Name ______________________ Class ______________ Date ________

18.1 Sequences of Transformations

Essential Question: **What happens when you apply more than one transformation to a figure?**

Resource Locker

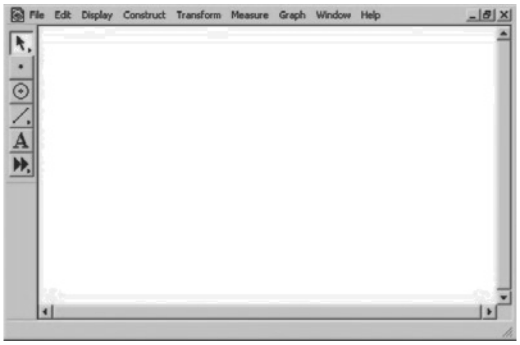
Explore Combining Rotations or Reflections

A transformation is a function that takes points on the plane and maps them to other points on the plane. Transformations can be applied one after the other in a sequence where you use the image of the first transformation as the preimage for the next transformation.

Find the image for each sequence of transformations.

(A) Using geometry software, draw a triangle and label the vertices A, B, and C. Then draw a point outside the triangle and label it P.

Rotate $\triangle ABC$ 30° around point P and label the image as $\triangle A'B'C'$. Then rotate $\triangle A'B'C'$ 45° around point P and label the image as $\triangle A''B''C''$. Sketch your result.



(B) Make a conjecture regarding a single rotation that will map $\triangle ABC$ to $\triangle A''B''C''$. Check your conjecture, and describe what you did.

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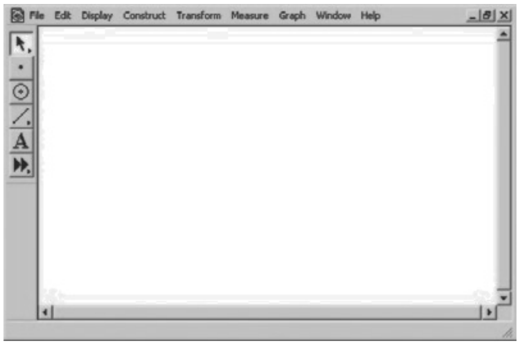
(C) Using geometry software, draw a triangle and label the vertices D, E, and F. Then draw two intersecting lines and label them j and k.

Reflect $\triangle DEF$ across line j and label the image as $\triangle D'E'F'$. Then reflect $\triangle D'E'F'$ across line k and label the image as $\triangle D''E''F''$. Sketch your result.



(D) Consider the relationship between $\triangle DEF$ and $\triangle D''E''F''$. Describe the single t ransformation that maps $\triangle DEF$ to $\triangle D''E''F''$. How can you check that you are correct?

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Reflect

1. Repeat Step A using other angle measures. Make a conjecture about what single transformation will describe a sequence of two rotations about the same center.

2. Make a conjecture about what single transformation will describe a sequence of three rotations about the same center.

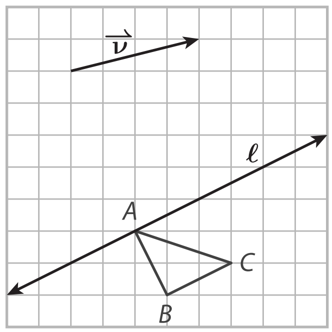
3. **Discussion** Repeat Step C, but make lines j and k parallel instead of intersecting. Make a conjecture about what single transformation will now map $\triangle DEF$ to $\triangle D''E''F''$. Check your conjecture and describe what you did.

Explain 1 Combining Rigid Transformations

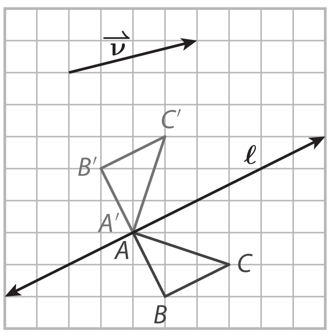
In the Explore, you saw that sometimes you can use a single transformation to describe the result of applying a sequence of two transformations. Now you will apply sequences of rigid transformations that cannot be described by a single transformation.

Example 1 **Draw the image of $\triangle ABC$ after the given combination of transformations.**

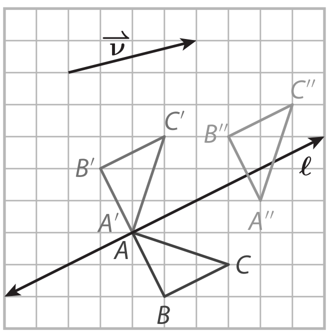
(A) Reflection over line ℓ then translation along $\vec{v}$



Step 1 Draw the image of $\triangle ABC$ after a reflection across line ℓ. Label the image $\triangle A'B'C'$.



Step 2 Translate $\triangle A'B'C'$ along $\vec{v}$. Label this image $\triangle A''B''C''$.

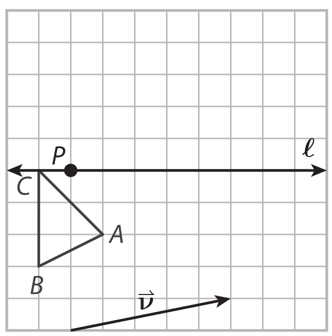


(B) 180° rotation around point P, then translation along $\vec{v}$, then reflection across line ℓ

Apply the rotation. Label the image $\triangle A'B'C'$.

Apply the translation to $\triangle A'B'C'$. Label the image $\triangle A''B''C''$.

Apply the reflection to $\triangle A''B''C''$. Label the image $\triangle A'''B'''C'''$.



Reflect

4. Are the images you drew for each example the same size and shape as the given preimage? In what ways do rigid transformations change the preimage?

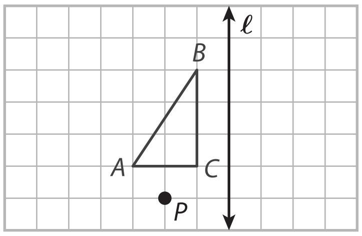
5. Does the order in which you apply the transformations make a difference? Test your conjecture by performing the transformations in Part B in a different order.

6. For Part B, describe a sequence of transformations that will take $\triangle A''B''C''$ back to the preimage.

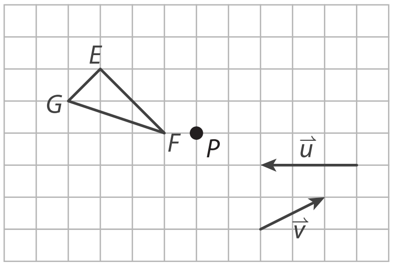
Your Turn

Draw the image of the triangle after the given combination of transformations.

7. Reflection across ℓ then 90° rotation around point P



8. Translation along $\vec{v}$ then 180° rotation around point P then translation along $\vec{u}$

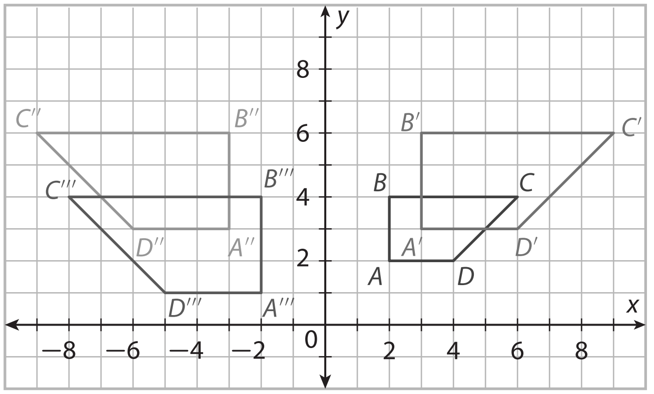


Explain 2 Combining Nonrigid Transformations

Example 2 **Draw the image of the figure in the plane after the given combination of transformations.**

(A) $(x, y) \rightarrow \left(\frac{3}{2}x, \frac{3}{2}y\right) \rightarrow (-x, y) \rightarrow (x + 1, y - 2)$

1. The first transformation is a dilation by a factor of $\frac{3}{2}$. Apply the dilation. Label the image $A'B'C'D'$.
2. Apply the reflection of $A'B'C'D'$ across the y-axis. Label this image $A''B''C''D''$.
3. Apply the translation of $A''B''C''D''$. Label this image $A'''B'''C'''D'''$.

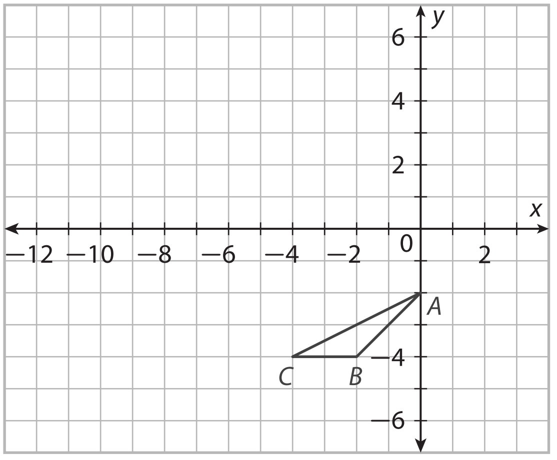


(B) $(x, y) \rightarrow (3x, y) \rightarrow \left(\frac{1}{2}x, -\frac{1}{2}y\right)$

1. The first transformation is a [horizontal/vertical] stretch by a factor of ______.

   Apply the stretch. Label the image ________.
2. The second transformation is a dilation by a factor of ________ combined with a reflection.

   Apply the transformation to ________. Label the image ________.



Reflect

9. If you dilated a figure by a factor of 2, what transformation could you use to return the figure back to its preimage? If you dilated a figure by a factor of 2 and then translated it right 2 units, write a sequence of transformations to return the figure back to its preimage.

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10. A student is asked to reflect a figure across the y-axis and then vertically stretch the figure by a factor of 2. Describe the effect on the coordinates. Then write one transformation using coordinate notation that combines these two transformations into one.

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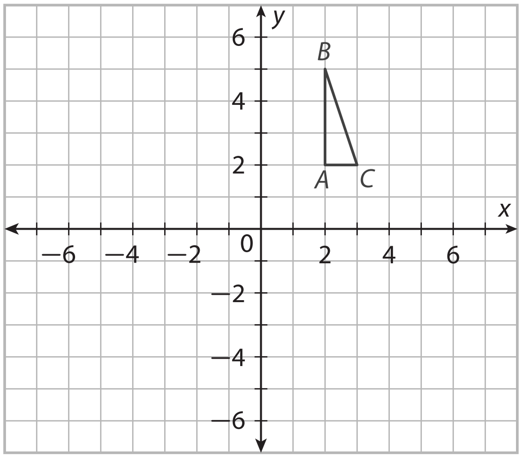
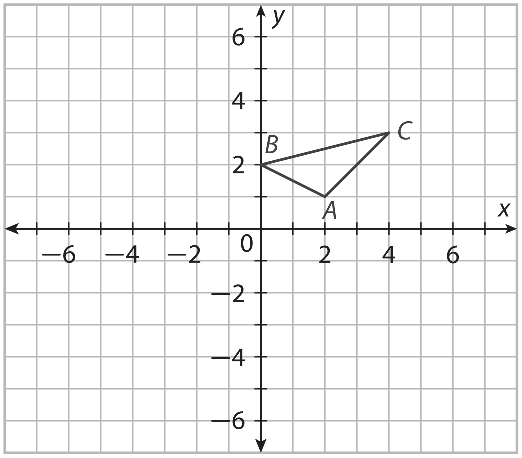
Your Turn

Draw the image of the figure in the plane after the given combination of transformations.

11. $(x, y) \rightarrow (x - 1, y - 1) \rightarrow (3x, y) \rightarrow (-x, -y)$ **12.** $(x, y) \rightarrow \left(\frac{3}{2}x, -2y\right) \rightarrow (x - 5, y + 4)$

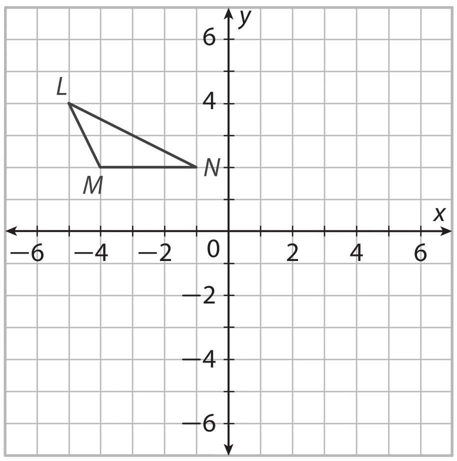




Explain 3 Predicting the Effect of Transformations

Example 3 **Predict the result of applying the sequence of transformations to the given figure.**

(A) $\triangle LMN$ is translated along the vector $\langle -2, 3\rangle$, reflected across the y-axis, and then reflected across the x-axis.



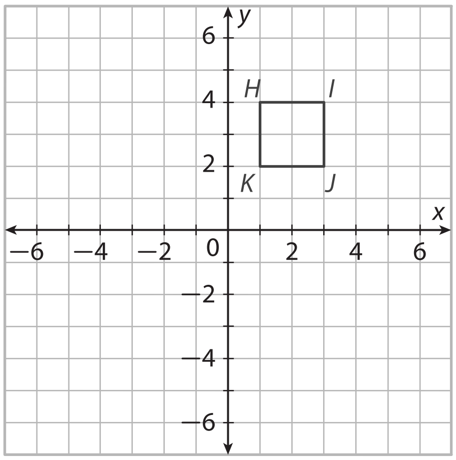
Predict the effect of the first transformation: A translation along the vector $\langle -2, 3\rangle$ will move the figure left 2 units and up 3 units. Since the given triangle is in Quadrant II, the translation will move it further from the x- and y-axes. It will remain in Quadrant II.

Predict the effect of the second transformation: Since the triangle is in Quadrant II, a reflection across the y-axis will change the orientation and move the triangle into Quadrant I.

Predict the effect of the third transformation: A reflection across the x-axis will again change the orientation and move the triangle into Quadrant IV. The two reflections are the equivalent of rotating the figure 180° about the origin.

The final result will be a triangle the same shape and size as $\triangle LMN$ in Quadrant IV. It has been rotated 180° about the origin and is farther from the axes than the preimage.

(B) Square $HIJK$ is rotated 90° clockwise about the origin and then dilated by a factor of 2, which maps $(x, y) \rightarrow (2x, 2y)$.



Predict the effect of the first transformation: ________

____

____

____

Predict the effect of the second transformation: ________

____

____

____

The final result will be ________

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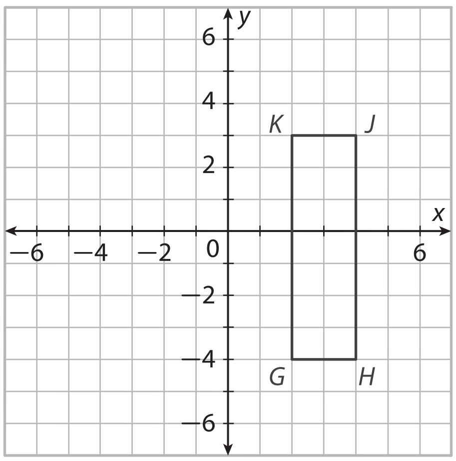
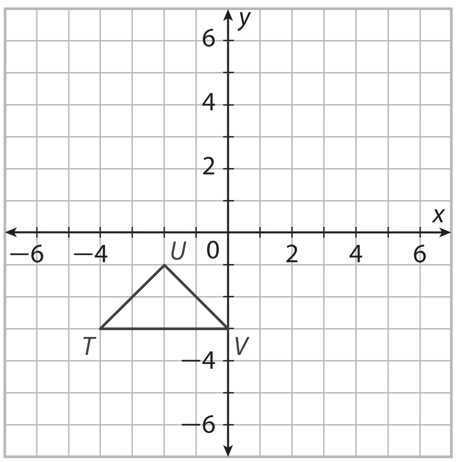
Your Turn

Predict the result of applying the sequence of transformations to the given figure.

13. Rectangle $GHJK$ is reflected across the y-axis and translated along the vector $\langle 5, 4 \rangle$.



14. $\triangle TUV$ is horizontally stretched by a factor of $\frac{3}{2}$, which maps $(x, y) \rightarrow \left(\frac{3}{2}x, y\right)$, and then translated along the vector $\langle 2, 1 \rangle$.



Elaborate

15. **Discussion** How many different sequences of rigid transformations do you think you can find to take a preimage back onto itself? Explain your reasoning.

16. Is there a sequence of a rotation and a dilation that will result in an image that is the same size and position as the preimage? Explain your reasoning.

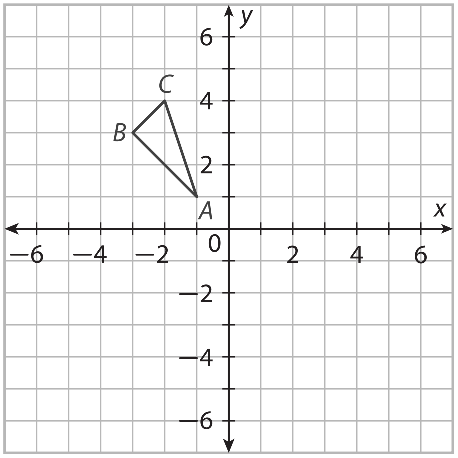
17. **Essential Question Check-In** In a sequence of transformations, the order of the transformations can affect the final image. Describe a sequence of transformations where the order does not matter. Describe a sequence of transformations where the order does matter.

Evaluate: Homework and Practice

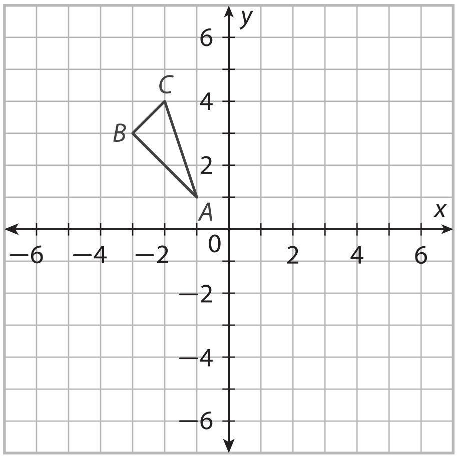
Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

Draw and label the final image of △ABC after the given sequence of transformations.

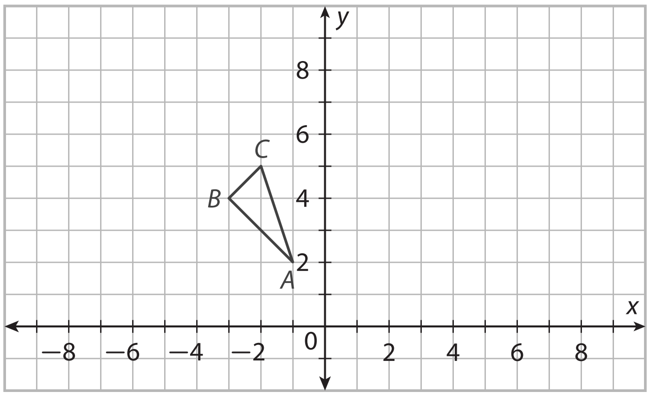
1. Reflect △ABC over the y-axis and then translate by $\langle 2, -3 \rangle$.



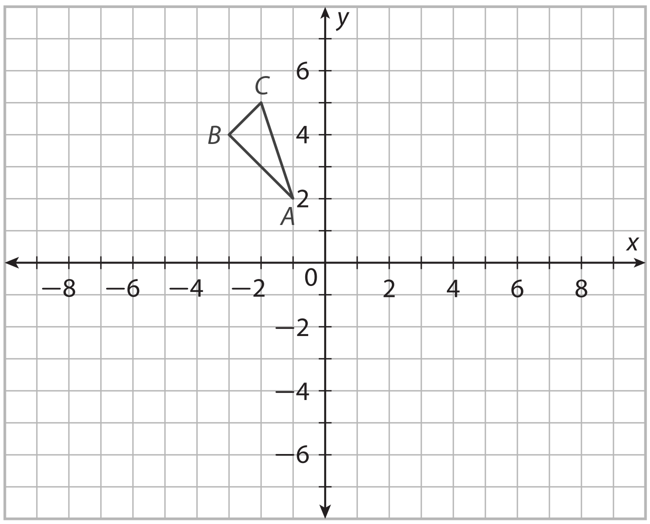
2. Rotate △ABC 90 degrees clockwise about the origin and then reflect over the x-axis.



3. Translate △ABC by $\langle 4, 4 \rangle$, rotate 90 degrees counterclockwise around A, and reflect over the y-axis.

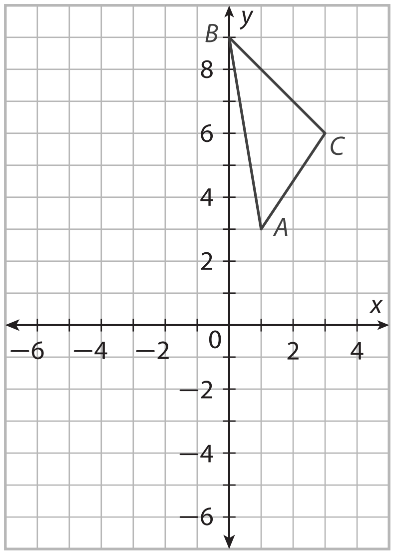


4. Reflect △ABC over the x-axis, translate by $\langle -3, -1 \rangle$, and rotate 180 degrees around the origin.

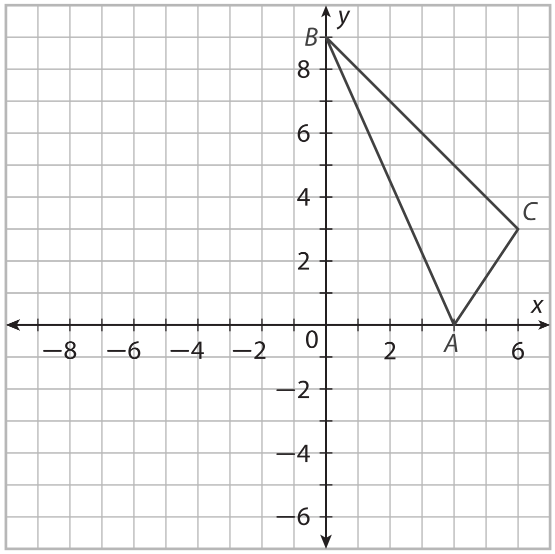


Draw and label the final image of $\triangle ABC$ after the given sequence of transformations.

5. $(x, y) \rightarrow \left(x, \frac{1}{3}y\right) \rightarrow (-2x, -2y)$

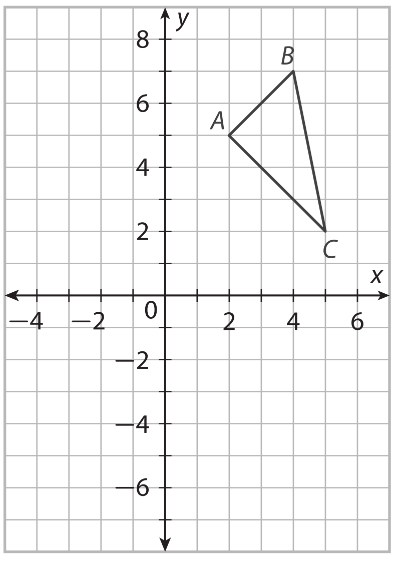


6. $(x, y) \rightarrow \left(-\frac{3}{2}x, \frac{2}{3}y\right) \rightarrow (x + 6, y - 4) \rightarrow \left(\frac{2}{3}x, -\frac{3}{2}y\right)$

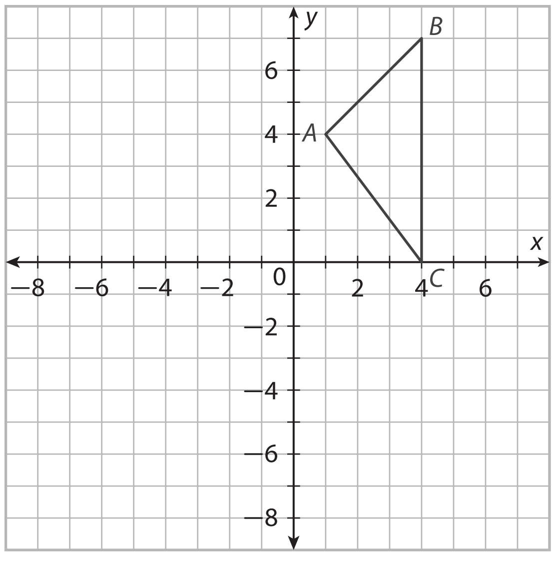


Predict the result of applying the sequence of transformations to the given figure.

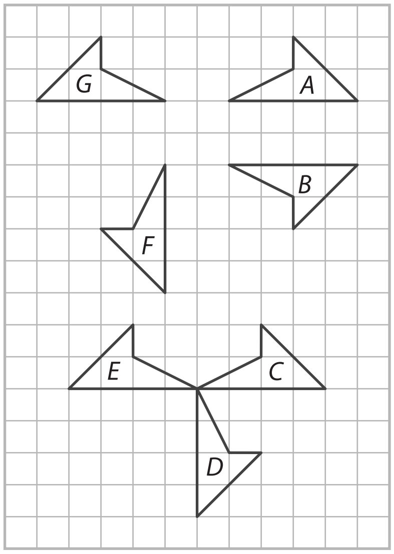
7. $\triangle ABC$ is translated along the vector $\langle -3, -1 \rangle$, reflected across the x-axis, and then reflected across the y-axis.



8. $\triangle ABC$ is translated along the vector $\langle -1, -3 \rangle$, rotated 180° about the origin, and then dilated by a factor of 2.



In Exercises 9–12, use the diagram. Fill in the blank with the letter of the correct image described.



9. _____ is the result of the sequence: G reflected over a vertical line and then a horizontal line.

10. _____ is the result of the sequence: D rotated 90° clockwise around one of its vertices and then reflected over a horizontal line.

11. _____ is the result of the sequence: E translated and then rotated 90° counterclockwise.

12. _____ is the result of the sequence: D rotated 90° counterclockwise and then translated.

Choose the correct word to complete a true statement.

13. A combination of two rigid transformations on a preimage will always/sometimes/never produce the same image when taken in a different order.

14. A double rotation can always/sometimes/never be written as a single rotation.

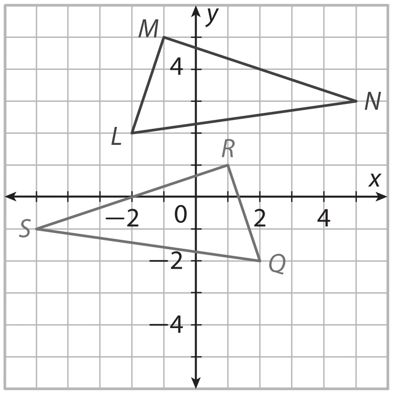
15. A sequence of a translation and a reflection always/sometimes/never has a point that does not change position.

16. A sequence of a reflection across the x-axis and then a reflection across the y-axis always/sometimes/never results in a 180° rotation of the preimage.

17. A sequence of rigid transformations will always/sometimes/never result in an image that is the same size and orientation as the preimage.

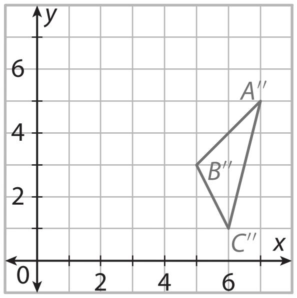
18. A sequence of a rotation and a dilation will always/sometimes/never result in an image that is the same size and orientation as the preimage.

19. $\triangle QRS$ is the image of $\triangle LMN$ under a sequence of transformations. Can each of the following sequences be used to create the image, $\triangle QRS$, from the preimage, $\triangle LMN$? Select yes or no.



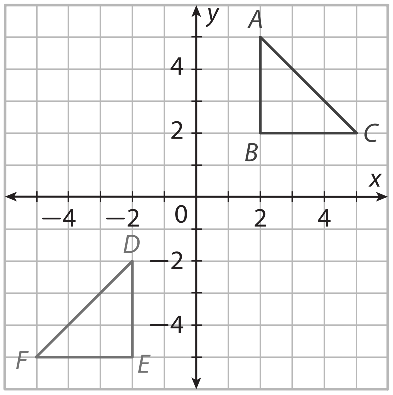
- **a.** Reflect across the y-axis and then translate along the vector $\langle 0, -4 \rangle$. ○ Yes ○ No
- **b.** Translate along the vector $\langle 0, -4 \rangle$ and then reflect across the y-axis. ○ Yes ○ No
- **c.** Rotate 90° clockwise about the origin, reflect across the x-axis, and then rotate 90° counterclockwise about the origin. ○ Yes ○ No
- **d.** Rotate 180° about the origin, reflect across the x-axis, and then translate along the vector $\langle 0, -4 \rangle$. ○ Yes ○ No

20. A teacher gave students this puzzle: "I had a triangle with vertex A at (1, 4) and vertex B at (3, 2). After two rigid transformations, I had the image shown. Describe and show a sequence of transformations that will give this image from the preimage."



H.O.T. Focus on Higher Order Thinking

21. Analyze Relationships What two transformations would you apply to $\triangle ABC$ to get $\triangle DEF$? How could you express these transformations with a single mapping rule in the form of $(x, y) \rightarrow (?, ?)$?



22. Multi-Step Muralists will often make a scale drawing of an art piece before creating the large finished version. A muralist has sketched an art piece on a sheet of paper that is 3 feet by 4 feet.

a. If the final mural will be 39 feet by 52 feet, what is the scale factor for this dilation?

b. The owner of the wall has decided to only give permission to paint on the lower half of the wall. Can the muralist simply use the transformation $(x, y) \rightarrow \left(x, \frac{1}{2}y\right)$ in addition to the scale factor to alter the sketch for use in the allowed space? Explain.

23. Communicate Mathematical Ideas As a graded class activity, your teacher asks your class to reflect a triangle across the y-axis and then across the x-axis. Your classmate gets upset because he reversed the order of these reflections and thinks he will have to start over. What can you say to your classmate to help him?

Lesson Performance Task

The photograph shows an actual snowflake. Draw a detailed sketch of the “arm” of the snowflake located at the top left of the photo (10:00 on a clock face). Describe in as much detail as you can any translations, reflections, or rotations that you see.

Then describe how the entire snowflake is constructed, based on what you found in the design of one arm.

Name ______________________ Class ____________ Date ________

18.2 Proving Figures are Congruent Using Rigid Motions

Essential Question: How can you determine whether two figures are congruent?

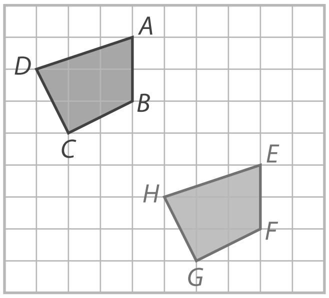
Resource Locker

Explore Confirming Congruence

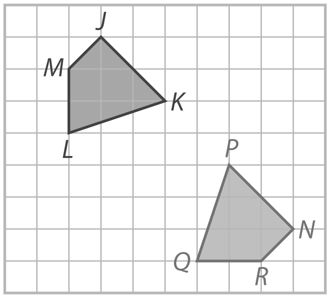
Two plane figures are congruent if and only if one can be obtained from the other by a sequence of rigid motions (that is, by a sequence of reflections, translations, and/or rotations).

A landscape architect uses a grid to design the landscape around a mall. Use tracing paper to confirm that the landscape elements are congruent.

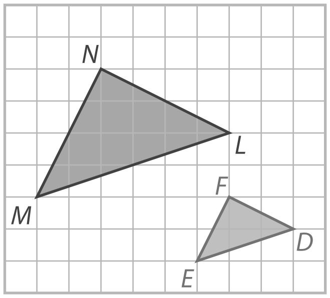
Ⓐ Trace planter *ABCD*. Describe a transformation you can use to move the tracing paper so that planter *ABCD* is mapped onto planter *EFGH*. What does this confirm about the planters?



Ⓑ Trace pools *JKLM* and *NPQR*. Fold the paper so that pool *JKLM* is mapped onto pool *NPQR*. Describe the transformation. What does this confirm about the pools?



Ⓒ Determine whether the lawns are congruent. Is there a rigid transformation that maps $\triangle LMN$ to $\triangle DEF$? What does this confirm about the lawns?



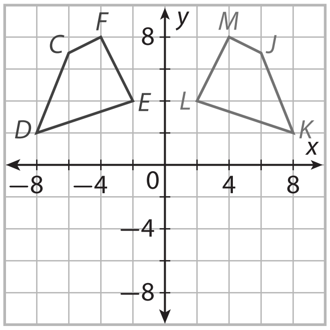
Reflect

1. How do the sizes of the pairs of figures help determine if they are congruent?

Explain 1 Determining if Figures are Congruent

Example 1 **Use the definition of congruence to decide whether the two figures are congruent. Explain your answer.**

(A)

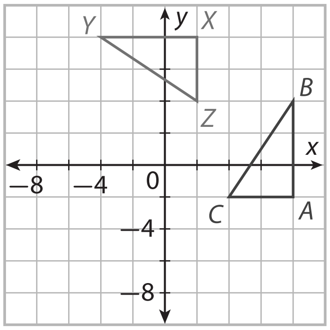


The two figures appear to be the same size and shape, so look for a rigid transformation that will map one to the other.

You can map *CDEF* onto *JKLM* by reflecting *CDEF* over the *y*-axis. This reflection is a rigid motion that maps *CDEF* to *JKLM*, so the two figures are congruent.

The coordinate notation for the reflection is $(x, y) \rightarrow (-x, y)$.

(B)



The two figures appear to be the same/different.

You can map $\triangle ABC$ to $\triangle XYZ$

by ______________________________.

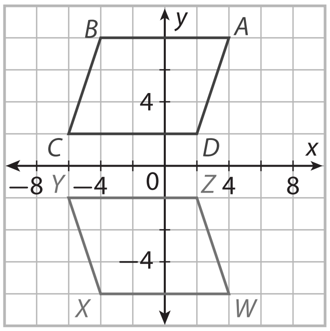
This is/is not a rigid motion that maps $\triangle ABC$ to $\triangle XYZ$, so the two figures are/are not congruent.

The coordinate notation for the rotation is ______________.

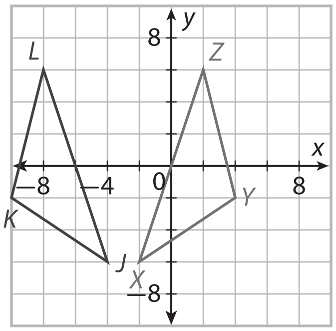
Your Turn

Use the definition of congruence to decide whether the two figures are congruent. Explain your answer.

2.



3.

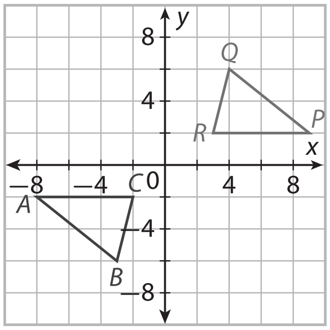


Explain 2 Finding a Sequence of Rigid Motions

The definition of congruence tells you that when two figures are known to be congruent, there must be some sequence of rigid motions that maps one to the other.

Example 2 **The figures shown are congruent. Find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.**

(A) $\triangle ABC \cong \triangle PQR$

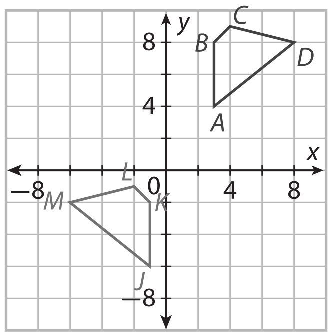


Map $\triangle ABC$ to $\triangle PQR$ with a rotation of 180° around the origin, followed by a horizontal translation.

Rotation: $(x, y) \rightarrow (-x, -y)$

Translation: $(x, y) \rightarrow (x + 1, y)$

(B) $ABCD \cong JKLM$



Map *ABCD* to *JKLM* with a

______________________________,

followed by a ______________.

______________: $(x, y) \rightarrow$ ______________

______________: $(x, y) \rightarrow$ ______________

Reflect

4. How is the orientation of the figure affected by a sequence of transformations?

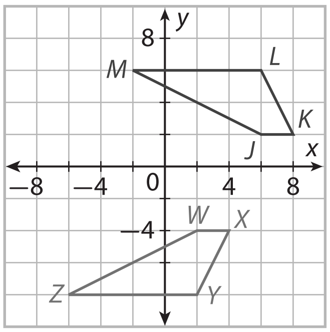
____

____

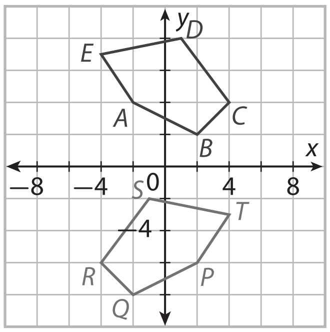
Your Turn

The figures shown are congruent. Find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.

5. $JKLM \cong WXYZ$



6. $ABCDE \cong PQRST$

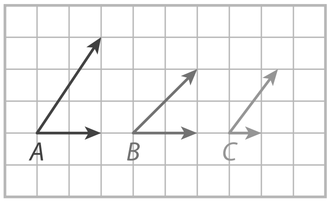


Explain 3 Investigating Congruent Segments and Angles

Congruence can refer to parts of figures as well as whole figures. Two angles are congruent if and only if one can be obtained from the other by rigid motions (that is, by a sequence of reflections, translations, and/or rotations.) The same conditions are required for two segments to be congruent to each other.

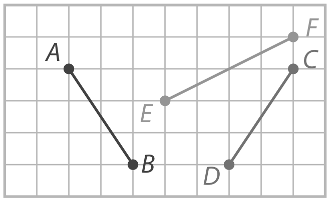
Example 3 **Determine which angles or segments are congruent. Describe transformations that can be used to verify congruence.**

Ⓐ



$\angle A$ and $\angle C$ are congruent. The transformation is a translation. There is no transformation that maps $\angle B$ to either of the other angles.

Ⓑ

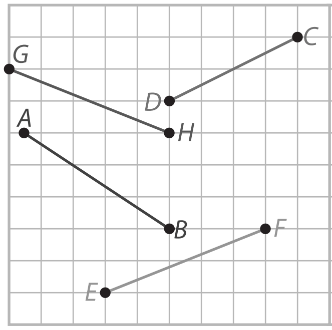


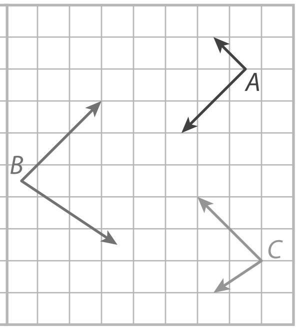
$\overline{AB}$ and ______ are congruent. A sequence of transformations is a ____________ and a translation.

There is no transformation that maps ______ to either of the other segments.

Your Turn

7. Determine which segments and which angles are congruent. Describe transformations that can be used to show the congruence.





Elaborate

8. Can you say two angles are congruent if they have the same measure but the segments that identify the rays that form the angle are different lengths?

____

____

9. **Discussion** Can figures have congruent angles but not be congruent figures?

____

____

10. **Essential Question Check-In** Can you use transformations to prove that two figures are not congruent?

____

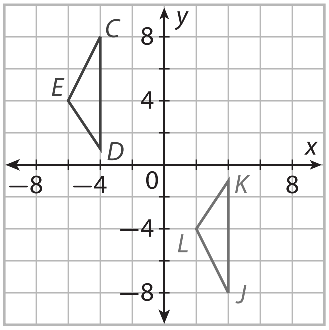
____

Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

Use the definition of congruence to decide whether the two figures are congruent. Explain your answer. Give coordinate notation for the transformations you use.

1.



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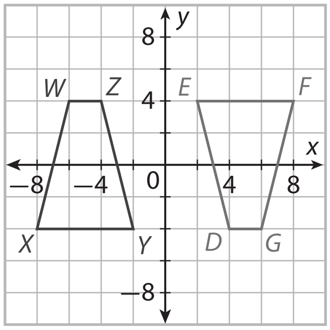

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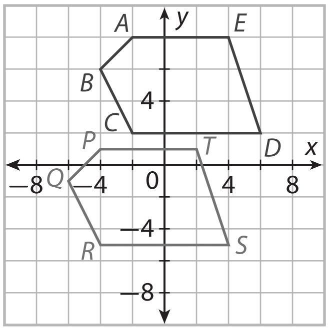

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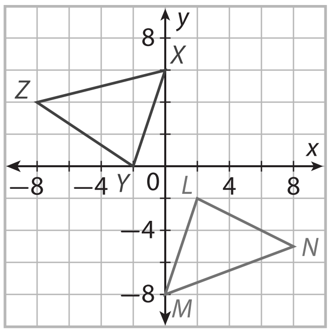

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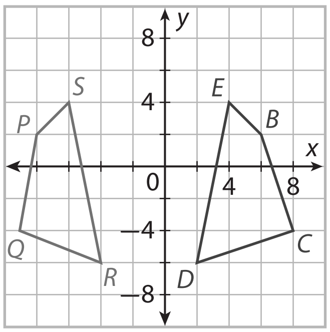
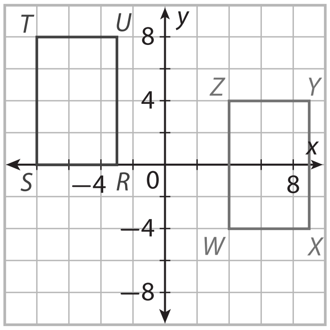
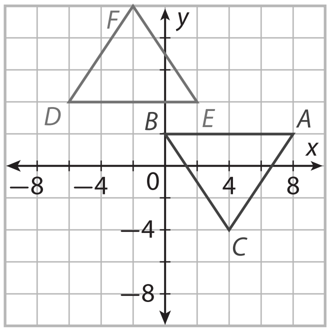


The figures shown are congruent. Find a sequence of rigid motions that maps one figure to the other. Give coordinate notation for the transformations you use.

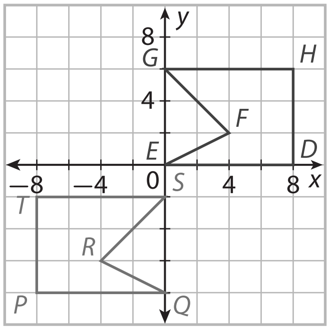
6. *RSTU* ≅ *WXYZ*



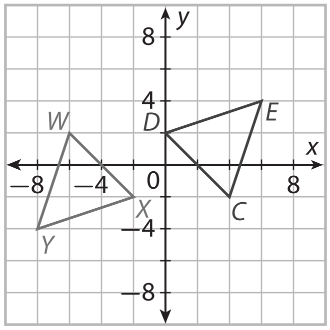
7. △*ABC* ≅ △*DEF*



8. *DEFGH* ≅ *PQRST*

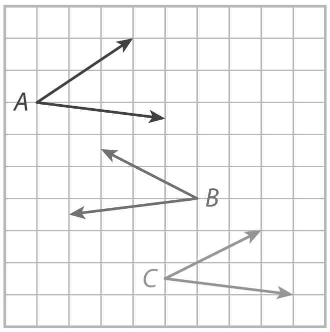


9. △*CDE* ≅ △*WXY*

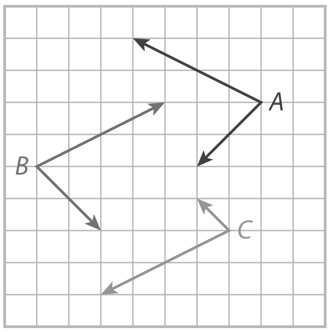


Determine which of the angles are congruent. Which transformations can be used to verify the congruence?

10.

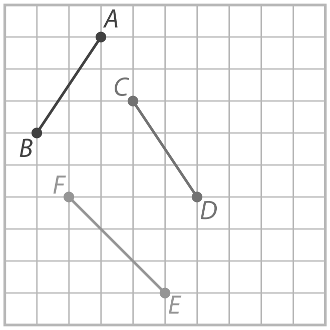


11.

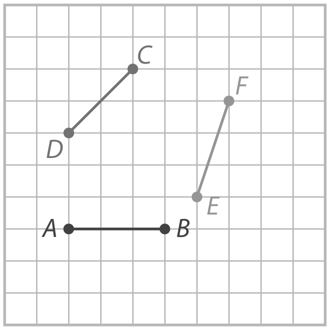


Determine which of the segments are congruent. Which transformations can be used to verify the congruence?

12.

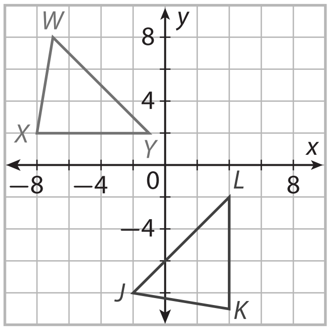


13.



Use the definition of congruence to decide whether the two figures are congruent. Explain your answer. Give coordinate notation for the transformations you use.

14.



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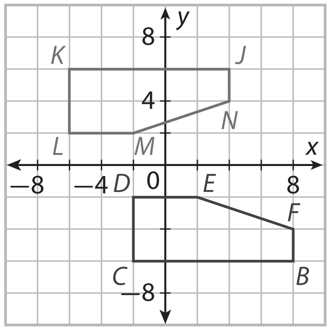

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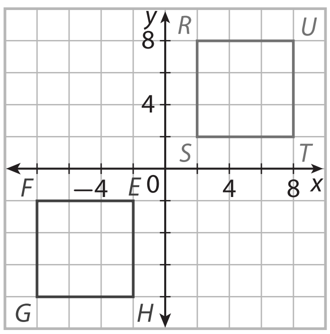
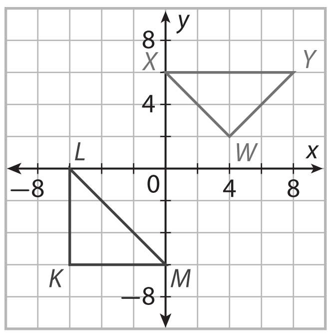
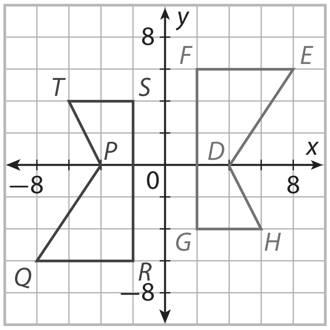


17.

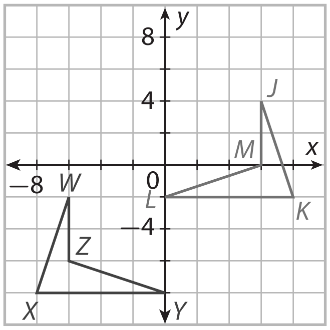


The figures shown are congruent. Find a sequence of transformations for the indicated mapping. Give coordinate notation for the transformations you use.

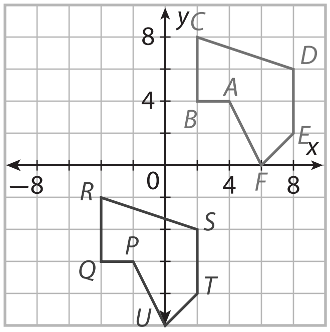
18. Map *PQRST* to *DEFGH*.



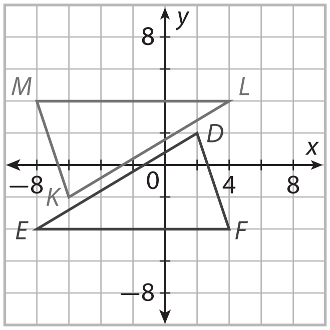
19. Map *WXYZ* to *JKLM*.



20. Map *PQRSTU* to *ABCDEF*.



21. Map △*DEF* to △*KLM*.



22. Determine whether each pair of angles is congruent or not congruent. Select the correct answer for each lettered part.

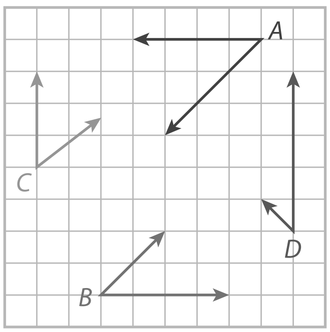
a. ∠*A* and ∠*B* ◯ Congruent ◯ Not congruent

b. ∠*A* and ∠*C* ◯ Congruent ◯ Not congruent

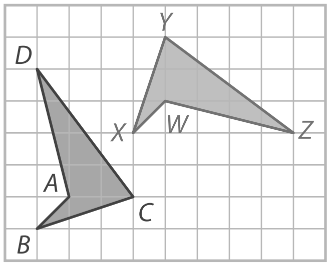
c. ∠*B* and ∠*C* ◯ Congruent ◯ Not congruent

d. ∠*B* and ∠*D* ◯ Congruent ◯ Not congruent

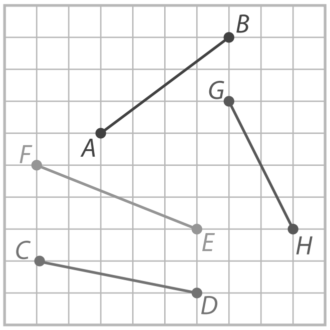
e. ∠*C* and ∠*D* ◯ Congruent ◯ Not congruent



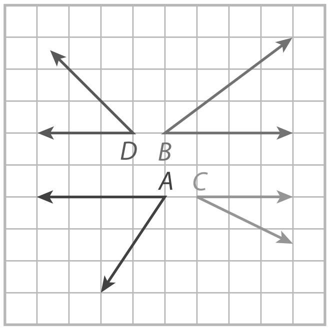
23. If $ABCD$ and $WXYZ$ are congruent, then $ABCD$ can be mapped to $WXYZ$ using a rotation and a translation. Determine whether the statement is true or false. Then explain your reasoning.



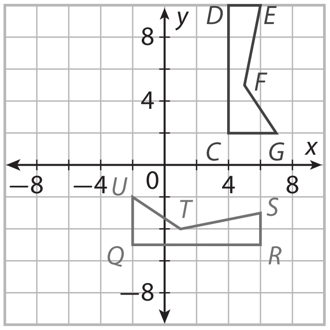
24. Which segments are congruent? Which are not congruent? Explain.



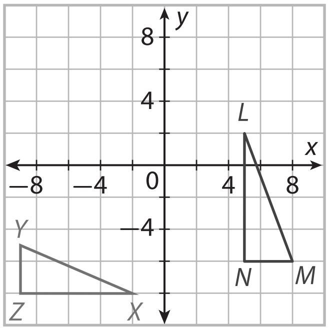
25. Which angles are congruent? Which are not congruent? Explain.



26. The figures shown are congruent. Find a sequence of transformations that will map $CDEFG$ to $QRSTU$. Give coordinate notation for the transformations you use.



27. The figures shown are congruent. Find a sequence of transformations that will map $\triangle LMN$ to $\triangle XYZ$. Give coordinate notation for the transformations you use.



28. Which sequence of transformations does not map a figure onto a congruent figure? Explain.

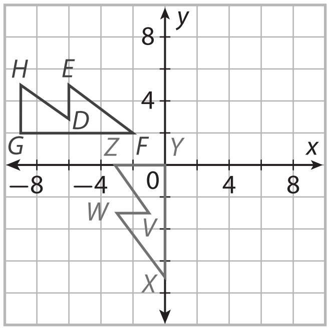
A. Rotation of 180° about the origin, reflection across the x-axis, horizontal translation $(x, y) \rightarrow (x + 4, y)$

B. Reflection across the y-axis, combined translation $(x, y) \rightarrow (x - 5, y + 2)$

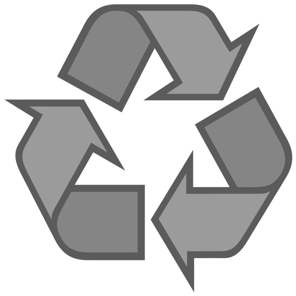
C. Rotation of 180° about the origin, reflection across the y-axis, dilation $(x, y) \rightarrow (2x, 2y)$

D. Counterclockwise rotation of 90° about the origin, reflection across the y-axis, combined translation $(x, y) \rightarrow (x - 11, y - 12)$

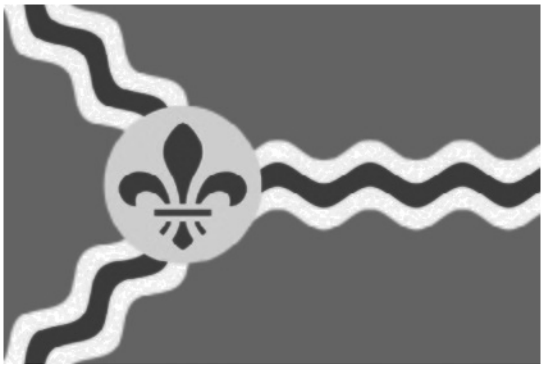
29. The figures shown are congruent. Find a sequence of transformations that will map *DEFGH* to *VWXYZ*. Give coordinate notation for the transformations you use.



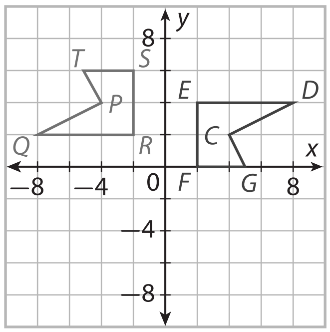
30. How can you prove that two arrows in the recycling symbol are congruent to each other?



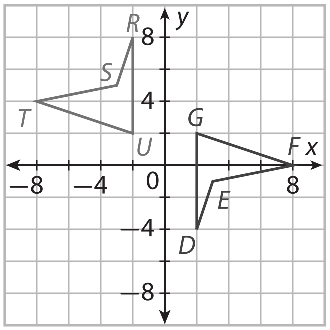
31. The city of St. Louis was settled by the French in the mid 1700s and joined the United States in 1803 as part of the Louisiana Purchase. The city flag reflects its French history by featuring the fleur-de-lis. How can you prove that the left and right petals are congruent to each other?



32. **Draw Conclusions** Two students are trying to show that the two figures are congruent. The first student decides to map *CDEFG* to *PQRST* using a rotation of 180° around the origin, followed by the translation $(x, y) \rightarrow (x, y + 6)$. The second student believes the correct transformations are a reflection across the y-axis, followed by the vertical translation $(x, y) \rightarrow (x, y - 2)$. Are both students correct, is only one student correct, or is neither student correct?



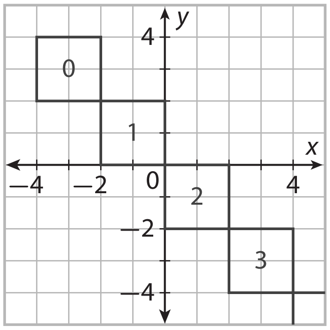
33. Justify Reasoning Two students are trying to show that the two figures are congruent. The first student decides to map *DEFG* to *RSTU* using a rotation of 180° about the origin, followed by the vertical translation $(x, y) \rightarrow (x, y + 4)$. The second student uses a reflection across the *x*-axis, followed by the vertical translation $(x, y) \rightarrow (x, y + 4)$, followed by a reflection across the *y*-axis. Are both students correct, is only one student correct, or is neither student correct?



H.O.T. Focus on Higher Order Thinking

34. Look for a Pattern Assume the pattern of congruent squares shown in the figure continues forever.

Write rules for rigid motions that map square 0 onto square 1, square 0 onto square 2, and square 0 onto square 3.



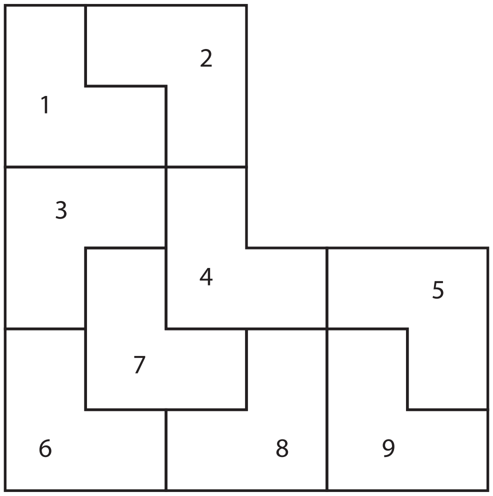
Write a rule for a rigid motion that maps square 0 onto square *n*.

35. Analyze Relationships Suppose you know that $\triangle ABC$ is congruent to $\triangle DEF$ and that $\triangle DEF$ is congruent to $\triangle GHJ$. Can you conclude that $\triangle ABC$ is congruent to $\triangle GHJ$? Explain.

36. Communicate Mathematical Ideas Ella plotted the points $A(0, 0)$, $B(4, 0)$, and $C(0, 4)$. Then she drew $\overline{AB}$ and $\overline{AC}$. Give two different arguments to explain why the segments are congruent.

Lesson Performance Task

The illustration shows how nine congruent shapes can be fitted together to form a larger shape. Each of the shapes can be formed from Shape #1 through a combination of translations, reflections, and/or rotations.



Describe how each of Shapes 2–9 can be formed from Shape #1 through a combination of translations, reflections, and/or rotations. Then design a figure like this one, using at least eight congruent shapes. Number the shapes. Then describe how each of them can be formed from Shape #1 through a combination of translations, reflections, and/or rotations.

Name ______________________ Class ______________ Date ________

18.3 Corresponding Parts of Congruent Figures Are Congruent

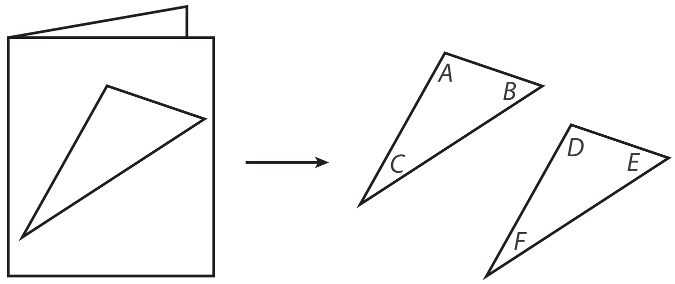
Resource Locker

Essential Question: What can you conclude about two figures that are congruent?

Explore — Exploring Congruence of Parts of Transformed Figures

You will investigate some conclusions you can make when you know that two figures are congruent.

Ⓐ Fold a sheet of paper in half. Use a straightedge to draw a triangle on the folded sheet. Then cut out the triangle, cutting through both layers of paper to produce two congruent triangles. Label them $\triangle ABC$ and $\triangle DEF$, as shown.



Ⓑ Place the triangles next to each other on a desktop. Since the triangles are congruent, there must be a sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$. Describe the sequence of rigid motions.

____

Ⓒ The same sequence of rigid motions that maps $\triangle ABC$ to $\triangle DEF$ maps parts of $\triangle ABC$ to parts of $\triangle DEF$. Complete the following.

$\overline{AB} \rightarrow$ ☐ $\overline{BC} \rightarrow$ ☐ $\overline{AC} \rightarrow$ ☐

$A \rightarrow$ ☐ $B \rightarrow$ ☐ $C \rightarrow$ ☐

Ⓓ What does Step C tell you about the corresponding parts of the two triangles? Why?

____

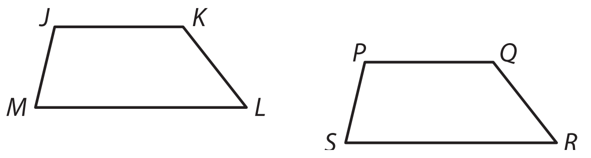
____

Reflect

1. If you know that $\triangle ABC \cong \triangle DEF$, what six congruence statements about segments and angles can you write? Why?

---

2. Do your findings in this Explore apply to figures other than triangles? For instance, if you know that quadrilaterals *JKLM* and *PQRS* are congruent, can you make any conclusions about corresponding parts? Why or why not?



Explain 1 Corresponding Parts of Congruent Figures Are Congruent

The following true statement summarizes what you discovered in the Explore.

Corresponding Parts of Congruent Figures Are Congruent

If two figures are congruent, then corresponding sides are congruent and corresponding angles are congruent.

Example 1 **$\triangle ABC \cong \triangle DEF$. Find the given side length or angle measure.**

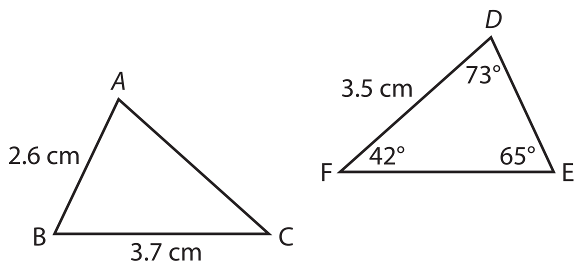
Ⓐ *DE*

Step 1 Find the side that corresponds to $\overline{DE}$.

Since $\triangle ABC \cong \triangle DEF$, $\overline{AB} \cong \overline{DE}$.

Step 2 Find the unknown length.

$DE = AB$, and $AB = 2.6$ cm, so $DE = 2.6$ cm.



Ⓑ $m\angle B$

Step 1 Find the angle that corresponds to $\angle B$.

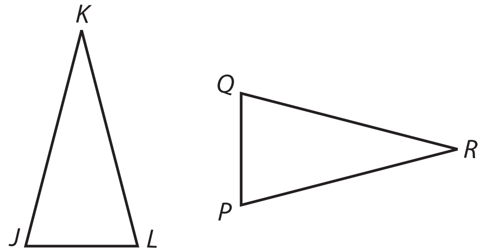
Since $\triangle ABC \cong \triangle DEF$, $\angle B \cong \angle$ ☐.

Step 2 Find the unknown angle measure.

$m\angle B = m\angle$ ☐, and $m\angle$ ☐ $=$ ☐°, so $m\angle B =$ ☐°.

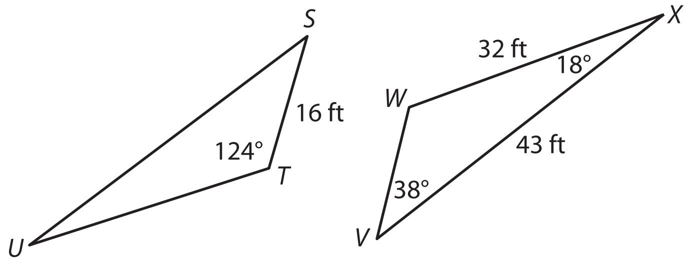
Reflect

3. **Discussion** The triangles shown in the figure are congruent. Can you conclude that $\overline{JK} \cong \overline{QR}$? Explain.



Your Turn

$\triangle STU \cong \triangle VWX$. Find the given side length or angle measure.



4. SU

5. $m\angle S$

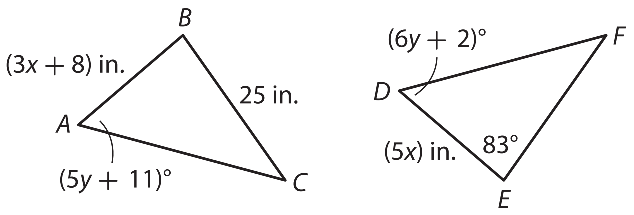
Explain 2 Applying the Properties of Congruence

Rigid motions preserve length and angle measure. This means that congruent segments have the same length, so $\overline{UV} \cong \overline{XY}$ implies $UV = XY$ and vice versa. In the same way, congruent angles have the same measure, so $\angle J \cong \angle K$ implies $m\angle J = m\angle K$ and vice versa.

Properties of Congruence	
Reflexive Property of Congruence	$\overline{AB} \cong \overline{AB}$
Symmetric Property of Congruence	If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AD}$.
Transitive Property of Congruence	If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

Example 2 **$\triangle ABC \cong \triangle DEF$. Find the given side length or angle measure.**

Ⓐ AB



Since $\triangle ABC \cong \triangle DEF$, $\overline{AB} \cong \overline{DE}$.
Therefore, $AB = DE$.

Write an equation. $3x + 8 = 5x$

Subtract $3x$ from each side. $8 = 2x$

Divide each side by 2. $4 = x$

So, $AB = 3x + 8 = 3(4) + 8 = 12 + 8 = 20$ in.

Ⓑ m∠D

Since $\triangle ABC \cong \triangle DEF$, ∠[ ] ≅ ∠$D$. Therefore, m∠[ ] = m∠$D$.

Write an equation. $5y +$ [ ] = [ ] $+ 2$

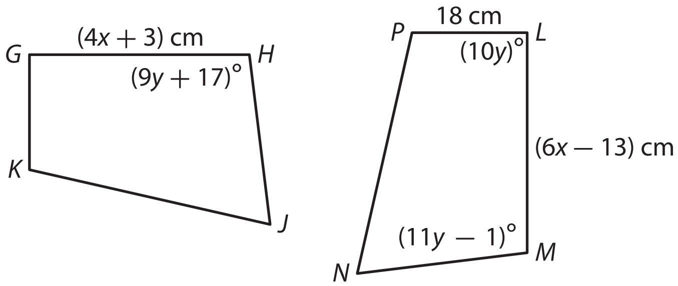
Subtract $5y$ from each side. $11 =$ [ ] $+ 2$

Subtract 2 from each side. [ ] = [ ]

So, $m\angle D = (6y + 2)° = (6 \cdot$ [ ] $+ 2)° =$ [ ]°.

Your Turn

Quadrilateral *GHJK* ≅ quadrilateral *LMNP*. Find the given side length or angle measure.

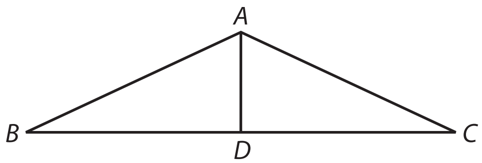


6. LM

7. m∠H

Explain 3 Using Congruent Corresponding Parts in a Proof

Example 3 **Write each proof.**



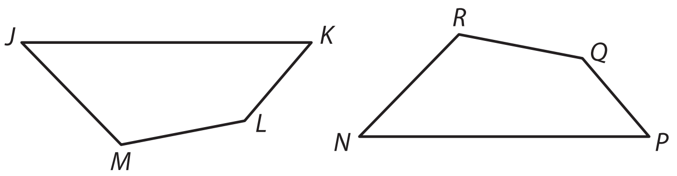
Ⓐ Given: $\triangle ABD \cong \triangle ACD$

Prove: D is the midpoint of $\overline{BC}$.

Statements	Reasons
1. $\triangle ABD \cong \triangle ACD$	**1.** Given
2. $\overline{BD} \cong \overline{CD}$	**2.** Corresponding parts of congruent figures are congruent.
3. D is the midpoint of $\overline{BC}$.	**3.** Definition of midpoint.

Ⓑ Given: Quadrilateral $JKLM \cong$ quadrilateral $NPQR$; $\angle J \cong \angle K$

Prove: $\angle J \cong \angle P$



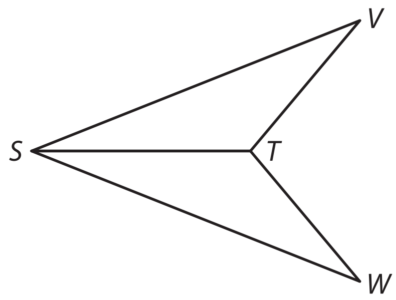
Statements	Reasons
1. Quadrilateral $JKLM \cong$ quadrilateral $NPQR$	**1.**
2. $\angle J \cong \angle K$	**2.**
3. $\angle K \cong \angle P$	**3.**
4. $\angle J \cong \angle P$	**4.**

Your Turn

Write each proof.

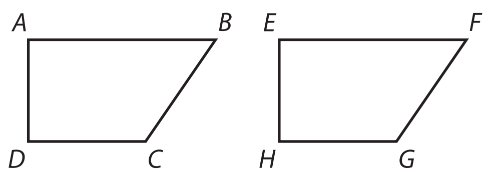
8. Given: $\triangle SVT \cong \triangle SWT$

Prove: $\overline{ST}$ bisects $\angle VSW$.



9. Given: Quadrilateral $ABCD \cong$ quadrilateral $EFGH$; $\overline{AD} \cong \overline{CD}$

Prove: $\overline{AD} \cong \overline{GH}$



Elaborate

10. A student claims that any two congruent triangles must have the same perimeter. Do you agree? Explain.

11. If $\triangle PQR$ is a right triangle and $\triangle PQR \cong \triangle XYZ$, does $\triangle XYZ$ have to be a right triangle? Why or why not?

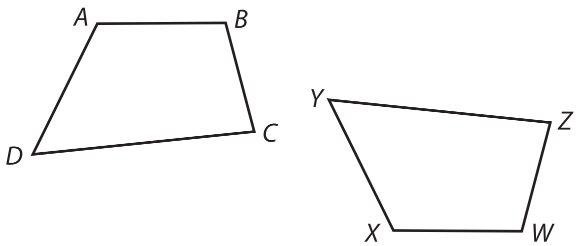
12. **Essential Question Check-In** Suppose you know that pentagon *ABCDE* is congruent to pentagon *FGHJK*. How many additional congruence statements can you write using corresponding parts of the pentagons? Explain.

Evaluate: Homework and Practice

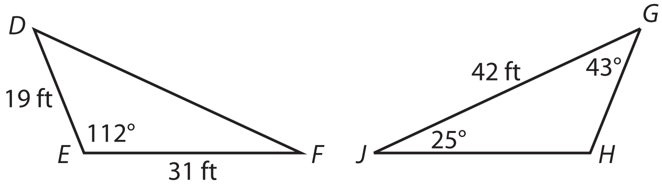
Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. Danielle finds that she can use a translation and a reflection to make quadrilateral *ABCD* fit perfectly on top of quadrilateral *WXYZ*. What congruence statements can Danielle write using the sides and angles of the quadrilaterals? Why?



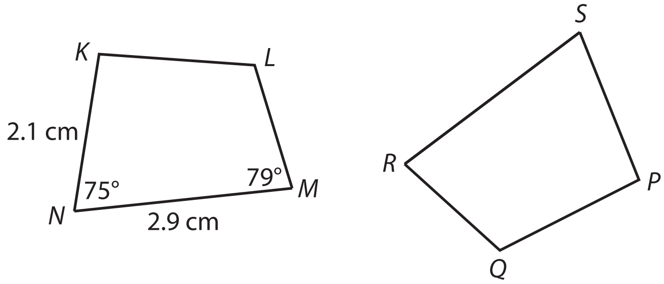
$\triangle DEF \cong \triangle GHJ$. Find the given side length or angle measure.



2. JH

3. $m\angle D$

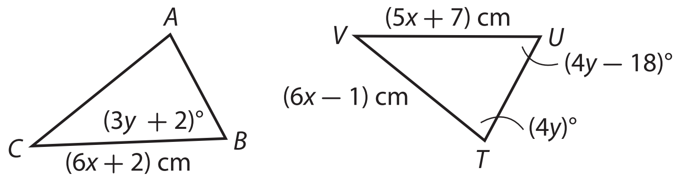
***KLMN* ≅ *PQRS*. Find the given side length or angle measure.**



4. m∠*R*

5. *PS*

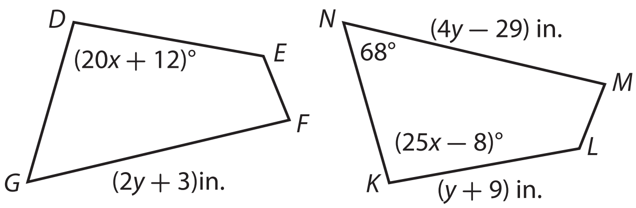
△*ABC* ≅ △*TUV*. Find the given side length or angle measure.



6. *BC*

7. m∠*U*

***DEFG* ≅ *KLMN*. Find the given side length or angle measure.**



8. *FG*

9. m∠*D*

△*GHJ* ≅ △*PQR* and △*PQR* ≅ △*STU*. Complete the following using a side or angle of △*STU*. Justify your answers.

10. $\overline{GH}$ ≅ ______________________________

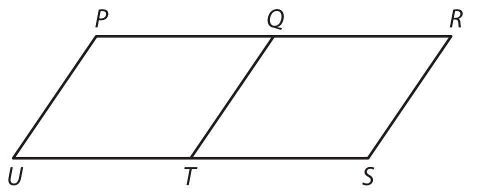
11. ∠*J* ≅ ______________________________

12. *GJ* = ______________________________

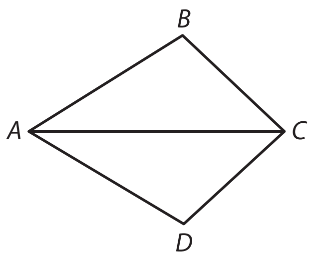
13. m∠*G* = ______________________________

Write each proof.

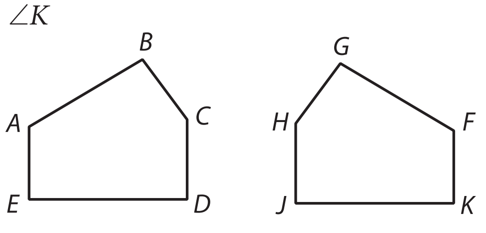
14. Given: Quadrilateral $PQTU \cong$ quadrilateral $QRST$
Prove: $\overline{QT}$ bisects $\overline{PR}$.



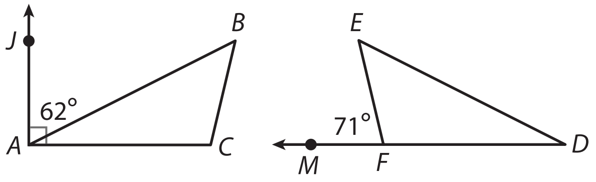
15. Given: $\triangle ABC \cong \triangle ADC$
Prove: $\overline{AC}$ bisects $\angle BAD$ and $\overline{AC}$ bisects $\angle BCD$.



16. Given: Pentagon $ABCDE \cong$ pentagon $FGHJK$; $\angle D \cong \angle E$
Prove: $\angle D \cong \angle K$



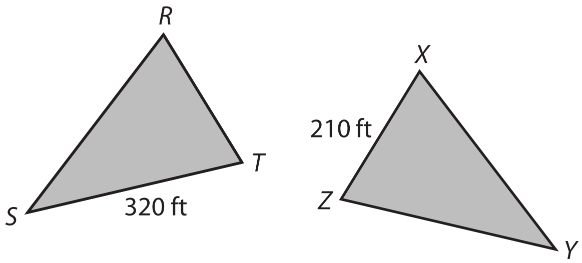
$\triangle ABC \cong \triangle DEF$. Find the given side length or angle measure.



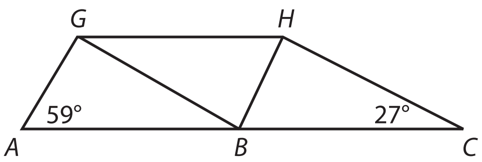
17. m$\angle D$

18. m$\angle C$

19. The figure shows the dimensions of two city parks, where $\triangle RST \cong \triangle XYZ$ and $\overline{YX} \cong \overline{YZ}$. A city employee wants to order new fences to surround both parks. What is the total length of the fences required to surround the parks?



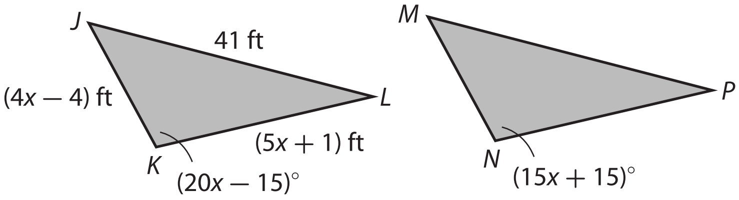
20. A tower crane is used to lift steel, concrete, and building materials at construction sites. The figure shows part of the horizontal beam of a tower crane, in which $\triangle ABG \cong \triangle BCH \cong \triangle HGB$



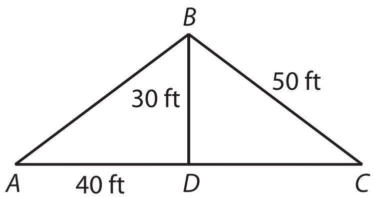
a. Is it possible to determine m$\angle GBH$? If so, how? If not, why not?

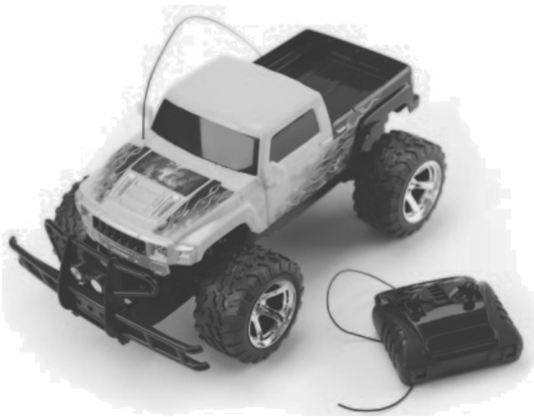
b. A member of the construction crew claims that $\overline{AC}$ is twice as long as $\overline{AB}$. Do you agree? Explain.

21. **Multi-Step** A company installs triangular pools at hotels. All of the pools are congruent and $\triangle JKL \cong \triangle MNP$ in the figure. What is the perimeter of each pool?

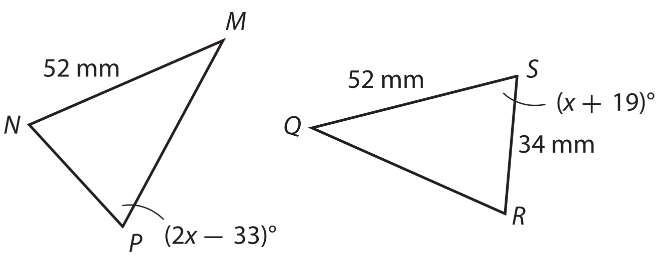


22. Kendall and Ava lay out the course shown below for their radio-controlled trucks. In the figure, $\triangle ABD \cong \triangle CBD$. The trucks travel at a constant speed of 15 feet per second. How long does it take a truck to travel on the course from A to B to C to D? Round to the nearest tenth of a second.





23. $\triangle MNP \cong \triangle QRS$. Determine whether each statement about the triangles is true or false. Select the correct answer for each lettered part.



a. $\triangle QRS$ is isosceles. ○ True ○ False

b. $\overline{MP}$ is longer than $\overline{MN}$. ○ True ○ False

c. $m\angle P = 52°$ ○ True ○ False

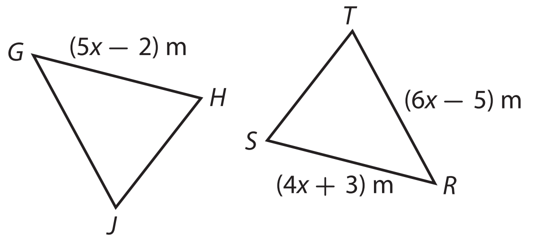
d. The perimeter of $\triangle QRS$ is 120 mm. ○ True ○ False

e. $\angle M \cong \angle Q$ ○ True ○ False

H.O.T. Focus on Higher Order Thinking

24. Justify Reasoning Given that $\triangle ABC \cong \triangle DEF$, $AB = 2.7$ ft, and $AC = 3.4$ ft, is it possible to determine the length of $\overline{EF}$? If so, find the length and justify your steps. If not, explain why not.

25. Explain the Error A student was told that $\triangle GHJ \cong \triangle RST$ and was asked to find GH. The student's work is shown below. Explain the error and find the correct answer.

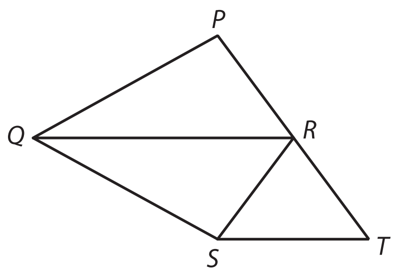


Student's Work

$$5x - 2 = 6x - 5$$
$$-2 = x - 5$$
$$3 = x$$
$$GH = 5x - 2 = 5(3) - 2 = 13 \text{ m}$$

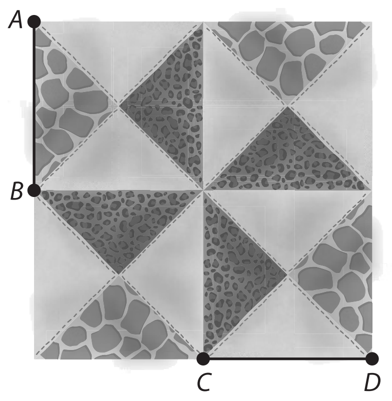
26. Critical Thinking In $\triangle ABC$, $m\angle A = 55°$, $m\angle B = 50°$, and $m\angle C = 75°$. In $\triangle DEF$, $m\angle E = 50°$, and $m\angle F = 65°$. Is it possible for the triangles to be congruent? Explain.

27. Analyze Relationships $\triangle PQR \cong \triangle SQR$ and $\overline{RS} \cong \overline{RT}$. A student said that point R appears to be the midpoint of $\overline{PT}$. Is it possible to prove this? If so, write the proof. If not, explain why not.



Lesson Performance Task

The illustration shows a "Yankee Puzzle" quilt.



a. Use the idea of congruent shapes to describe the design of the quilt.

b. Explain how the triangle with base $\overline{AB}$ can be transformed to the position of the triangle with base $\overline{CD}$.

c. Explain how you know that $CD = AB$.