Janelle alternates between running and walking. She begins by walking for a short period, and then runs for the same amount of time. She takes a break before beginning to walk again. Consider the graphs shown.



5. Which graph best represents the given situation?

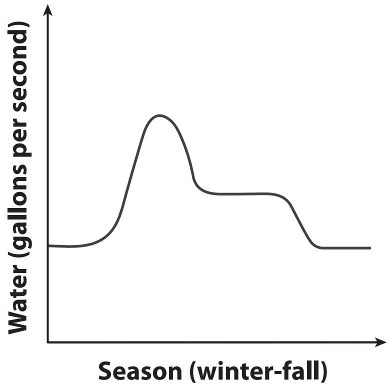
6. Describe the other three graphs.

7. What if Janelle began by running, then slowed to a walk, stopped, and then began running again. Which graph would represent this situation?

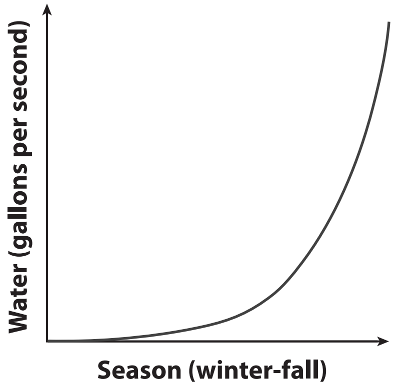
8. What are possible situations for graphs A and D?

Use Graphs A–D for Exercises 9–11. During the winter, the amount of water that flows down a river remains at a low constant. In the spring, when the snow melts, the flow of water increases drastically, until it decreases to a steady rate in the summer. The flow then slowly decreases through the fall into the winter. Consider the graphs shown.

A.

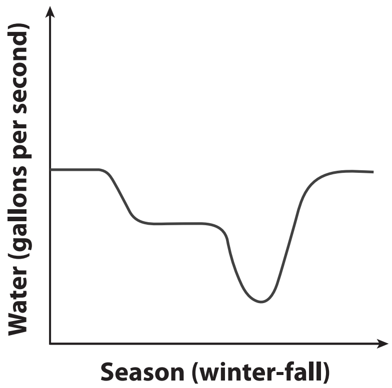


B.



C.

D.

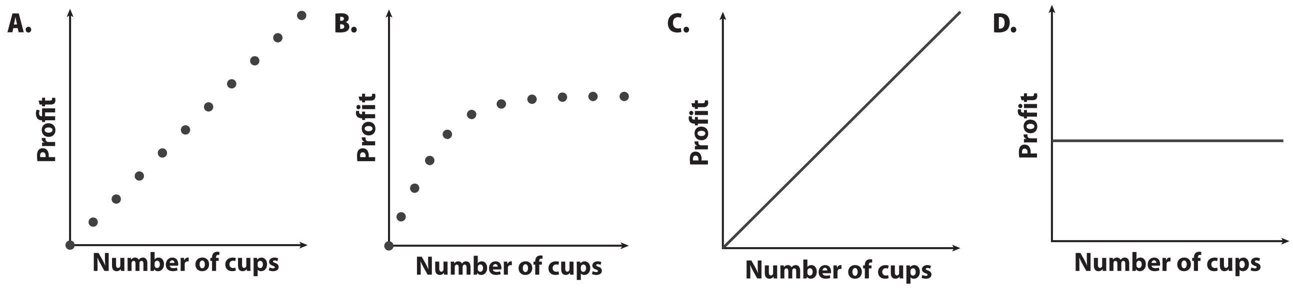


9. Which graph best represents the given situation?

10. Describe the other three graphs.

11. What are possible situations for graphs B, C, and D?

Two children are selling lemonade. They are charging $1 for a cup. They only sell 10 cups. Consider the graphs shown.



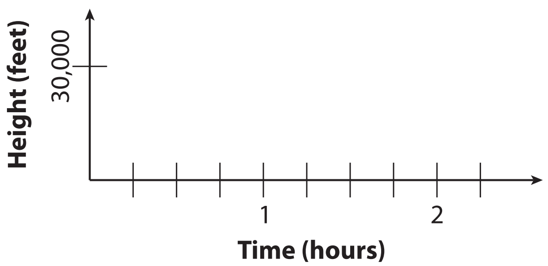
12. Which graph best represents the given situation?

13. What situations could the other graphs represent?

14. Is the graph that represents the given situation discrete or continuous?

A plane takes off and climbs steadily for 15 minutes until it reaches 30,000 feet. It travels at that altitude for 2 hours until it begins to descend to land, which it takes 15 minutes at a constant rate.

15. Sketch a graph of the situation.

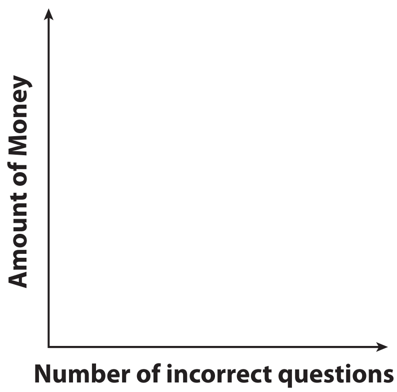


16. Is the graph discrete or continuous?

17. Determine the domain and range.

A contestant on a game show is given $100 and is asked five questions. The contestant loses $20 for every wrong answer.

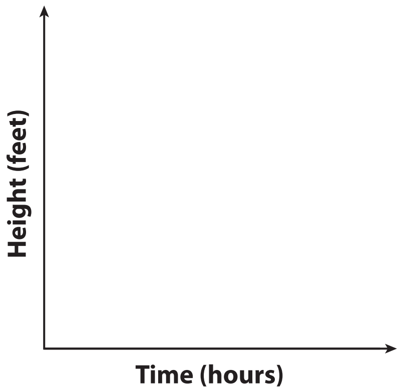
18. Sketch a graph of the situation.



19. Is the graph discrete or continuous?

20. Determine the domain and range.

You decide to hike up a mountain. You climb steadily for 2 hours, then take a 30 minute break for lunch. Then you continue to climb, faster than before. When you make it to the summit, you enjoy the view for an hour. Finally, you decide to climb down the mountain, but stop halfway down for a short break. Then you continue down at a slower pace than before.

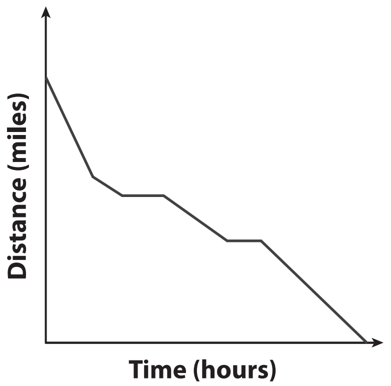


21. Sketch a graph of the situation.

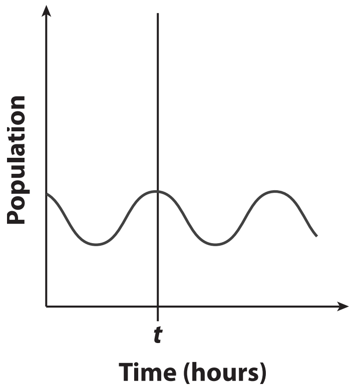
22. Is the graph discrete or continuous?

H.O.T. Focus on Higher Order Thinking

23. Analyze Relationships Write a possible situation for the graph shown.

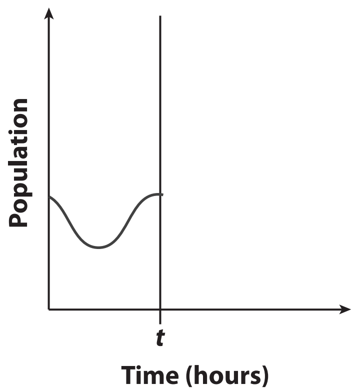


24. Represent Real-World Problems Scientists are conducting an experiment on a bacteria colony that causes its population to fluctuate. The population of a bacteria colony is shown in the graph.



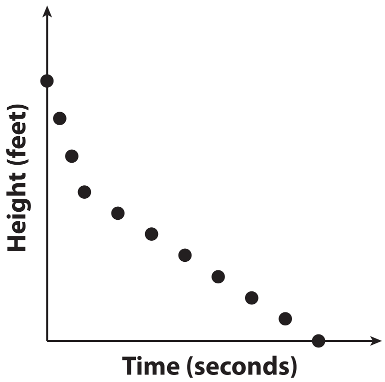
a. What happened to the bacteria colony before time t?

b. Suppose at time t, a second colony of bacteria is added to the first. Draw a new graph to show how this action might affect the population after time t.



c. Suppose at some point after time t, scientists add a substance to the colony that destroys some of the bacteria. Describe how your graph from part b might change.

25. **Explain the Error** A student is told to draw a graph of the situation which represents the height of a skydiver with respect to time. He drew the following graph. Explain the student's error and draw the correct graph.



Lesson Performance Task

A digital rain gauge has an outdoor sensor that collects rainfall and transmits data to an indoor display. Assume you produced a graph of all the data collected by the rain gauge over a 24-hour period.

a. Would that graph be a discrete graph or a continuous graph? Explain your reasoning.

b. Describe the general shape of the graph assuming it rained at a rate of 0.1 inch per hour for the entire 24-hour period.

c. Describe the general shape of the graph assuming it rained 0.1 inch per hour for 6 hours, stopped raining for 6 hours, and then rained 0.2 inch for 12 hours.

Name ______________________ Class ____________ Date ________

3.2 Understanding Relations and Functions

Essential Question: How do you represent relations and functions?

Resource Locker

Explore Understanding Relations

A **relation** is a set of ordered pairs (x, y) where x is the input value and y is the output value. The **domain** is all possible inputs of a relation, and the **range** is all possible outputs of a relation. For example, the given relation represents the number of whole-wheat cracker boxes sold and the money earned.

$\{(1, 4), (2, 8), (3, 12), (4, 16)\}$.

Domain: $\{1, 2, 3, 4\}$ Range: $\{2, 8, 12, 16\}$

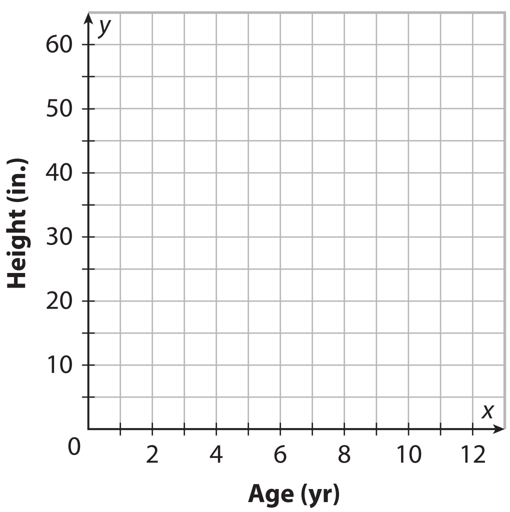
(A) For the following relation, the input, x, is the ages of boys and the output, y, is their corresponding height, in inches.

$\{(7, 41), (8, 45), (9, 49), (10, 52), (10, 53), (11, 55), (12, 59)\}$

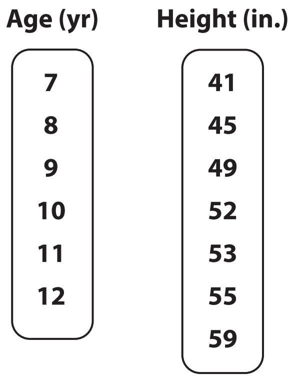
(B) Fill the values in the table.

x	y

(C) Plot the points on the graph.



Ⓓ Complete the mapping diagram.



Ⓔ State the domain of the relation.

____

Ⓕ State the range of the relation.

____

Reflect

1. **Discussion** The number 10 appears twice in the *x* column of the table. How many times is it written in the domain? Explain.

____

____

Explain 1 Recognizing Functions

A **function** is a type of relation in which there is only one output value for each input value.

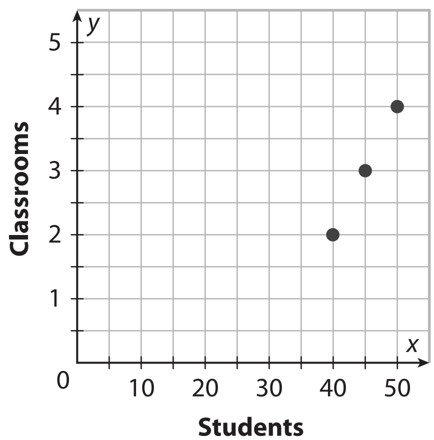
For every input value, there is a unique output value.

Example: $y = x^2$. When $x = 3$, y will always be equal to 9.

Example 1 **Give the domain and range of each relation. State the corresponding outputs for the given inputs in context and explain whether the relation is a function.**

Ⓐ The given relation represents the number of students and the number of classrooms the school has to have for the corresponding number of students.

Students *x*	Classrooms *y*
40	2
45	3
50	4



Domain: {40, 45 50}

The domain represents the number of students.

Range: {2, 3, 4}

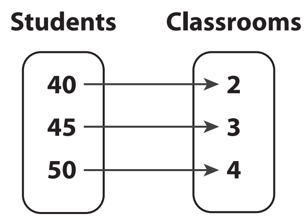
The range represents the number of classrooms.

For an input of 40 students, there is an output of 2 classrooms.

For an input of 45 students, there is an output of 3 classrooms.

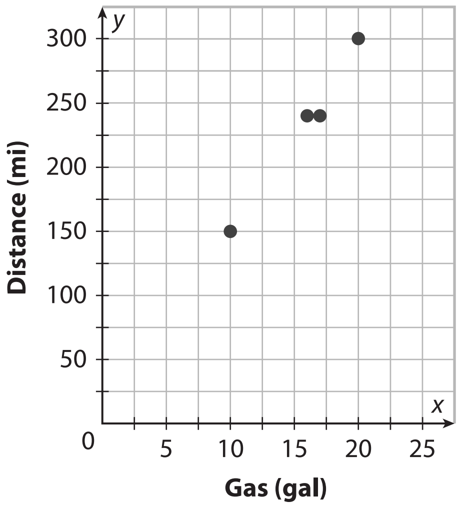
For an input of 50 students, there is an output of 4 classrooms.

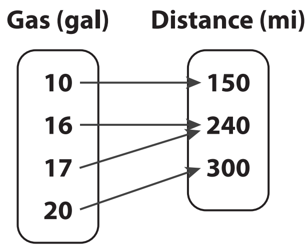
This relation is a function. Each domain value is paired with exactly one range value.



Ⓑ The given relation represents the amount of gas in gallons and the distance traveled in miles from that amount of gas.

Gas (gal)	Distance (mi)
10	150
16	240
17	240
20	300





Domain: ____________

The domain represents ____________________

Range: ____________

The range represents ________________.

For an input of ____________, there is an output of ____________.

For an input of ____________, there is an output of ____________.

For an input of ____________, there is an output of ____________.

For an input of ____________, there is an output of ____________.

This relation __________ a function. Each domain value is paired with __________ range value.

Reflect

2. If each month in a year was paired with all the possible numbers of days in the month, will the result be a function? Explain.

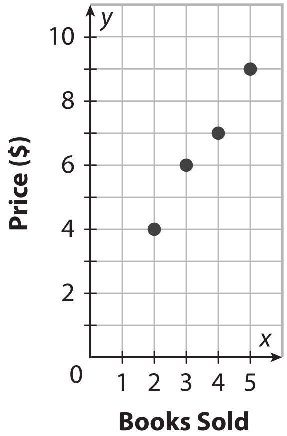
____

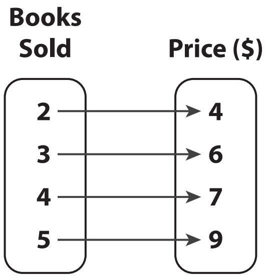
Your Turn

Give the domain and range of each relation and interpret them in context. State the corresponding outputs for the given inputs in context and explain whether the relation is a function.

3. The relation represents the number of books sold and the price for the corresponding number of books.

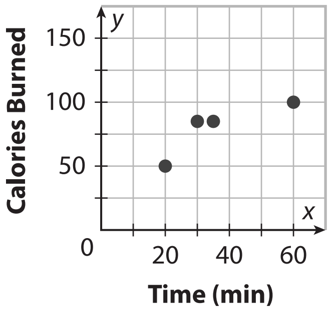
Number of books sold	Price ($)
2	4
3	6
4	7
5	9

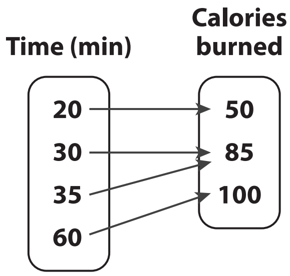




4. The relation represents the time spent exercising and the number of calories burned during that time.

Time (min)	Calories burned
20	50
30	85
35	85
60	100





Explain 2 Understanding the Vertical Line Test

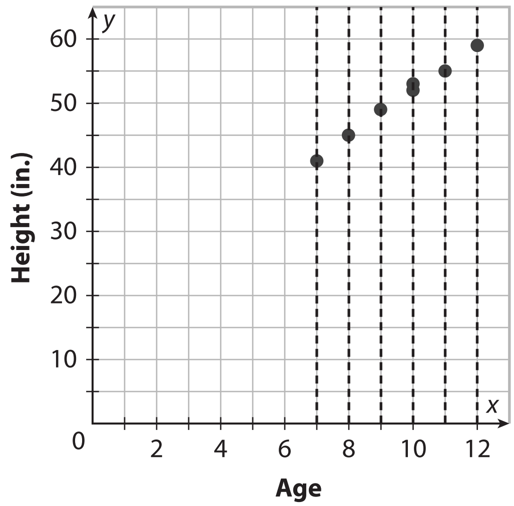
A test, called the *vertical line test*, can be used to determine if a relation is a function. The **vertical line test** states that a relation is a function if and only if a vertical line does not pass through more than one point on the graph of the relation.

Example 2 **Use the vertical line test to determine if each relation is a function. Explain.**

Ⓐ Draw a vertical line through each point of the graph.

Does any vertical line touch more than one point? Yes

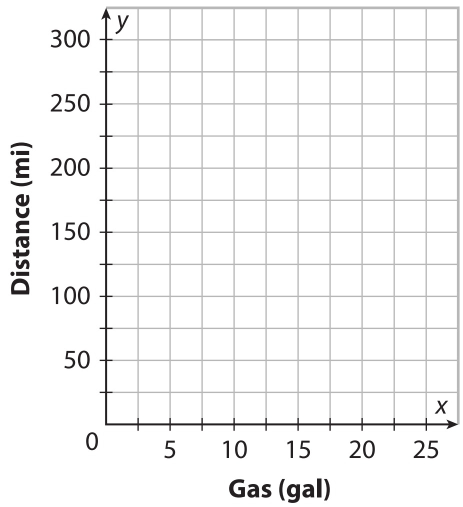
Since a vertical line does pass through more than one point, the graph fails the vertical line test. So, the relation is not a function.



Ⓑ Draw a vertical line through each point of the graph.

Does any vertical line touch more than one point? __________

Since a vertical line __________ pass through more than one point, the graph __________ the vertical line test. So, the relation __________ a function.



Reflect

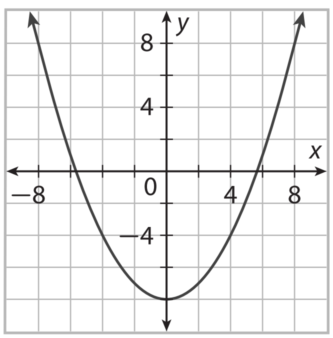
5. Why does the vertical line test work?

____

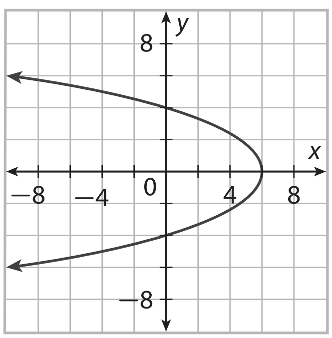
Your Turn

Use the vertical line test to determine if each relation is a function.

6.



7.



Elaborate

8. How can you use a mapping diagram to determine the domain and the range of a relation?

9. **Discussion** For a discrete function, can the number of elements in the range be greater than the number of elements in the domain? Explain.

10. Is a relation a function if its graph intersects the y-axis twice?

11. **Essential Question Check-In** You are asked to determine if the relation $y = x^2 - 8x + 4$ is a function. What would be the best way to represent this relation in order to determine if it is a function or not? Explain.

Evaluate: Homework and Practice

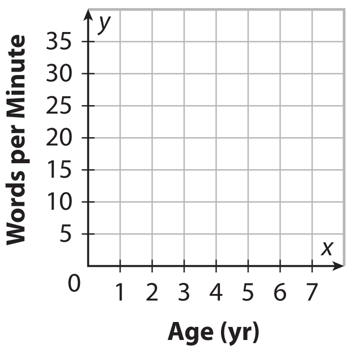
Personal Math Trainer

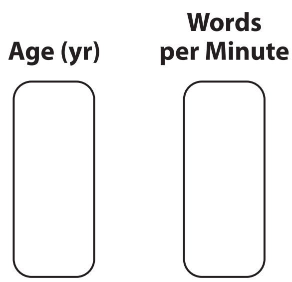
- Online Homework
- Hints and Help
- Extra Practice

Express each relation as a table, as a graph, and as a mapping diagram.

1. The relation represents ages of students and the number of words they can write per minute.
$\{(5, 10), (6, 20), (6, 23), (7, 35)\}$

x	y



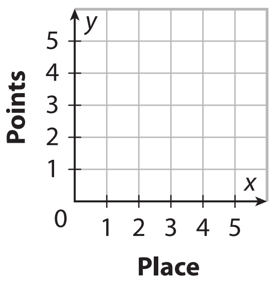


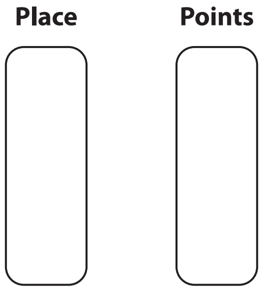
Express each relation as a table, as a graph, and as a mapping diagram.

2. The relation represents the place won in a track meet and the number of points that place finish is worth.

$\{(1, 5), (2, 3), (3, 2), (4, 1), (5, 0)\}$

x	*y*



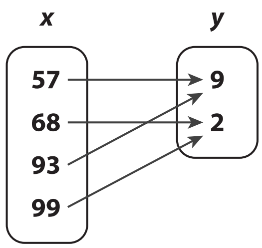


State the domain and range of each relation.

3.

x	*y*
2	5
7	8
8	15
11	12
15	19

4.



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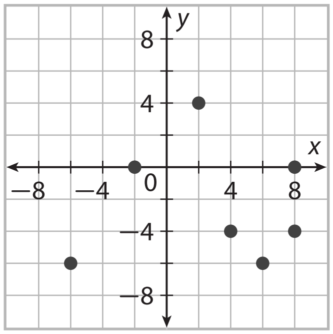
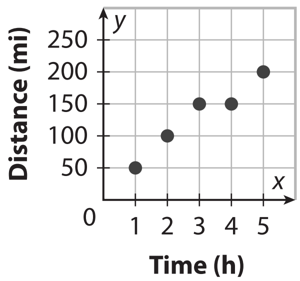


State the domain and range of each relation, interpret in context, and explain if it is a function or not.

6. The relation represents the age of each student and the number of pets the student has.

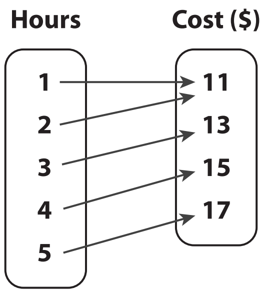
Age	Number of Pets
6	3
8	2
9	0
11	1
11	2

7. The relation represents time driven in hours and the number of miles traveled at the end of each hour.

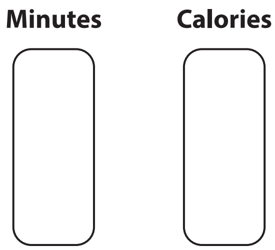


State the domain and range of each relation, interpret in context, and explain if it is a function or not.

8. The relation represents the number of hours a person is able to rent a canoe and the cost of renting the canoe for that many hours.



9. A person can burn about 6 calories per minute bicycling. Let *x* represent the number of minutes bicycled, and let *y* represent the number of calories burned. Create a mapping diagram to show the number of calories burned by bicycling for 60, 120, 180, or 240 minutes.



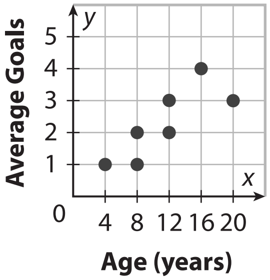
10. The table represents a sample of ages of people and their shoe size.

Age	Shoe Size
x	*y*
11	7
12	8
13	10
15	10
15	10.5
16	11

11. An electrician charges a base fee of $75 plus $50 for each hour of work. The minimum the electrician charges is $175. Create a table that shows the amount the electrician charges for 1, 2, 3, and 4 hours of work.

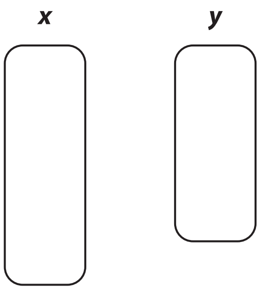
x	*y*

12. The graph represents the average soccer goals scored for players of different ages. Determine the domain and range of the relation in context and explain whether or not this represents a function.

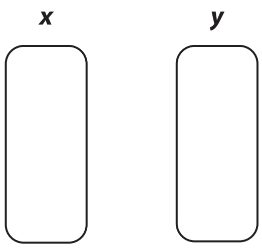


Express each relation as a mapping diagram and explain whether or not the relation represents a function.

13. $\{(13, 33), (17, 25), (22, 22), (25, 17), (33, 17)\}$

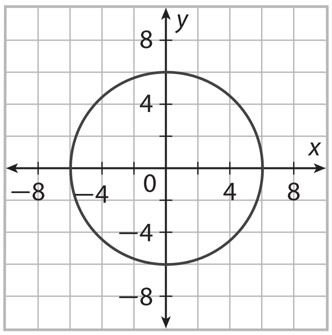


14. $\{(1, 2), (5, 2), (5, 4), (7, 6), (11, 6)\ (11, 8)\}$



Use the vertical line test to determine if each relation is a function.

15.



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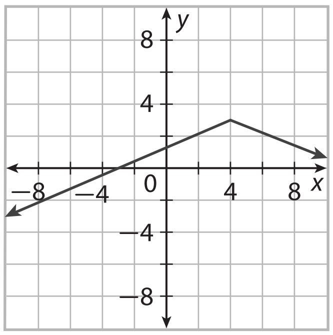
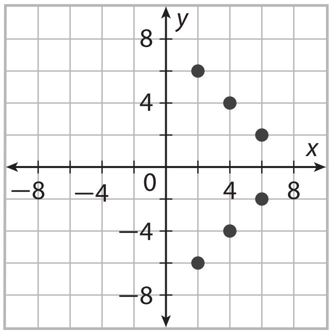


Use the vertical line test to determine if each relation is a function.

17.



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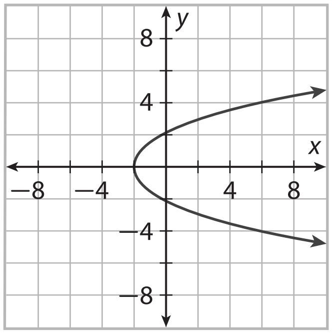

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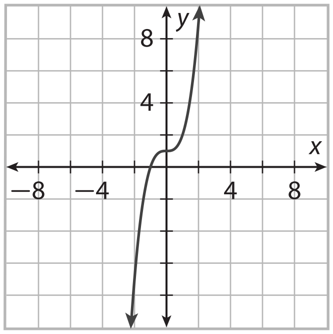

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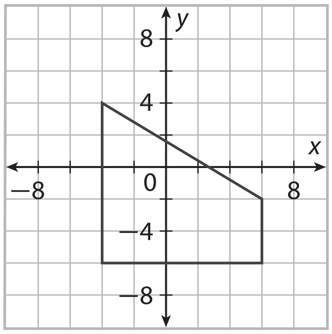

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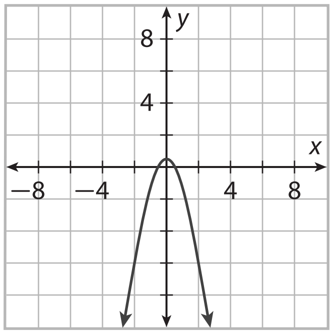

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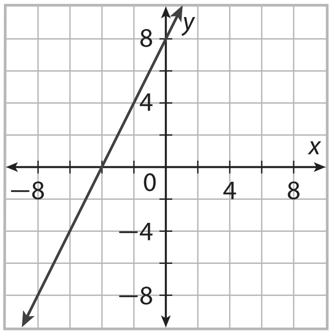
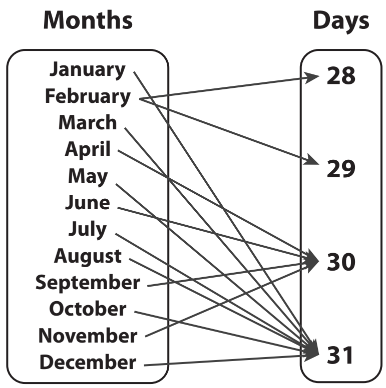


H.O.T. Focus on Higher Order Thinking

23. Draw Conclusions Examine the mapping diagram. The first set is the months of the year, and the second set is the possible number of days per month. Is the relation a function? Explain.

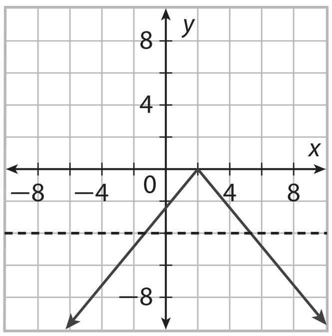


24. Justify Reasoning Tell whether each situation represents a function. Explain your reasoning. If the situation represents a function, give the domain and range.

a. Each U.S. coin is mapped to its monetary value.

b. A \$1, \$5, \$10, \$20, \$50, or \$100 bill is mapped to all the sets of coins that are the same as the total value of the bill.

25. Explain the Error A student was given a graph and asked to use the vertical line test to determine if the relation was a function or not. The student said that the relation failed the vertical line test and the graph was not a function. What error did the student make? Explain the error and give the correct answer.



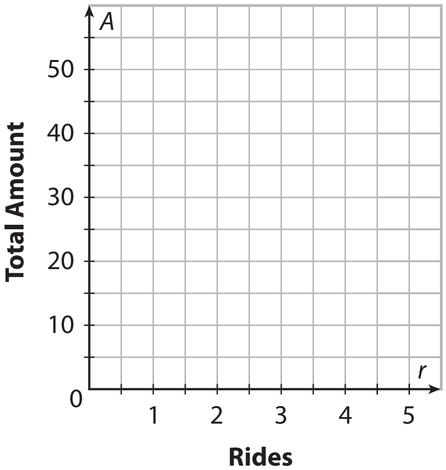
Lesson Performance Task

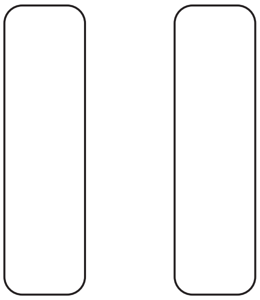
At an amusement park, a person spends \$30 on admission and food, and then goes on r number of rides that cost \$2 each.

a. Write an equation to represent the total amount A spent at the amusement park if a person goes on anywhere from 0 to 5 rides.

b. Represent the relation as a table, as a graph, and as a mapping diagram.

c. Find the domain and range, and then determine whether the relation is a function or not.





Name ______________________ Class ______________ Date ________

3.3 Modeling with Functions

Essential Question: What is function notation and how can you use functions to model real-world situations?

Resource Locker

Explore 1 Identifying Independent and Dependent Variables

The input of a function is the **independent variable**. The output of the function is the **dependent variable**. The value of the dependent variable depends on, or is a function of, the value of the independent variable.

Identify dependent and independent variables in each situation.

In the winter, more electricity is used when the outside temperature goes down, and less is used when the outside temperature rises.

(A) The ______________ depends on the ______________.

(B) Dependent: ______________

Independent: ______________

(C) The cost of shipping a package is based on its weight.

The ______________ depends on the ______________.

(D) Dependent: ______________ Independent: ______________

(E) The faster Tom walks, the quicker he gets home.

The ______________ depends on the ______________.

(F) Dependent: ______________ Independent: ______________

Reflect

1. **Discussion** Give a situation where "time" is the dependent variable and "distance" is the independent variable.

____

____

2. In Explore 1, explain how you know that the amount of electricity used is not the independent variable.

____

Explore 2 Applying Function Notation

If x is the independent variable and y is the dependent variable, then you can use **function notation** to write $y = f(x)$, which is read "y equals f of x," where f names the function. When an equation in two variables describes a function, you always can use function notation to write it.

The dependent variable	is	a function of	the independent variable.
y	is	a function of	x.
y	$=$	f	(x)

Write an equation in function notation.

Amanda babysits and charges $5 per hour.

Time Worked in Hours (x)	1	2	3	4
Amount Earned in Dollars (y)	5	10	15	20

(A) The ______________ is $5 times the ______________.

(B) An algebraic expression that defines a function is a **function rule**. Write an equation using two variables to show this relationship.

Amount earned	is	$5	times	the number of hours worked.
↓	↓	↓	↓	↓
____	$=$	5	•	____

(C) The dependent variable is a function of the independent variable. Write the equation in function notation.

Amount earned	is	$5	times	the number of hours worked.
↓	↓	↓	↓	↓
y	$=$	5	•	x
____	$=$	5	•	x

Reflect

3. **Discussion** Can y be used instead of $f(x)$ in function notation? If so, tell why. If not, give an example of a function not written in function notation and the same function written in function notation.

Explain 1 Modeling Using Function Notation

The value of the dependent variable depends on, or is a function of, the value of the independent variable. If x is the independent variable and y is the dependent variable, then the function notation for y will read "f of x," where f names the function. When an equation in two variables describes a function, you can use function notation to write it.

Example 1 **For each example identify the independent and dependent variables. Write an equation in function notation for each situation, and then use the equation to solve the problem.**

Ⓐ A lawyer's fee is $180 per hour for his services. How much does the lawyer charge for 5 hours?

The fee for the lawyer depends on how many hours he works.

Dependent: fee; Independent: hours

Let h represent the number of hours the lawyer works.

The function for the lawyer's fee is $f(h) = 180h$.

$f(h) = 180h$

$f(5) = 180(5)$ Substitute 5 for h.

$= 900$ Simplify.

The lawyer charges $900 for 5 hours of work.

Ⓑ The admission fee at a carnival is $9. Each ride costs $1.75. How much does it cost to go to the carnival and then go on 12 rides?

The __________ depends on the ______________, plus $9.

Dependent: __________ Independent: ________________

Let r represent the ______________. The function for the total cost of the carnival is

$f(r) =$ ______________.

Substitute 12 for r into the function for the total cost of the carnival, and find the total cost.

$f(\square) = \square$

$f(\square) = \square$

It costs ______ to go to the carnival and go on ______ rides.

Your Turn

For each example identify the independent and dependent variables. Write an equation in function notation for each situation. Then use the equation to solve the problem.

4. Kate earns $7.50 per hour. How much money will she earn after working 8 hours?

Explain 2 Choosing a Reasonable Domain and Range

When a function describes a real-world situation, every real number is not always a reasonable choice for the domain and range. For example, a number representing the length of an object cannot be negative, and only whole numbers can represent a number of people.

Example 2 **Write a function in function notation for each situation. Find a reasonable domain and range for each function.**

(A) Manuel has already sold \$20 worth of tickets to the school play. He has 4 tickets left to sell at \$2.50 per ticket. Write a function for the total amount collected from ticket sales.

Let t represent the number of tickets to sell.

Total amount collected from ticket sales	is	\$2.50	per	ticket	plus	tickets already sold
$f(t)$	=	\$2.50	•	t	+	20

Manuel has only 4 tickets left to sell, so a reasonable domain is {0, 1, 2, 3, 4}.

Substitute these values into the function rule $2.50t + 20$ to find the range values.

The range is {\$20, \$22.50, \$25, \$27.50, \$30}.

(B) A telephone company charges \$0.25 per minute for the first 5 minutes of a call plus a \$0.45 connection fee per call. Write a function for the total cost in dollars of making a call.

Let m represent the number of minutes used.

Total cost for one call	is	\$0.25	per	minute	plus	\$0.45 fee.
$f(m)$	=	______	•	______	+	______

The charges only occur if a call is made, so a reasonable domain is {______________}.

Substitute these values into the function rule $0.25m + 0.45$ to find the range values.

The range is ______________________________.

Your Turn

Write a function in function notation for each situation. Find a reasonable domain and range for each function.

5. The temperature early in the morning is 17 °C. The temperature increases by 2 °C for every hour for the next 5 hours. Write a function for the temperature in degrees Celsius.

6. Takumi earns \$8.50 per hour proofreading advertisements at a local newspaper. He works no more than 5 hours a day. Write a function for the temperature in degrees Celsius.

Elaborate

7. How can you identify the independent variable and the dependent variable given a situation?

8. Describe how to write $3x + 2y = 12$ in function notation. Assume that y represents the dependent variable.

9. **Discussion** What is the advantage of using function notation instead of using y?

10. **Essential Question Check-In** Explain how to find reasonable domain values for a function.

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Identify the dependent and independent variables in each situation.

1. Identify the dependent and independent variables in each situation.

A. The total cost of running a business is based on its expenses.

B. The price of a house depends on its area.

C. The time it takes you to run a certain distance depends on the distance.

D. The number of items in a carton depends on the size of the carton.

2. Charles will babysit for up to 4 hours and charges $7 per hour.

Write a function in function notation for this situation.

For each situation, identify the independent and dependent variables. Write a function in function notation. Then use the function to solve the problem.

3. Almira earns \$50 an hour. How much does she earn in 6 hours?

4. Stan, a local delivery driver, is paid \$3.50 per mile driven plus a daily amount of \$75. On Monday, he is assigned a route that is 30 miles long. How much is he being paid for that day?

5. Bruce owns a small grocery store and charges \$4.75 per pound of produce. If a customer orders 5 pounds of produce, how much does Bruce charge the customer?

6. Georgia, a florist, charges \$10.95 per flower bundle plus a \$15 delivery charge per order. If Charlie orders 8 bundles of flowers and has them delivered, how much does Georgia charge Charlie?

7. Allison owns a music store and sells DVDs at \$17.75 per DVD. If Craig orders 5 DVDs, how much does it cost?

8. Anne buys used cars at auction for \$2000 per car. There is a \$150 fee to take part in the auction. If Anne buys 13 used cars, how much does she pay in total?

9. Harold, a real estate developer, sells houses at \$250,000 per house. If he sells 9 houses, how much does he earn?

10. Gordon buys 3 HD TVs for \$1200 each. There is a shipping charge of \$90 to have the TVs delivered to his house. How much does Gordon pay in total?

11. Cindy is buying jackets for her local community charity's auction. Each jacket costs \$50. If Cindy bought 23 jackets, what is the total cost?

12. Autumn sells laptop computers for \$600 each. If she sells 68 computers, how much money does she earn?

Write a function using function notation to describe each situation. Find a reasonable domain and range for each function.

13. Elijah has already sold \$40 worth of tickets for a local raffle.
He has 5 tickets left to sell at \$5 per ticket.

14. Mary has already sold \$55 worth of tickets to the benefit concert. She has 3 tickets left to sell at \$7 per ticket.

15. A law firm charges \$100 per hour for the first 3 hours plus a \$300 origination fee for its services.

16. A pay-for-service Internet company charges \$5 per hour for the first 3 hours of service plus a \$10 connection fee.

17. A high definition radio station charges \$200 per year in addition to \$50 per month for the first 3 months to receive its broadcast.

18. A newspaper charges \$3 per line for the first 4 lines plus a \$20 fee to advertise.

19. Matt has already sold \$72 worth of tickets to the benefit concert. He has 6 tickets left to sell at \$9 per ticket.

20. Sarah has sold \$33 worth of ticket sales to the comedy show. She has 4 tickets left to sell at \$11 per ticket.

H.O.T. Focus on Higher Order Thinking

21. Justify Reasoning The function $f(x) = -6x + 11$ has a range given by {–37, –25, –13, –1}. Select the domain values of the function from the list 1, 2, 3, 4, 5, 6, 7, 8. Explain how you arrived at your answer.

22 a. Represent Real-World Problems Victor needs to find the volume of 6 cube-shaped boxes with sides lengths of between 2 feet and 7 feet. The side lengths of the boxes can only be whole numbers. The volume of a cube-shaped box with a side length of s is given by the function $V(s) = s^3$.

What is a reasonable domain for this situation? Explain.

b. What is a reasonable range for this situation? Explain.

23 a. Represent Real-World Problems Tanya is printing a report. There are 100 sheets of paper in the printer, and the number of sheets of paper p left after t minutes of printing is given by the function $p(t) = -8t + 100$.

How many minutes would it take the printer to use all 100 sheets of paper? Show your work.

b. What is a reasonable domain for this situation? Explain.

c. What is a reasonable range for this situation? Explain.

Lesson Performance Task

Jenna's parents have given her an interest-free loan of $100 to buy a new pair of running shoes. She plans to pay back the loan with monthly payments of $20 each.

a. Write a function rule for the balance function $B(p)$, where p represents the number of payments that Jenna has made.

b. After how many payments will Jenna have paid back more than half the loan? Explain your reasoning.

c. Suppose the loan amount were $120 and the monthly payments were $15. Write a rule for the new balance function and use it to determine how long it would take Jenna to pay off the loan.

Name ________________ Class ________ Date ______

3.4 Graphing Functions

Essential Question: How do you graph functions?

Resource Locker

Explore: Graphing Functions Using a Given Domain

Recall that the domain of a function is the set of input values, or x-values, of the function and that the range is the set of corresponding output values, or y-values, of the function. One way to understand a function and its features is to graph it. You can graph a function by finding ordered pairs that satisfy the function.

Graph the function for the given domain.

$x + 3y = 15$ D: {0, 3, 6, 9}

(A) You have been given the input values, x, of the domain. You need to solve the function for y.

$x + 3y = 15$

$-x \qquad -x$ Subtract x from both sides.

$3y = \square$

$\frac{3y}{3} = \frac{\square}{3}$ Since y is multiplied by 3, divide both sides by 3.

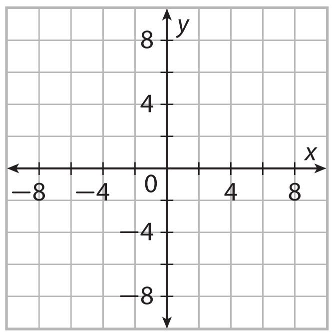
$y = \square + \square$ Rewrite the right side as two separate fractions.

$y = \square + \square$ Simplify.

(B) Substitute the given values of the domain for x to find the values of y.

x	$y = -\frac{1}{3}x + 5$	(x, y)
0	$y = -\frac{1}{3}(0) + 5 = \square$	$(0, \square)$
3	$y = -\frac{1}{3}(\square) + 5 = 4$	$(\square, 4)$
6	$y = -\frac{1}{3}(6) + 5 = \square$	$(6, \square)$
9	$y = -\frac{1}{3}(\square) + 5 = 2$	$(\square, \square)$

(C) Graph the ordered pairs.



Reflect

1. **Discussion** Why do you not connect the points of the graph?

2. **Discussion** How would the graph be different if the domain was $0 \leq x \leq 9$?

Explain 1 Graphing Functions Using a Domain of All Real Numbers

If the domain of a function is all real numbers, any number can be used as an input value producing an infinite number of ordered pairs that satisfy the function. Arrowheads are drawn at both ends of a smooth line or curve to represent the infinite number of ordered pairs. If a domain is not provided, it should be assumed that the domain is all real numbers.

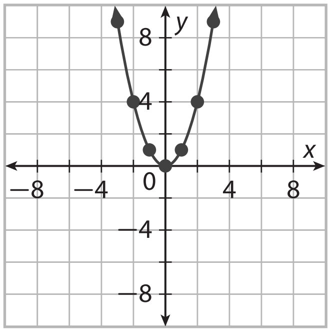
Graphing Functions Using a Domain of All Real Numbers	
Step 1	Use the function to generate ordered pairs by choosing several values of *x*.
Step 2	Plot enough points to see a pattern for the graph.
Step 3	Connect the points with a line or smooth curve.

Example 1 **Graph each function.**

(A) $y = x^2$

Use several values of *x* to generate ordered pairs. Plot the points from the table, and draw a smooth curve through the points. Include an arrowhead at each end.

x	$y = x^2$	(x, y)
−3	$y = (-3)^2 = 9$	$(-3, 9)$
−2	$y = (-2)^2 = 4$	$(-2, 4)$
−1	$y = (-1)^2 = 1$	$(-1, 1)$
0	$y = (0)^2 = 0$	$(0, 0)$
1	$y = (1)^2 = 1$	$(1, 1)$
2	$y = (2)^2 = 4$	$(2, 4)$
3	$y = (3)^2 = 9$	$(3, 9)$



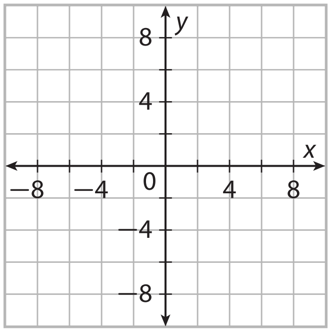
Ⓑ $f(x) = 3x - 5$

Use several values of x to generate ordered pairs.

x	$f(x) = 3x - 5$	$(x, f(x))$
-1	$f(-1) = 3(-1) - 5 =$	$(-1, \ )$
0	$f(\ ) = 3(\ ) - 5 = -5$	$(\ , -5)$
1	$f(1) = 3(1) - 5 = (\ )$	$(1, \ )$
2	$f(\ ) = 3(\ ) - 5 = 1$	$(\ , 1)$
3	$f(3) = 3(3) - 5 = (\ )$	$(3, \ )$
4	$f(\ ) = 3(\ ) - 5 = 7$	$(\ , \ )$

Plot the points from the table to see a pattern.

The points appear to form a ______________. Draw a ______________ through all the points to show the ordered pairs that satisfy the function. Draw ______________ on both ends of the graph.



Reflect

3. When graphing a function, does it matter if the function is written in function notation? Explain.

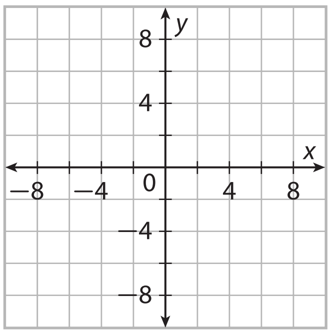
____

____

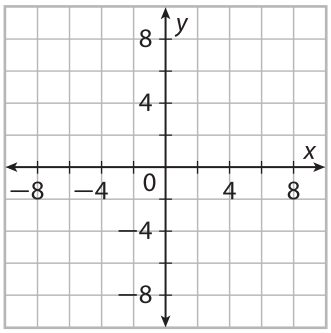
Your Turn

Graph each function.

4. $y = -x^2$



5. $y = -4x + 2$



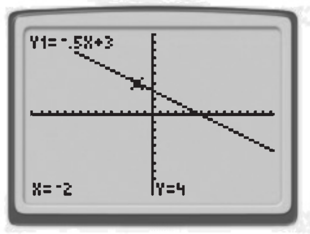
Explain 2 Using a Graph to Find Values

To find the value of a function for a given value of x using a graph, locate the value of x on the x-axis, move up or down to the graph of the function, and then move left or right to the y-axis to find the corresponding value of y.

Example 2 **Use a graph to find the value of $f(x)$ when $x = -2$ for each function.**

(A) $f(x) = -\frac{1}{2}x + 3$

Use a graphing calculator to graph $y = -\frac{1}{2}x + 3$, and use TRACE to find the function value when $x = -2$.

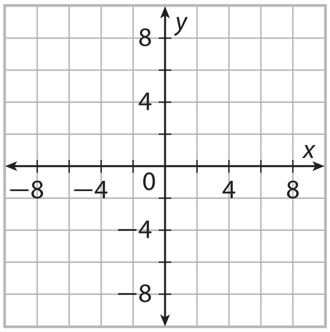


Therefore, the value of y is 4 when x is -2.

(B) $f(x) = \frac{3}{2}x - 4$

First graph the line. Locate ______________ on the x-axis. Draw a vertical line segment

from ______________ on the x-axis to the graph of the function and a horizontal line segment from the graph of the function to the y-axis.

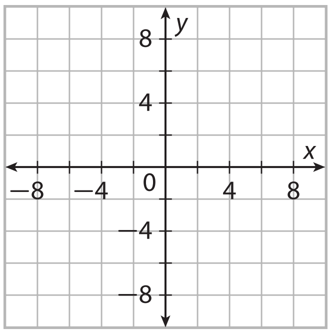


The value of y on the y-axis is the value of the function. Therefore,

the value of $f(x)$ is ______________ when x is –2.

Your Turn

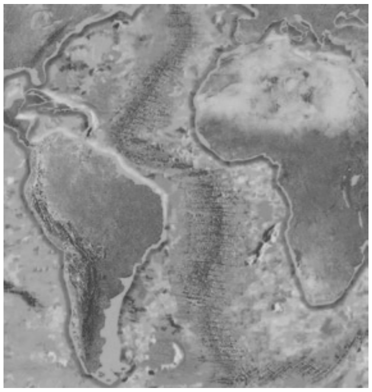
6. Use a graph to find the value of $f(x)$ when $x = 3$ for the function $f(x) = -x + 7$.



Explain 3 Modeling Using a Function Graph

The domain of a real-world situation may have to be limited in order to have reasonable answers. Only nonnegative numbers can be used to represent quantities such as time, distance, and the number of people. When both the domain and the range of a function are limited to nonnegative values, the function is graphed only in Quadrant I.

Example 3 **The Mid-Atlantic Ridge separates the North and South American Plates from the Eurasian and African Plates. The function $y = 2.5x$ relates the number of centimeters y the Mid-Atlantic Ridge spreads after x years. Graph the function and use the graph to estimate how many centimeters the Mid-Atlantic Ridge spreads in 4.5 years.**



Analyze Information

Identify the important information:

- The function ______ describes how many centimeters the Mid-Atlantic Ridge spreads after ______ years.

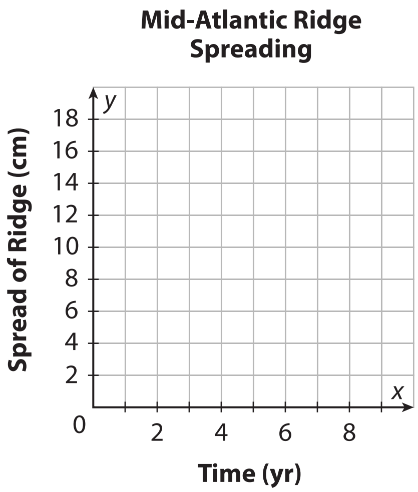
Formulate a Plan

Only use ______ values of x and y. Use a graph to find the value of ______ when x is ______.

Solve

Choose several values of x that are in the domain of the function to find values of y.

x	$y = 2.5x$	(x, y)
0	$y = 2.5(\square) = 10$	$(\square, 0)$
2	$y = 2.5(2) = \square$	$(2, \square)$
4	$y = 2.5(\square) = 10$	$(\square, 10)$
5	$y = 2.5(5) = \square$	$(5, \square)$



Plot the points that represent the ordered pairs on the graph. Draw a ______ through all of the points because the points appear to form a ______.

Use the graph to estimate the y-value when x is 4.5.

The Mid-Atlantic Ridge spreads about ______ centimeters after 4.5 years.

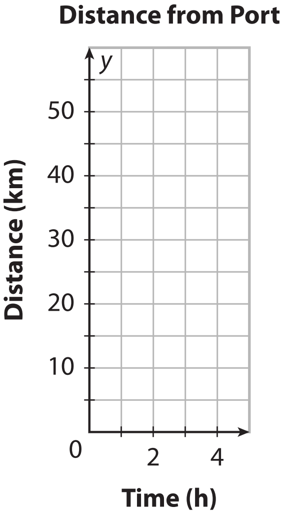
Justify and Evaluate

The distance of the Mid-Atlantic Ridge spread ______ as the number of years increases, so the graph is reasonable. When x is between 4 and 5, y is between ______ and ______. Since 4.5 is between 4 and 5, it is reasonable to estimate y to be ______ when x is 4.5.

Your Turn

7. A cruise ship is currently 5 kilometers away from its port and is traveling away from the port at 15 kilometers per hour. The function $y = 15x + 5$ relates the number of kilometers y the ship will be from its port x hours from now. How far will the cruise ship be from its port 2.5 hours from now?

x	$y = 15x + 5$	(x, y)



Elaborate

8. Is it enough to plot two points to see a pattern for a graph? Explain.

9. **Discussion** When you use a graph to find the value of a function for a specific value of x, do you always get an exact answer? Explain.

10. **Essential Question Check-In** How do you graph a function that has a domain of all real numbers?

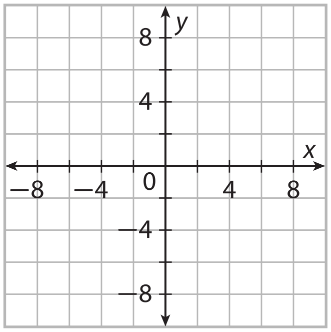
Evaluate: Homework and Practice

Personal Math Trainer

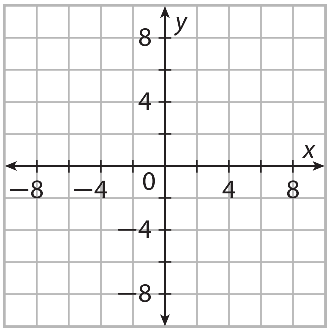
- Online Homework
- Hints and Help
- Extra Practice

Graph each function for the given domain.

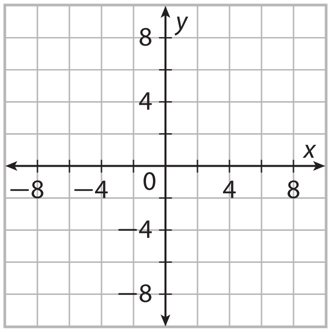
1. $y = 2x$ D:$\{-2, 0, 2, 4\}$



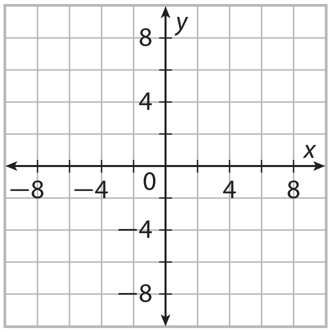
2. $y = \frac{1}{4}x + 5$ D:$\{-8, -4, 0, 4\}$



3. $-3x - 5y = 20$ D:$\{-10, -5, 0, 5\}$

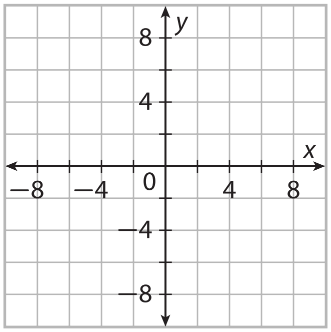


4. $y = x^2 - 3$ D:$\{-2, -1, 0, 1, 2\}$

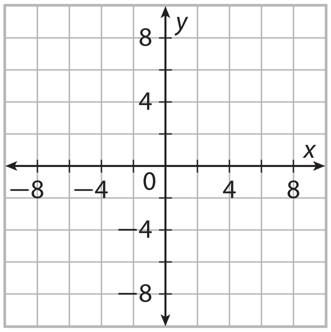


Graph each function.

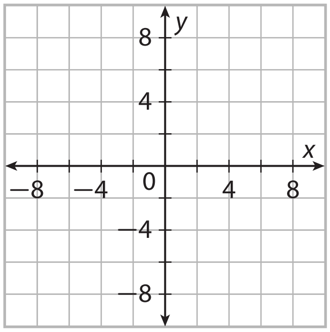
5. $y = -x^2 + 5$



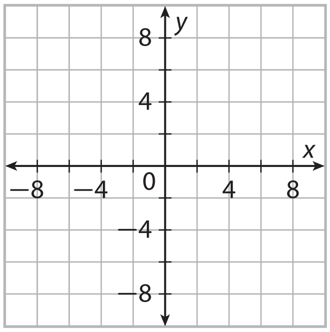
6. $y = \frac{2}{3}x - 1$



7. $x + y = 0$



8. $y = \frac{1}{4}x^2 - 8$



Use a graphing calculator to find the value of $f(x)$ when $x = 3$ for each function.

9. $f(x) = \frac{1}{3}x - 2$

10. $f(x) = -x^2 - 4$

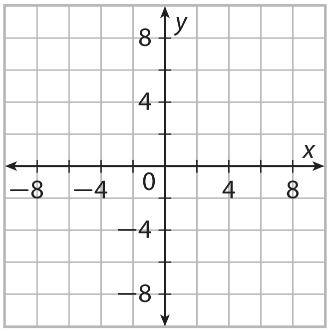
Use a graphing calculator to find the value of $f(x)$ when $x = -4$ for each function.

11. $fx = x^2 - 3$

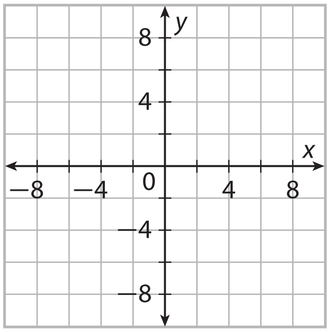
12. $f(x) = -4x - \frac{3}{2}$

13. $f(x) = -\frac{9}{2}x^2 - 5$

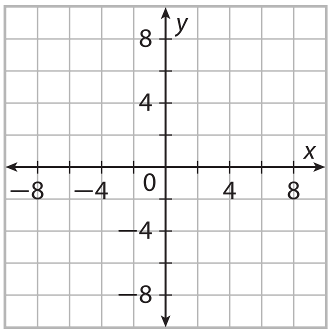
14. Graph $f(x) = 8 + 2x$. Then find the value of $f(x)$ when $x = \frac{1}{2}$.



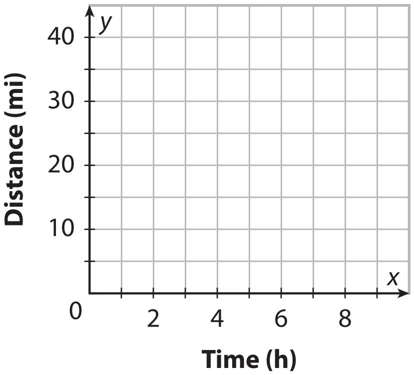
15. Graph $f(x) = 0.5 - 2x^2$. Then find the value of $f(x)$ when $x = 0$.



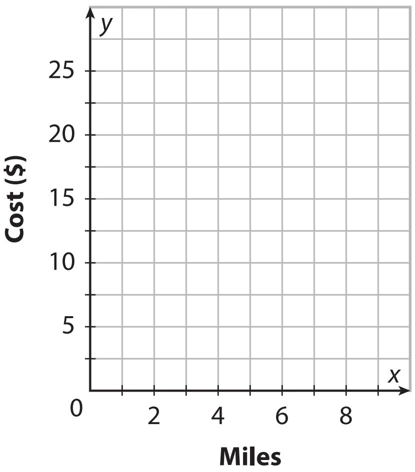
16. Graph $f(x) = \frac{1}{4}x^2$. Then find the value of $f(x)$ when $x = -6$.



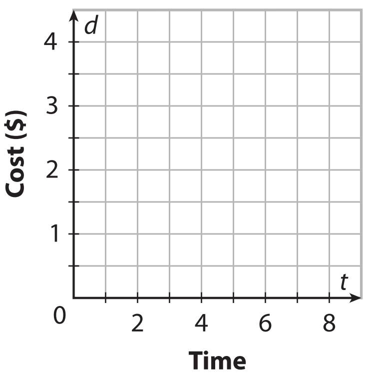
17. The fastest recorded Hawaiian lava flow moved at an average speed of 6 miles per hour. The function $y = 6x$ describes the distance y the lava moved on average in x hours. Graph the function. Use the graph to estimate how many miles the lava moved after 4.5 hours.



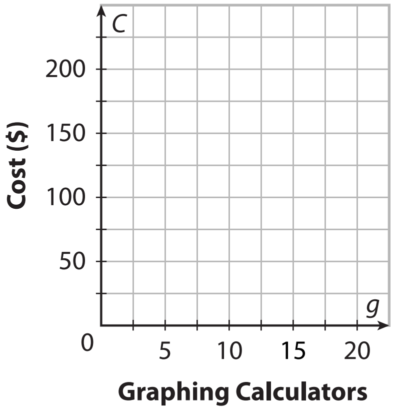
18. The total cost of a cab ride can be represented by the function $f(x) = 3x + 2.5$, where x is the number of miles driven. Graph the function. Use the graph to estimate how much the cab will cost if the cab ride is 8 miles.



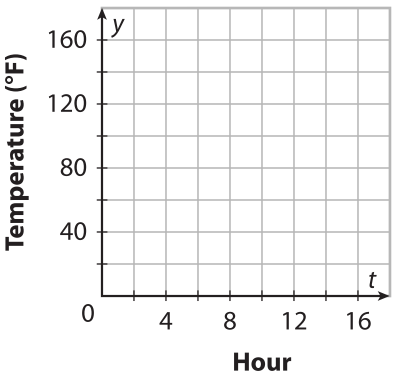
19. Joshua is driving to the store. The average distance d in miles he travels over t minutes is given by the function $d(t) = 0.5t$. Graph the function. Use the graph to estimate how many miles he drove after 5 minutes.



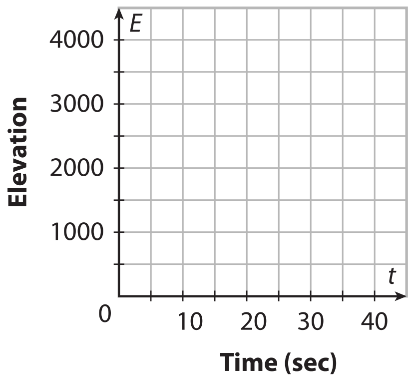
20. The production cost for g graphing calculators is $C(g) = 15g$. Graph the function and then evaluate it when $g = 15$. What does the value of the function at $g = 15$ represent?



21. The temperature, in degrees Fahrenheit, of a liquid that is increasing can be represented by the equation $f(t) = 64 + 4t$, where t is the time in hours. Graph the function to show the temperatures over the first 10 hours. Use the graph to find the temperature after 7 hours.

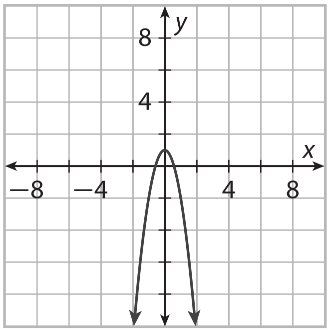


22. A snowboarder's elevation, in feet, can be represented by the function $E(t) = 3000 - 70t$, where t is in seconds. Graph the function and find the elevation of the snowboarder after 30 seconds.



H.O.T. Focus on Higher Order Thinking

23. Explain the Error Student A and student B were given the following graph and asked to find the value of $f(x)$ when $x = 1$. Student A gave an answer of 0 while student B gave an answer of -2. Who is incorrect? Explain the error.



24. Justify Reasoning Without graphing, tell which statement(s) are true for the graph of the function $y = x^2 + 1$. Explain your choices.

I. All points on the graph are above the origin.

II. All points on the graph have positive x-values.

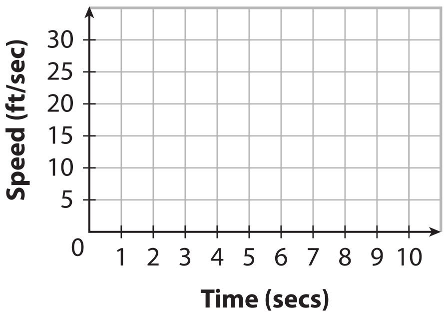
III. All points on the graph have positive y-values.

Lesson Performance Task

The Japanese Shinkansen, or bullet train, can accelerate rapidly to reach its maximum traveling speed of about 170 miles per hour. The table gives the speed of the train in feet per second at several different times.

Time (seconds)	Speed (feet per second)
0	0
1	2.5
4	10
6	15
10	25

a. Convert the data from the table to a set of ordered pairs and graph them on a coordinate grid. Connect the points with a line. What does the line represent?



b. What is the slope of the line? What does the slope represent?

c. If the acceleration remains constant, how long will it take the train to reach its maximum speed of 170 miles per hour (mph)? [1 mph equals about 1.5 feet per second]