Name ______________________ Class ____________ Date ________

17.1 Translations

Essential Question: How do you draw the image of a figure under a translation?

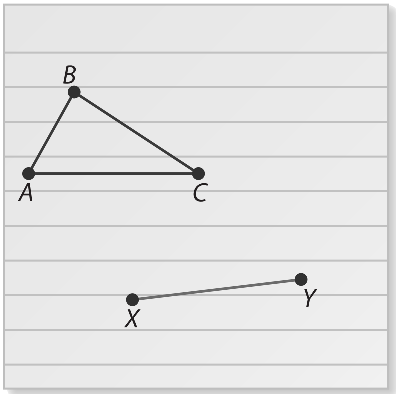
Resource Locker

Explore Exploring Translations

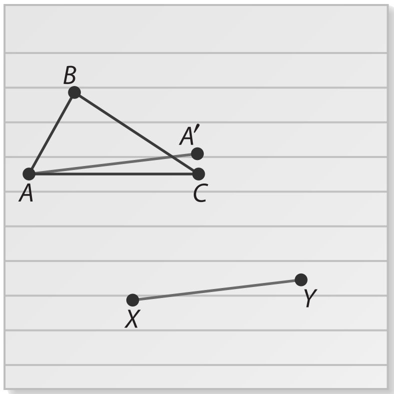
A translation slides all points of a figure the same distance in the same direction.

You can use tracing paper to model translating a triangle.

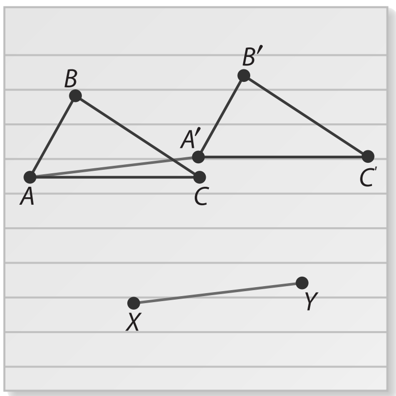
(A) First, draw a triangle on lined paper. Label the vertices A, B, and C. Then draw a line segment XY. An example of what your drawing may look like is shown.



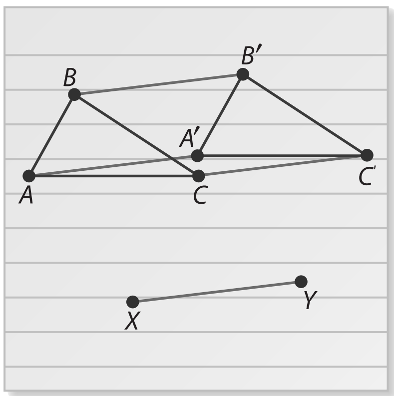
(B) Use tracing paper to draw a copy of triangle ABC. Then copy $\overline{XY}$ so that the point X is on top of point A. Label the point made from Y as A'.



(C) Using the same piece of tracing paper, place A' on A and draw a copy of $\triangle ABC$. Label the corresponding vertices B' and C'. An example of what your drawing may look like is shown.



(D) Use a ruler to draw line segments from each vertex of the preimage to the corresponding vertex on the new image.



(E) Measure the distances AA', BB', CC', and XY. Describe how AA', BB', and CC' compare to the length XY.

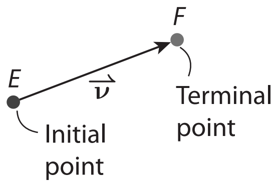
Reflect

1. Are BB', AA', and CC' parallel, perpendicular, or neither? Describe how you can check that your answer is reasonable.

2. How does the angle BAC relate to the angle $B'A'C'$? Explain.

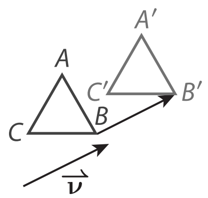
Explain 1 Translating Figures Using Vectors

A **vector** is a quantity that has both direction and magnitude.
The **initial point** of a vector is the starting point.
The **terminal point** of a vector is the ending point. The vector shown may be named $\overrightarrow{EF}$ or $\vec{v}$.



Translation

It is convenient to describe translations using vectors. A **translation** is a transformation along a vector such that the segment joining a point and its image has the same length as the vector and is parallel to the vector.



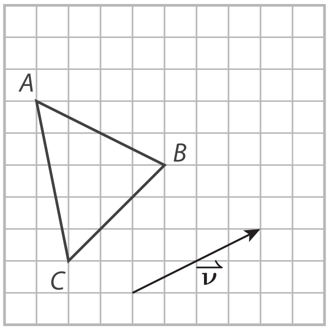
For example, BB' is a line segment that is the same length as and is parallel to vector $\vec{v}$.

You can use these facts about parallel lines to draw translations.

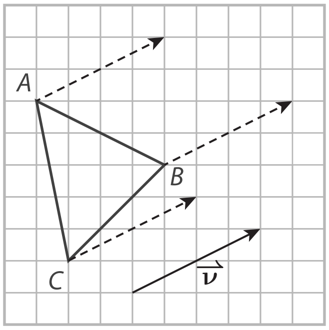
- Parallel lines are always the same distance apart and never intersect.
- Parallel lines have the same slope.

Example 1 **Draw the image of $\triangle ABC$ after a translation along $\vec{v}$.**

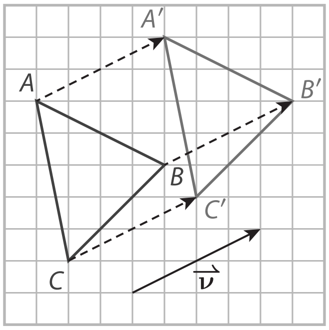
Ⓐ



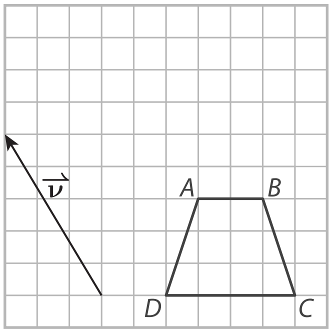
Draw a copy of $\vec{v}$ with its initial point at vertex A of $\triangle ABC$. The copy must be the same length as $\vec{v}$, and it must be parallel to $\vec{v}$. Repeat this process at vertices B and C.



Draw segments to connect the terminal points of the vectors. Label the points A', B', and C'. $\triangle A'B'C'$ is the image of $\triangle ABC$.



Ⓑ



Draw a vector from the vertex A that is the same length as and ___________ vector $\vec{v}$. The terminal point A' will be ___________ units up and 3 units ___________.

Draw three more vectors that are parallel from _______, _______, and _______ with terminal points B', C', and D'.

Draw segments connecting A', B', C', and D' to form ______________________.

Reflect

3. How is drawing an image of quadrilateral $ABCD$ like drawing an image of $\triangle ABC$? How is it different?

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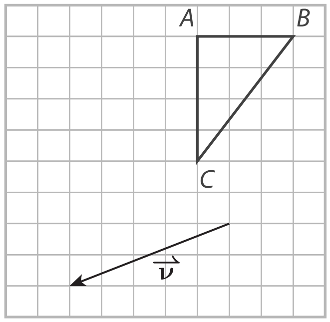
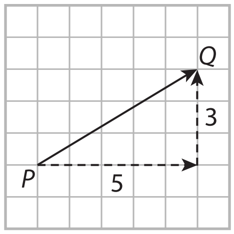
Your Turn

4. Draw the image of $\triangle ABC$ after a translation along $\vec{v}$.



Explain 2 Drawing Translations on a Coordinate Plane

A vector can also be named using component form, $\langle a, b \rangle$, which specifies the horizontal change a and the vertical change b from the initial point to the terminal point. The component form for $\overline{PQ}$ is $\langle 5, 3 \rangle$.



You can use the component form of the vector to draw coordinates for a new image on a coordinate plane. By using this vector to move a figure, you are moving the x-coordinate 5 units to the right. So, the new x-coordinate would be 5 greater than the x-coordinate in the preimage. Using this vector you are also moving the y-coordinate up 3 units. So, the new y-coordinate would be 3 greater than the y-coordinate in the preimage.

Rules for Translations on a Coordinate Plane	
Translation a units to the right	$\langle x, y \rangle \to \langle x + a, y \rangle$
Translation a units to the left	$\langle x, y \rangle \to \langle x - a, y \rangle$
Translation b units up	$\langle x, y \rangle \to \langle x, y + b \rangle$
Translation b units down	$\langle x, y \rangle \to \langle x, y - b \rangle$

So, when you move an image to the right a units and up b units, you use the rule $\langle x, y \rangle \to \langle x + a, y + b \rangle$ which is the same as moving the image along vector $\langle a, b \rangle$.

Example 2 **Calculate the vertices of the image figure. Graph the preimage and the image.**

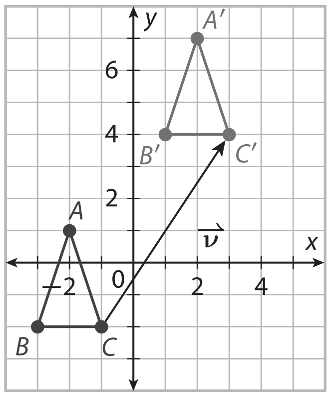
Ⓐ Preimage coordinates: $(-2, 1)$, $(-3, -2)$, and $(-1, -2)$. Vector: $\langle 4, 6 \rangle$

Predict which quadrant the new image will be drawn in: 1st quadrant.

Use a table to record the new coordinates. Use vector components to write the transformation rule.

Preimage coordinates (x, y)	Image $(x + 4,\ y + 6)$
$(-2, 1)$	$(2, 7)$
$(-3, -2)$	$(1, 4)$
$(-1, -2)$	$(3, 4)$

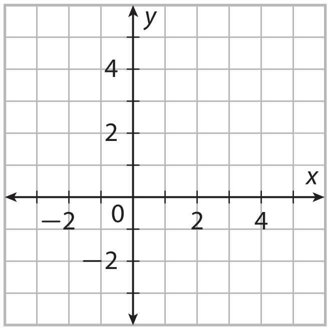
Then use the preimage coordinates to draw the preimage, and use the image coordinates to draw the new image.



(B) Preimage coordinates: $A(3, 0)$, $B(2, -2)$, and $C(4, -2)$. Vector $\langle -2, 3\rangle$

Prediction: The image will be in Quadrant ____.

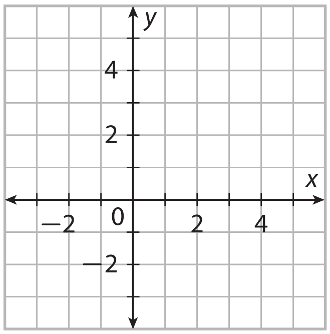
Preimage coordinates (x, y)	Image $(x - \square, y + \square)$
$(3, 0)$	$(\square, \square)$
$(2, -2)$	$(\square, \square)$
$(4, -2)$	$(\square, \square)$



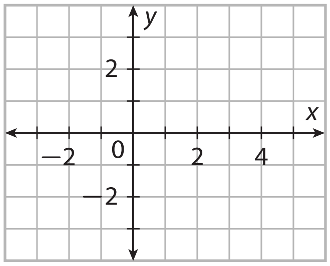
Your Turn

Draw the preimage and image of each triangle under a translation along $\langle -4, 1\rangle$.

5. Triangle with coordinates: $A(2, 4)$, $B(1, 2)$, $C(4, 2)$.



6. Triangle with coordinates: $P(2, -1)$, $Q(2, -3)$, $R(4, -3)$.

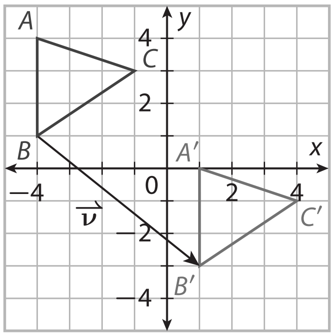


Explain 3 Specifying Translation Vectors

You may be asked to specify a translation that carries a given figure onto another figure. You can do this by drawing the translation vector and then writing it in component form.

Example 3 **Specify the component form of the vector that maps $\triangle ABC$ to $\triangle A'B'C'$.**

(A)



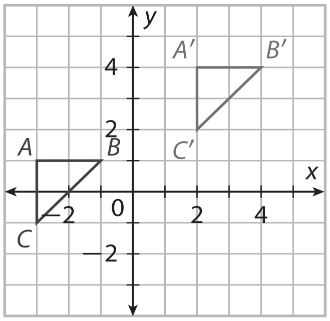
Determine the components of $\vec{v}$.

The horizontal change from the initial point $(-4, 1)$ to the terminal point $(1, -3)$ is $1 - (-4) = 5$.

The vertical change from the initial point $(-4, 1)$ to the terminal point $(1, -3)$ is $-3 - 1 = -4$

Write the vector in component form.

$$\vec{v} = \langle 5, -4\rangle$$

Ⓑ 

Draw the vector $\vec{v}$ from a vertex of $\triangle ABC$ to its image in $\triangle A'B'C'$.

Determine the components of $\vec{v}$.

The horizontal change from the initial point $(-3, 1)$ to the terminal point $(2, 4)$ is ____ − ____ = ____.

The vertical change from the initial point to the terminal point is ____ − ____ = ____

Write the vector in component form. $\vec{v} = \langle \square, \square \rangle$

Reflect

7. What is the component form of a vector that translates figures horizontally? Explain.

Your Turn

8. In Example 3A, suppose $\triangle A'B'C'$ is the preimage and $\triangle ABC$ is the image after translation. What is the component form of the translation vector in this case? How is this vector related to the vector you wrote in Example 3A?

Elaborate

9. How are translations along the vectors $\langle a, -b \rangle$ and $\langle -a, b \rangle$ similar and how are they different?

10. A translation along the vector $\langle -2, 7 \rangle$ maps point P to point Q. The coordinates of point Q are $(4, -1)$. What are the coordinates of point P? Explain your reasoning.

11. A translation along the vector $\langle a, b \rangle$ maps points in Quadrant I to points in Quadrant III. What can you conclude about a and b? Justify your response.

12. **Essential Question Check-In** How does translating a figure using the formal definition of a translation compare to the previous method of translating a figure?

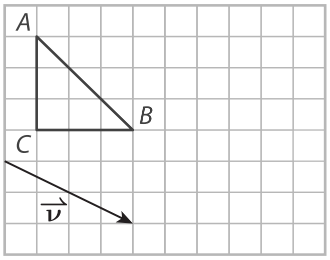
Evaluate: Homework and Practice

Personal Math Trainer

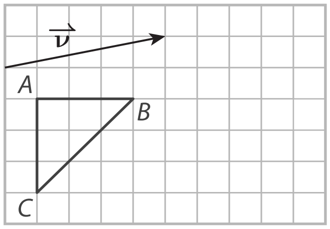
- Online Homework
- Hints and Help
- Extra Practice

Draw the image of $\triangle ABC$ after a translation along $\vec{v}$.

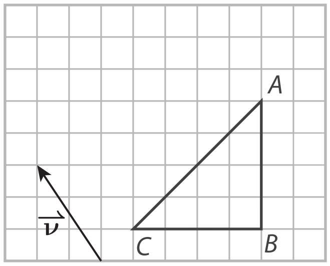
1.



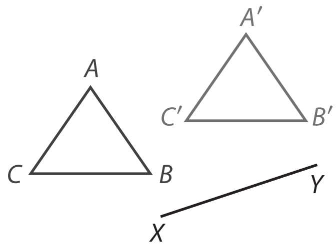
2.



3.

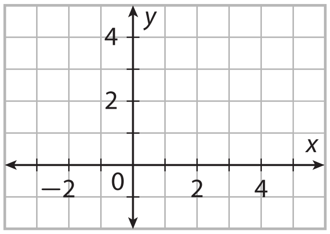


4. Line segment $\overline{XY}$ was used to draw a copy of $\triangle ABC$. $\overline{XY}$ is 3.5 centimeters long. What is the length of $AA' + BB' + CC'$?

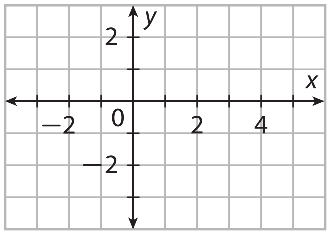


Draw the preimage and image of each triangle under the given translation.

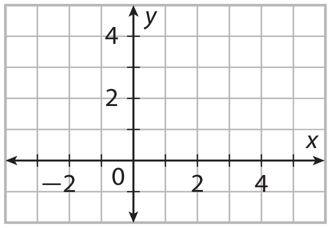
5. Triangle: $A(-3, -1)$; $B(-2, 2)$; $C(0, -1)$; Vector: $\langle 3, 2\rangle$



6. Triangle: $P(1, -3)$; $Q(3, -1)$; $R(4, -3)$; Vector: $\langle -1, 3\rangle$



7. Triangle: $X(0, 3)$; $Y(-1, 1)$; $Z(-3, 4)$; Vector: $\langle 4, -2\rangle$

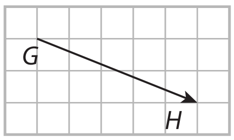


8. Find the coordinates of the image under the transformation $\langle 6, -11\rangle$.

$(x, y) \rightarrow$ $\qquad$ $(2, -3) \rightarrow$

$(3, 1) \rightarrow$ $\qquad$ $(4, -3) \rightarrow$

9. Name the vector. Write it in component form.



10. Match each set of coordinates for a preimage with the coordinates of its image after applying the vector $\langle 3, -8\rangle$. Indicate a match by writing a letter for a preimage on the line in front of the corresponding image.

A. $(1, 1)$; $(10, 1)$; $(6, 5)$ ________ $(6, -10)$; $(6, -4)$; $(9, -3)$

B. $(0, 0)$; $(3, 8)$; $(4, 0)$; $(7, 8)$ ________ $(1, -6)$; $(5, -6)$; $(-1, -8)$; $(7, -8)$

C. $(3, -2)$; $(3, 4)$; $(6, 5)$ ________ $(4, -7)$; $(13, -7)$; $(9, -3)$

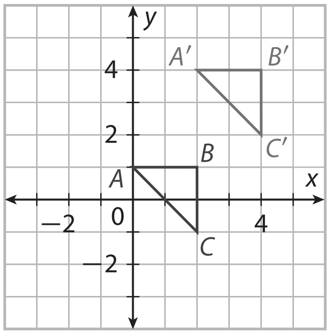
D. $(-2, 2)$; $(2, 2)$; $(-4, 0)$; $(4, 0)$ ________ $(3, -8)$; $(6, 0)$; $(7, -8)$; $(10, 0)$

11. Persevere in Problem Solving Emma and Tony are playing a game. Each draws a triangle on a coordinate grid. For each turn, Emma chooses either the horizontal or vertical value for a vector in component form. Tony chooses the other value, alternating each turn. They each have to draw a new image of their triangle using the vector with the components they chose and using the image from the prior turn as the preimage. Whoever has drawn an image in each of the four quadrants first wins the game.

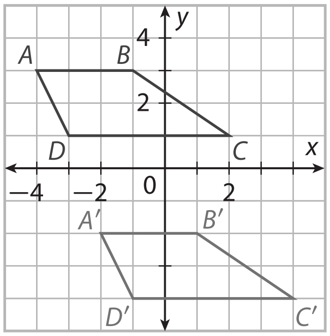
Emma's initial triangle has the coordinates $(-3, 0)$, $(-4, -2)$, $(-2, -2)$ and Tony's initial triangle has the coordinates $(2, 4)$, $(2, 2)$, $(4, 3)$. On the first turn the vector $\langle 6, -5 \rangle$ is used and on the second turn the vector $\langle -10, 8 \rangle$ is used. What quadrant does Emma need to translate her triangle to in order to win? What quadrant does Tony need to translate his triangle to in order to win?

Specify the component form of the vector that maps each figure to its image.

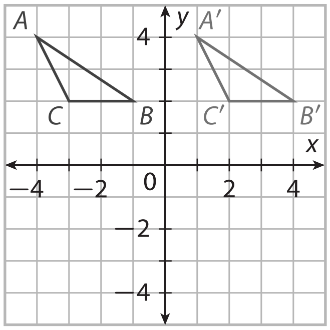
12.



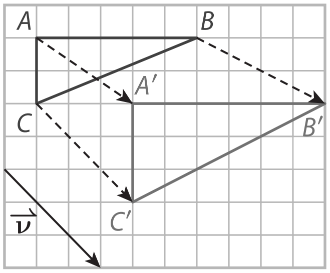
13.



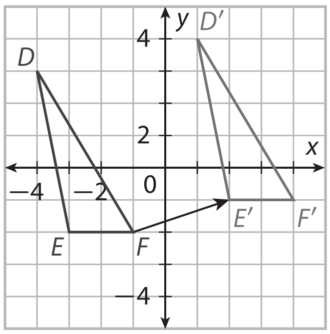
14.



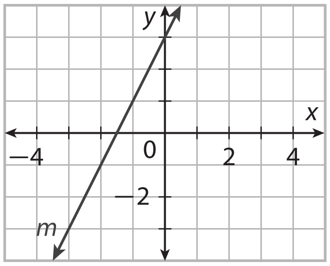
15. Explain the Error Andrew is using vector $\vec{v}$ to draw a copy of $\triangle ABC$. Explain his error.



16. Explain the Error Marcus was asked to identify the vector that maps $\triangle DEF$ to $\triangle D'E'F'$. He drew a vector as shown and determined that the component form of the vector is $\langle 3, 1 \rangle$. Explain his error.



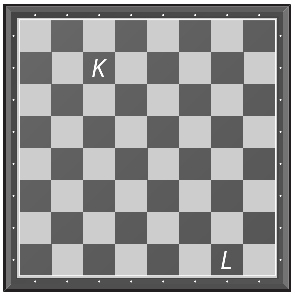
17. **Algebra** A cartographer is making a city map. Line m represents Murphy Street. The cartographer translates points on line m along the vector $\langle 2, -2 \rangle$ to draw Nolan Street. Draw the line for Nolan Street on the coordinate plane and write its equation. What is the image of the point $(0, 3)$ in this situation?



H.O.T. Focus on Higher Order Thinking

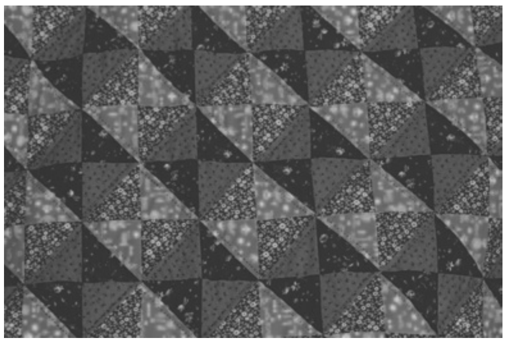
18. **Represent Real-World Problems** A builder is trying to level out some ground with a front-end loader. He picks up some excess dirt at (9, 16) and then maneuvers through the job site along the vectors $\langle -6, 0 \rangle$, $\langle 2, 5 \rangle$, $\langle 8, 10 \rangle$ to get to the spot to unload the dirt. Find the coordinates of the unloading point. Find a single vector from the loading point to the unloading point.

19. **Look for a Pattern** A checker player's piece begins at K and, through a series of moves, lands on L. What translation vector represents the path from K to L?



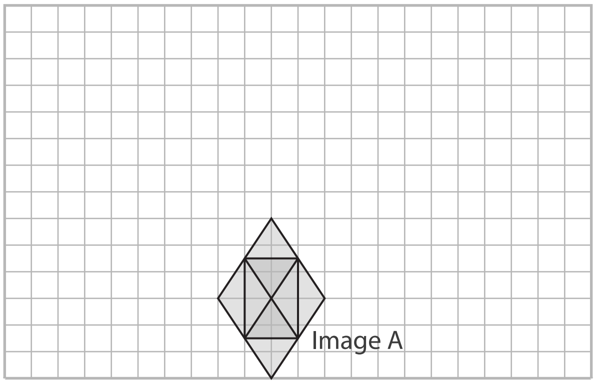
20. **Represent Real-World Problems** A group of hikers walks 2 miles east and then 1 mile north. After taking a break, they then hike 4 miles east to their final destination. What vector describes their hike from their starting position to their final destination? Let 1 unit represent 1 mile.

21. **Communicate Mathematical Ideas** In a quilt pattern, a polygon with vertices $(-4, -2)$, $(-3, -1)$, $(-2, -2)$, and $(-3, -3)$ is translated repeatedly along the vector $\langle 2, 2 \rangle$. What are the coordinates of the third polygon in the pattern? Explain how you solved the problem.



Lesson Performance Task

A contractor is designing a pattern for tiles in an entryway, using a sun design called Image *A* for the center of the space. The contractor wants to duplicate this design three times, labeled Image *B*, Image *C*, and Image *D*, above Image *A* so that they do not overlap. Identify the three vectors, labeled $\overrightarrow{m}$, $\overrightarrow{n}$, and $\overrightarrow{p}$ that could be used to draw the design, and write them in component form. Draw the images on grid paper using the vectors you wrote.



Name ______________________ Class ______________ Date ________

17.2 Reflections

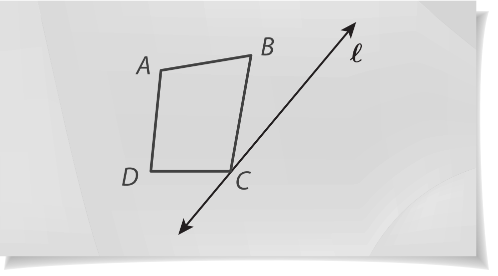
Essential Question: How do you draw the image of a figure under a reflection?

Resource Locker

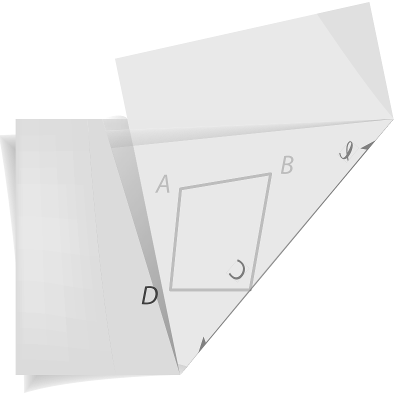
Explore Exploring Reflections

Use tracing paper to explore reflections.

(A) Draw and label a line ℓ on tracing paper. Then draw and label a quadrilateral $ABCD$ with vertex C on line ℓ.



(B) Fold the tracing paper along line ℓ. Trace the quadrilateral. Then unfold the paper and draw the image of the quadrilateral. Label it $A'\ B'\ C'\ D'$.



(C) Draw segments to connect each vertex of quadrilateral $ABCD$ with its image. Use a protractor to measure the angle formed by each segment and line ℓ. What do you notice?

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(D) Use a ruler to measure each segment and the two shorter segments formed by its intersection with line ℓ. What do you notice?

____

Reflect

1. In this activity, the fold line (line ℓ) is the line of reflection. What happens when a point is located on the line of reflection?

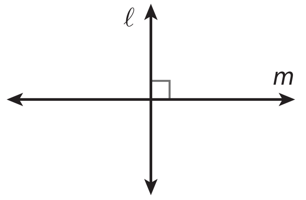
____

____

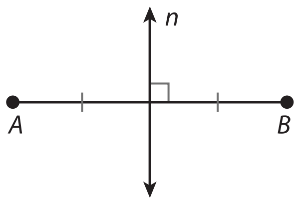
2. **Discussion** A student claims that a figure and its reflected image always lie on opposite sides of the line of reflection. Do you agree? Why or why not?

Explain 1 Reflecting Figures Using Graph Paper

Perpendicular lines are lines that intersect at right angles. In the figure, line ℓ is perpendicular to line m. The right angle mark in the figure indicates that the lines are perpendicular.

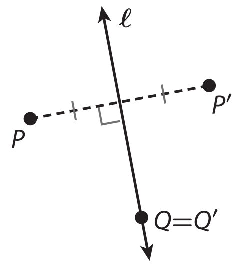


The **perpendicular bisector** of a line segment is a line perpendicular to the segment at the segment's midpoint. In the figure, line n is the perpendicular bisector of $\overline{AB}$.



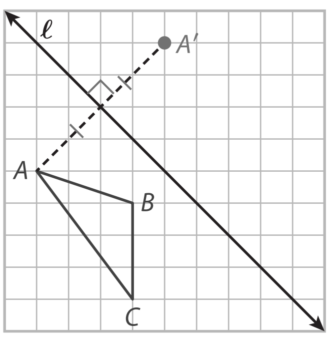
A **reflection** across line ℓ maps a point P to its image P'.

- If P is not on line ℓ, then line ℓ is the perpendicular bisector of $\overline{PP'}$.
- If P is on line ℓ, then $P = P'$.



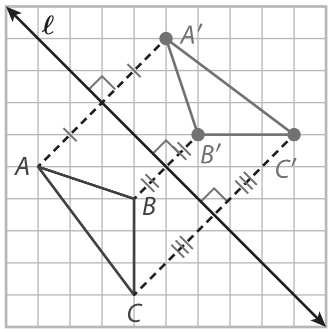
Example 1 **Draw the image of $\triangle ABC$ after a reflection across line ℓ.**

Ⓐ **Step 1** Draw a segment with an endpoint at vertex A so that the segment is perpendicular to line ℓ and is bisected by line ℓ. Label the other endpoint of the segment A'.

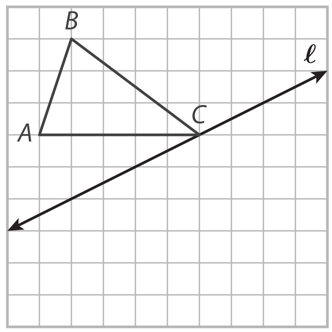


Step 2 Repeat Step 1 at vertices B and C.

Step 3 Connect points A', B', and C'.
$\triangle A'B'C'$ is the image of $\triangle ABC$.



Ⓑ Draw the image of $\triangle ABC$ after a reflection across line ℓ.



Step 1 Draw a segment with an endpoint at vertex A so that the segment is perpendicular to line ℓ and is bisected by line ℓ. Label the other endpoint of the segment A'.

Step 2 Repeat Step 1 at vertex B.

Notice that C and C' are the same point because C is on the line of reflection.

Step 3 Connect points A', B', and C'. $\triangle A'B'C'$ is the image of $\triangle ABC$.

Reflect

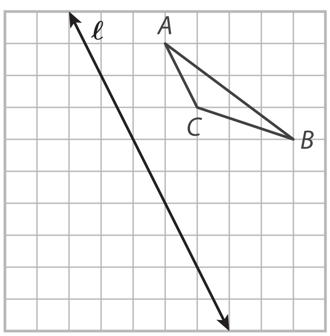
3. How can you check that you drew the image of the triangle correctly?

4. In Part A, how can you tell that $\overline{AA'}$ is perpendicular to line ℓ?

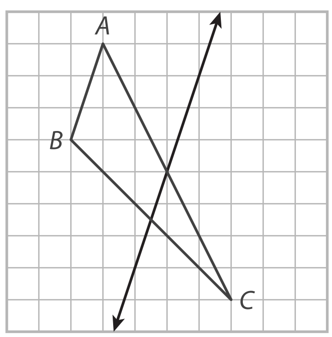
Your Turn

Draw the image of $\triangle ABC$ after a reflection across line ℓ.

5.



6.



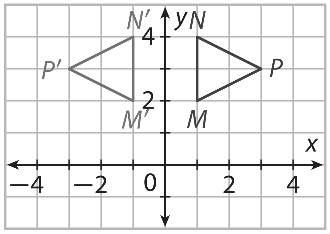
Explain 2 Drawing Reflections on a Coordinate Plane

The table summarizes coordinate notation for reflections on a coordinate plane.

Rules for Reflections on a Coordinate Plane	
Reflection across the x-axis	$(x, y) \to (x, -y)$
Reflection across the y-axis	$(x, y) \to (-x, y)$
Reflection across the line $y = x$	$(x, y) \to (y, x)$
Reflection across the line $y = -x$	$(x, y) \to (-y, -x)$

Example 2 **Reflect the figure with the given vertices across the given line.**

Ⓐ $M(1, 2)$, $N(1, 4)$, $P(3, 3)$; y-axis



Step 1 Find the coordinates of the vertices of the image.

$A(x, y) \quad \to \quad A'(-x, y).$

$M(1, 2) \quad \to \quad M'(-1, 2)$

$N(1, 4) \quad \to \quad N'(-1, 4)$

$P(3, 3) \quad \to \quad P'(-3, 3)$

Step 2 Graph the preimage.

Step 3 Predict the quadrant in which the image will lie. Since $\triangle MNP$ lies in Quadrant I and the triangle is reflected across the y-axis, the image will lie in Quadrant II.

Graph the image.

Ⓑ $D(2, 0)$, $E(2, 2)$, $F(5, 2)$, $G(5, 1)$; $y = x$

Step 1 Find the coordinates of the vertices of the image.

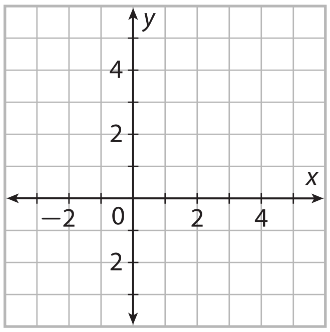
$A(x, y) \to A'(\boxed{}, \boxed{})$

$D(2, 0) \to D'(\boxed{}, \boxed{})$

$E(2, 2) \to E'(\boxed{}, \boxed{})$

$F(5, 2) \to F'(\boxed{}, \boxed{})$

$G(5, 1) \to G'(\boxed{}, \boxed{})$



Step 2 Graph the preimage.

Step 3 Since $DEFG$ lies in Quadrant I and the quadrilateral is reflected across the line $y = x$,

the image will lie in Quadrant ______.

Graph the image.

Reflect

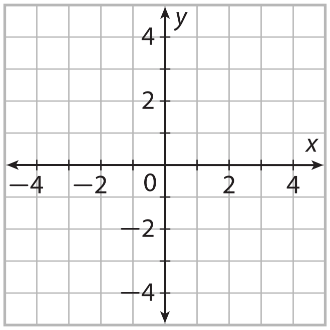
7. How would the image of ΔMNP be similar to and different from the one you drew in Part A if the triangle were reflected across the x-axis?

8. A classmate claims that the rule $(x, y) \to (-x, y)$ for reflecting a figure across the y-axis only works if all the vertices are in the first quadrant because the values of x and y must be positive. Explain why this reasoning is not correct.

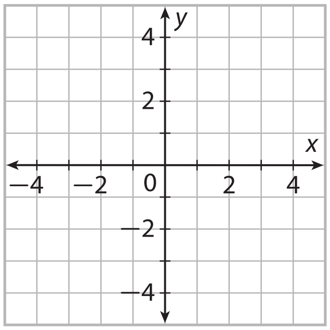
Your Turn

Reflect the figure with the given vertices across the given line.

9. $S(3, 4)$, $T(3, 1)$, $U(-2, 1)$, $V(-2, 4)$; x-axis



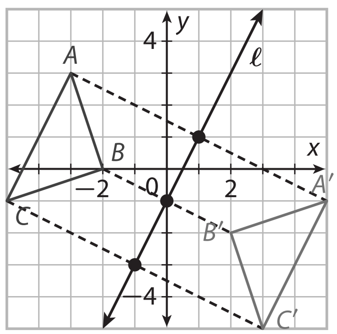
10. $A(-4, -2)$, $B(-1, -1)$, $C(-1, -4)$; $y = -x$



Explain 3 Specifying Lines of Reflection

Example 3 **Given that $\triangle A'B'C'$ is the image of $\triangle ABC$ under a reflection, draw the line of reflection.**

Ⓐ Draw the segments $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$.



Find the midpoint of each segment.

The midpoint of $\overline{AA'}$ is $\left(\frac{-3+5}{2}, \frac{3+(-1)}{2}\right) = (1, 1)$.

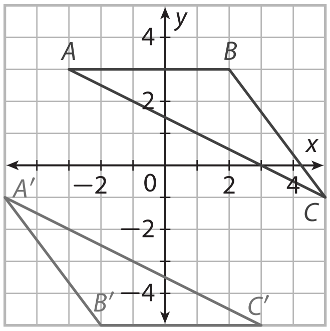
The midpoint of $\overline{BB'}$ is $\left(\frac{-2+2}{2}, \frac{0+(-2)}{2}\right) = (0, -1)$.

The midpoint of $\overline{CC'}$ is $\left(\frac{-5+3}{2}, \frac{-1+(-5)}{2}\right) = (-1, -3)$.

Plot the midpoints. Draw line ℓ through the midpoints.

Line ℓ is the line of reflection.

Ⓑ Draw $\overline{AA'}$, $\overline{BB'}$, and $\overline{CC'}$. Find the midpoint of each segment.



The midpoint of $\overline{AA'}$ is $\left(\frac{\square + \square}{2}, \frac{\square + \square}{2}\right) = (\square, \square)$.

The midpoint of $\overline{BB'}$ is $\left(\frac{\square + \square}{2}, \frac{\square + \square}{2}\right) = (\square, \square)$.

The midpoint of $\overline{CC'}$ is $\left(\frac{\square + \square}{2}, \frac{\square + \square}{2}\right) = (\square, \square)$.

Plot the midpoints. Draw line ℓ through the midpoints. Line ℓ is the line of reflection.

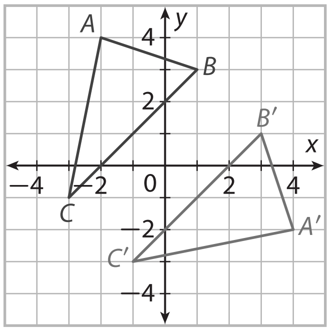
Reflect

11. How can you use a ruler and protractor to check that line ℓ is the line of reflection?

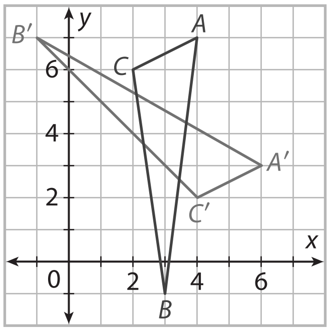
Your Turn

Given that $\triangle A'B'C'$ is the image of $\triangle ABC$ under a reflection, draw the line of reflection.

12.



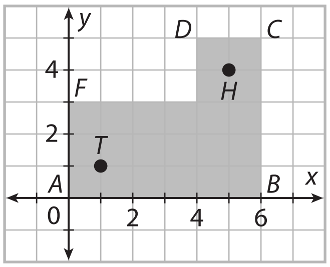
13.



Explain 4 Applying Reflections

Example 4

The figure shows one hole of a miniature golf course. It is not possible to hit the ball in a straight line from the tee T to the hole H. At what point should a player aim in order to make a hole in one?



Understand the Problem

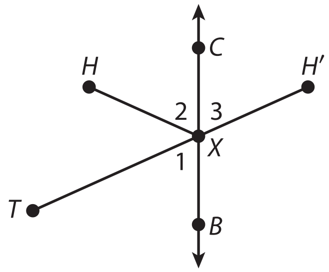
The problem asks you to locate point X on the wall of the miniature golf hole so that the ball can travel in a straight line from T to X and from X to H.

Make a Plan

In order for the ball to travel directly from T to X to H, the angle of the ball's path as it hits the wall must equal the angle of the ball's path as it leaves the wall. In the figure, $m\angle 1$ must equal $m\angle 2$.

Let H' be the reflection of point H across $\overline{BC}$.

Reflections preserve angle measure, so $m\angle 2 = m\angle \boxed{}$. Therefore, $m\angle 1$ is equal to $m\angle 2$ when $m\angle 1$ is equal to $m\angle 3$. This occurs when T, $\boxed{}$, and H' are collinear.



Solve

Reflect H across $\overline{BC}$ to locate H'.

The coordinates of H' are $\left(\boxed{}, \boxed{}\right)$.

Draw $\overline{TH'}$ and locate point X where $\overline{TH'}$ intersects $\overline{BC}$.

The coordinates of point X are $\left(\boxed{}, \boxed{}\right)$.

The player should aim at this point.

Look Back

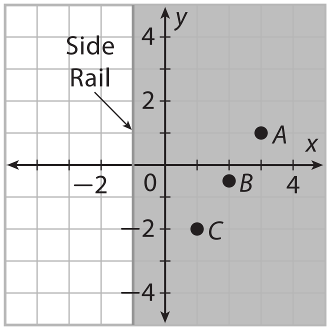
To check that the answer is reasonable, plot point X using the coordinates you found. Then use a protractor to check that the angle of the ball's path as it hits the wall at point X is equal to the angle of the ball's path as it leaves the wall from point X.

Reflect

14. Is there another path the ball can take to hit a wall and then travel directly to the hole? Explain.

Your Turn

15. Cara is playing pool. She wants to use the cue ball C to hit the ball at point A without hitting the ball at point B. To do so, she has to bounce the cue ball off the side rail and into the ball at point A. Find the coordinates of the exact point along the side rail that Cara should aim for.



Elaborate

16. Do any points in the plane have themselves as images under a reflection? Explain.

17. If you are given a figure and its image under a reflection, how can you use paper folding to find the line of reflection?

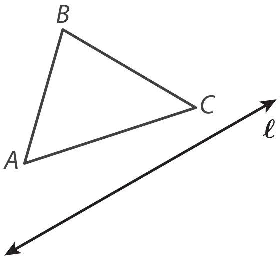
18. **Essential Question Check-In** How do you draw the image of a figure under a reflection across the x-axis?

Evaluate: Homework and Practice

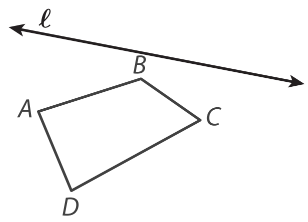
- Online Homework
- Hints and Help
- Extra Practice

Use tracing paper to copy each figure and line ℓ. Then fold the paper to draw and label the image of the figure after a reflection across line ℓ.

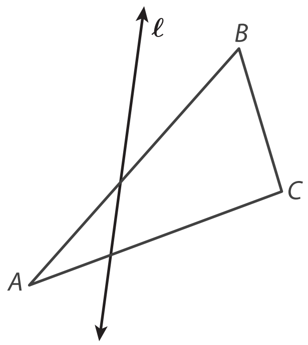
1.



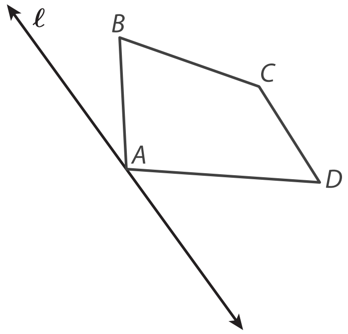
2.



3.

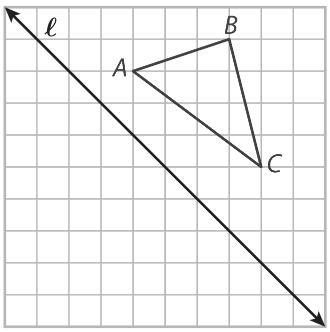


4.

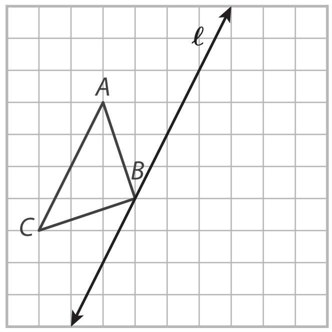


Draw the image of $\triangle ABC$ after a reflection across line ℓ.

5.



6.



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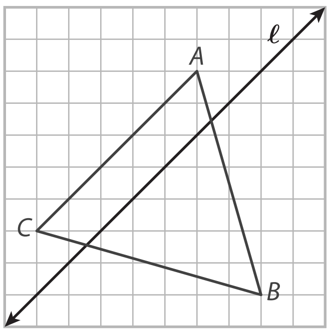
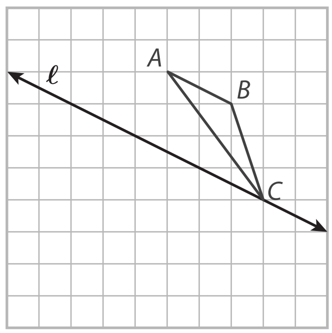
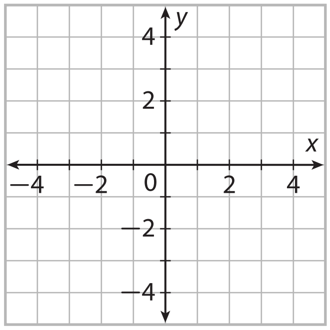


8.

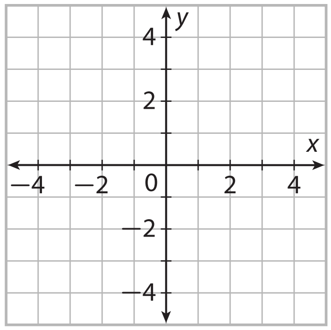


Reflect the figure with the given vertices across the given line.

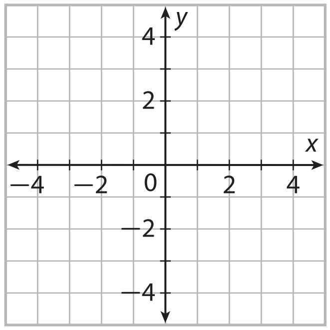
9. $P(-2, 3)$, $Q(4, 3)$, $R(-1, 0)$, $S(-4, 1)$; x-axis



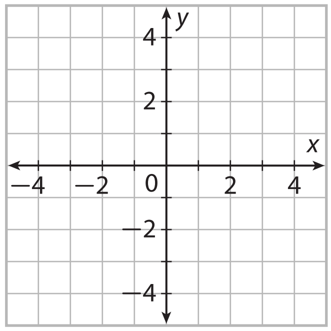
10. $A(-3, -3)$, $B(1, 3)$, $C(3, -1)$; y-axis



11. $J(-1, 2)$, $K(2, 4)$, $L(4, -1)$; $y = -x$

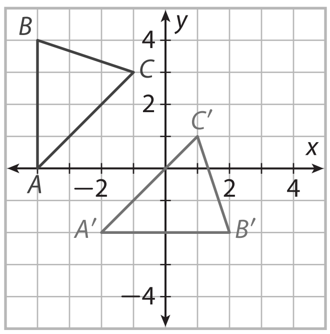


12. $D(-1, 1)$, $E(3, 2)$, $F(4, -1)$, $G(-1, -3)$; $y = x$

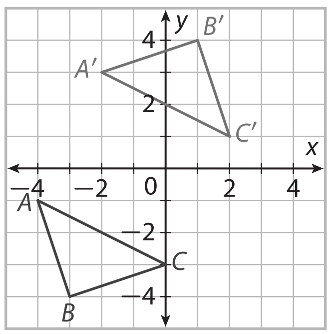


Given that $\triangle A'B'C'$ is the image of $\triangle ABC$ under a reflection, draw the line of reflection.

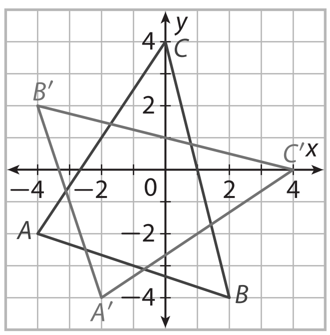
13.



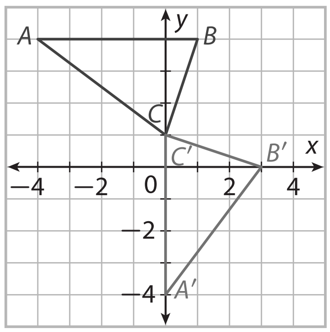
14.



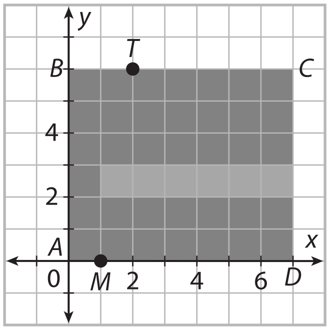
15.



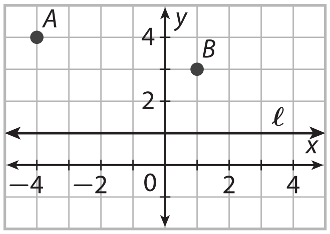
16.



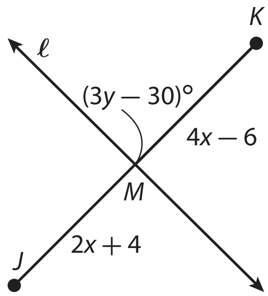
17. Jamar is playing a video game. The object of the game is to roll a marble into a target. In the figure, the shaded rectangular area represents the video screen and the striped rectangle is a barrier. Because of the barrier, it is not possible to roll the marble *M* directly into the target *T*. At what point should Jamar aim the marble so that it will bounce off a wall and roll into the target?



18. A trail designer is planning two trails that connect campsites *A* and *B* to a point on the river, line ℓ. She wants the total length of the trails to be as short as possible. At what point should the trails meet the river?



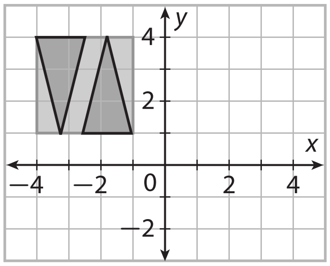
Algebra **In the figure, point *K* is the image of point *J* under a reflection across line ℓ. Find each of the following.**

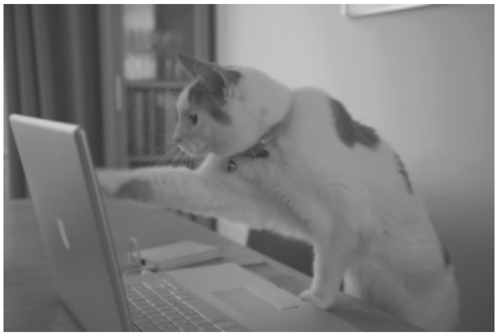


19. *JM*

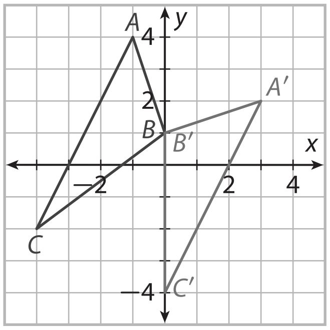
20. *y*

21. Make a Prediction Each time Jenny presses the tab key on her keyboard, the software reflects the logo she is designing across the x-axis. Jenny's cat steps on the keyboard and presses the tab key 25 times. In which quadrant does the logo end up? Explain.

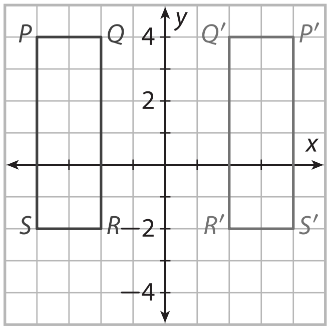




22. Multi-Step Write the equation of the line of reflection.



23. Communicate Mathematical Ideas
The figure shows rectangle $PQRS$ and its image after a reflection across the y-axis. A student said that $PQRS$ could also be mapped to its image using the translation $(x, y) \to (x + 6, y)$. Do you agree? Explain why or why not.



24. Which of the following transformations map $\triangle ABC$ to a triangle that intersects the x-axis? Select all that apply.

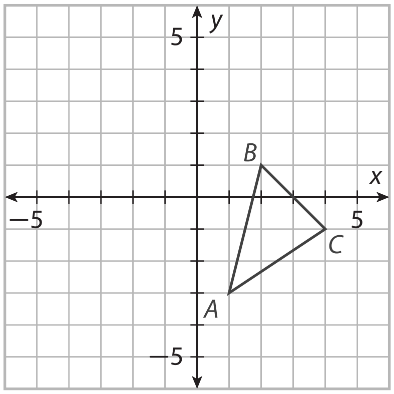
A. $(x, y) \to (-x, y)$

B. $(x, y) \to (x, -y)$

C. $(x, y) \to (y, x)$

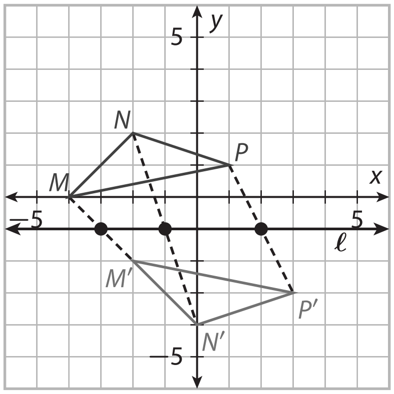
D. $(x, y) \to (-y, -x)$

E. $(x, y) \to (x, y + 1)$

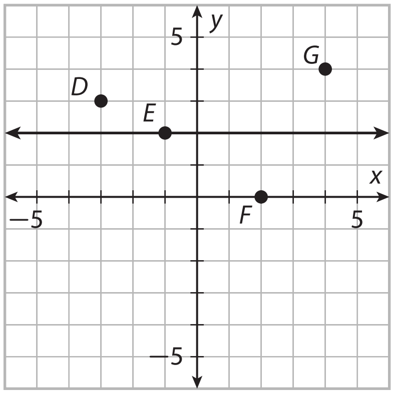


H.O.T. Focus on Higher Order Thinking

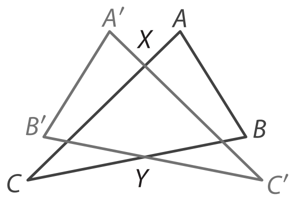
25. Explain the Error $\triangle M'N'P'$ is the image of $\triangle MNP$. Casey draws $\overline{MM'}$, $\overline{NN'}$, and $\overline{PP'}$. Then she finds the midpoint of each segment and draws line ℓ through the midpoints. She claims that line ℓ is the line of reflection. Do you agree? Explain.



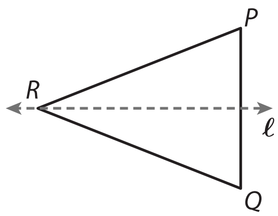
26. Draw Conclusions Plot the images of points D, E, F, and G after a reflection across the line $y = 2$. Then write an algebraic rule for the reflection.



27. Critique Reasoning Mayumi wants to draw the line of reflection for the reflection that maps $\triangle ABC$ to $\triangle A'B'C'$. She claims that she just needs to draw the line through the points X and Y. Do you agree? Explain.



28. Justify Reasoning Point Q is the image of point P under a reflection across line ℓ. Point R lies on line ℓ. What type of triangle is $\triangle PQR$? Justify your answer.



Lesson Performance Task

In order to see the entire length of your body in a mirror, do you need a mirror that is as tall as you are? If not, what is the length of the shortest mirror you can use, and how should you position it on a wall?

a. Let the x-axis represent the floor and let the y-axis represent the wall on which the mirror hangs. Suppose the bottom of your feet are at $F(3, 0)$, your eyes are at $E(3, 7)$, and the top of your head is at $H(3, 8)$. Plot these points and the points that represent their reflection images. (*Hint:* When you look in a mirror, your reflection appears to be as far behind the mirror as you are in front of it.) Draw the lines of sight from your eyes to the reflection of the top of your head and to the reflection of the bottom of your feet. Determine where these lines of sight intersect the mirror.

b. Experiment by changing your distance from the mirror, the height of your eyes, and/or the height of the top of your head. Use your results to determine the length of the shortest mirror you can use and where it should be positioned on the wall so that you can see the entire length of your body in the mirror.

Name ______________________ Class ______________ Date ________

17.3 Rotations

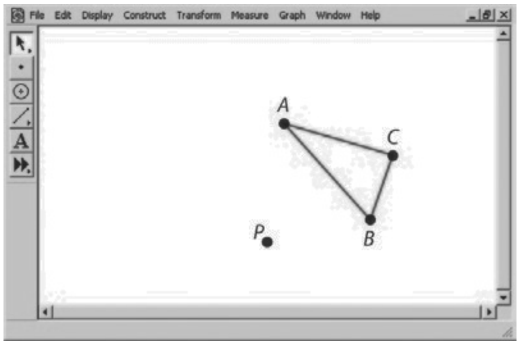
Essential Question: How do you draw the image of a figure under a rotation?

Resource Locker

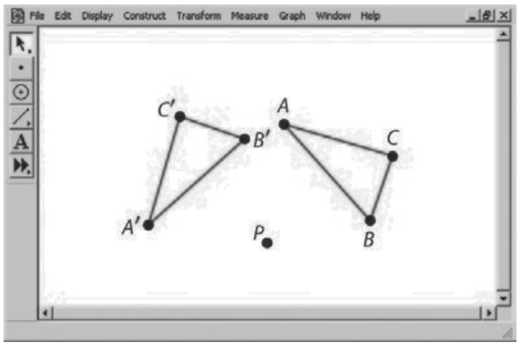
Explore Exploring Rotations

You can use geometry software or an online tool to explore rotations.

(A) Draw a triangle and label the vertices A, B, and C. Then draw a point P. Mark P as a center. This will allow you to rotate figures around point P.



(B) Select $\triangle ABC$ and rotate it 90° around point P. Label the image of $\triangle ABC$ as $\triangle A'B'C'$. Change the shape, size, or location of $\triangle ABC$ and notice how $\triangle A'B'C'$ changes.



(C) Draw $\angle APA'$, $\angle BPB'$, and $\angle CPC'$. Measure these angles. What do you notice? Does this relationship remain true as you move point P? What happens if you change the size and shape of $\triangle ABC$?

____

____

(D) Measure the distance from A to P and the distance from A' to P. What do you notice? Does this relationship remain true as you move point P? What happens if you change the size and shape of $\triangle ABC$?

____

____

Reflect

1. What can you conclude about the distance of a point and its image from the center of rotation?

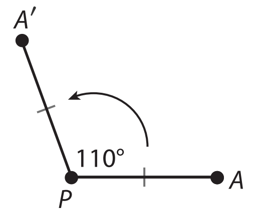
2. What are the advantages of using geometry software or an online tool rather than tracing paper or a protractor and ruler to investigate rotations?

Explain 1 Rotating Figures Using a Ruler and Protractor

A **rotation** is a transformation around point P, the **center of rotation**, such that the following is true.

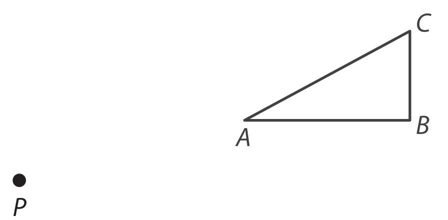
- Every point and its image are the same distance from P.
- All angles with vertex P formed by a point and its image have the same measure. This angle measure is the **angle of rotation**.

In the figure, the center of rotation is point P and the angle of rotation is 110°.

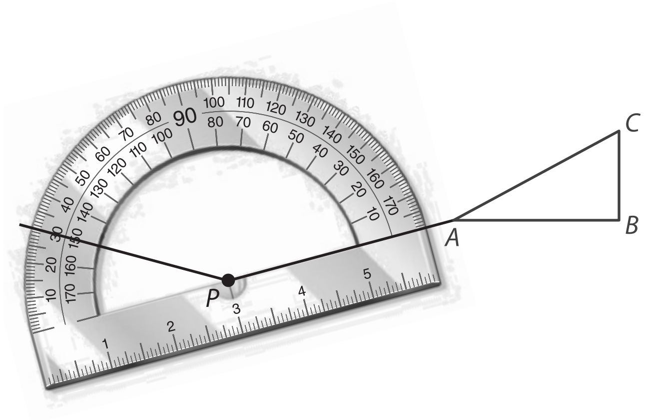


Example 1 **Draw the image of the triangle after the given rotation.**

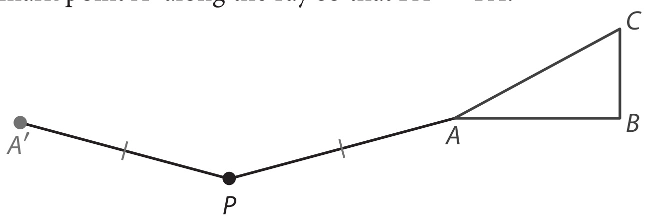
(A) Counterclockwise rotation of 150° around point P



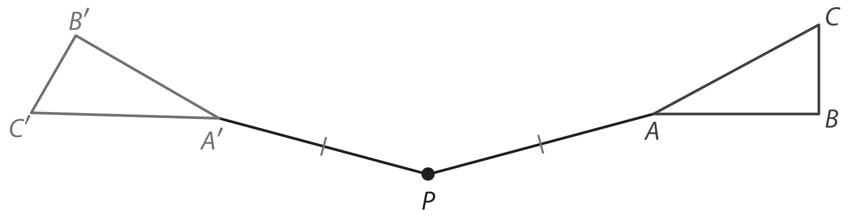
Step 1 Draw $\overline{PA}$. Then use a protractor to draw a ray that forms a 150° angle with $\overline{PA}$.



Step 2 Use a ruler to mark point A' along the ray so that $PA' = PA$.



Step 3 Repeat Steps 1 and 2 for points B and C to locate points B' and C'. Connect points A', B', and C' to draw $\triangle A'B'C'$.

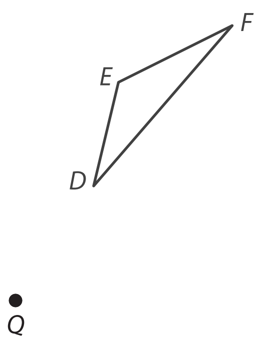


(B) Clockwise rotation of 75° around point Q

Step 1 Draw $\overline{QD}$. Use a protractor to draw a ray forming a clockwise 75° angle with $\overline{QD}$.

Step 2 Use a ruler to mark point D' along the ray so that $QD' = QD$.

Step 3 Repeat Steps1 and 2 for points E and F to locate points E' and F'. Connect points D', E', and F' to draw $\triangle D'E'F'$.



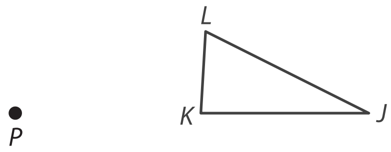
Reflect

3. How could you use tracing paper to draw the image of $\triangle ABC$ in Part A?

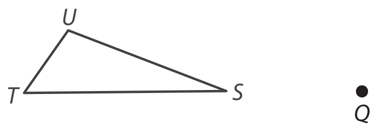
Your Turn

Draw the image of the triangle after the given rotation.

4. Counterclockwise rotation of 40° around point P

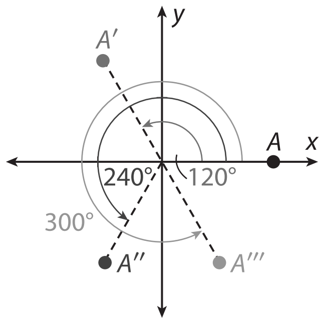


5. Clockwise rotation of 125° around point Q



Explain 2 Drawing Rotations on a Coordinate Plane

You can rotate a figure by more than 180°. The diagram shows counterclockwise rotations of 120°, 240°, and 300°. Note that a rotation of 360° brings a figure back to its starting location.



When no direction is specified, you can assume that a rotation is counterclockwise. Also, a counterclockwise rotation of $x°$ is the same as a clockwise rotation of $(360 - x)°$.

The table summarizes rules for rotations on a coordinate plane.

Rules for Rotations Around the Origin on a Coordinate Plane	
90° rotation counterclockwise	$(x, y) \rightarrow (-y, x)$
180° rotation	$(x, y) \rightarrow (-x, -y)$
270° rotation counterclockwise	$(x, y) \rightarrow (y, -x)$
360° rotation	$(x, y) \rightarrow (x, y)$

Example 2 **Draw the image of the figure under the given rotation.**

(A) Quadrilateral $ABCD$; 270°

The rotation image of (x, y) is $(y, -x)$.

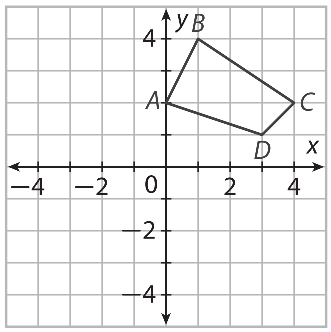
Find the coordinates of the vertices of the image.

$A(0, 2) \rightarrow A'(2, 0)$

$B(1, 4) \rightarrow B'(4, -1)$

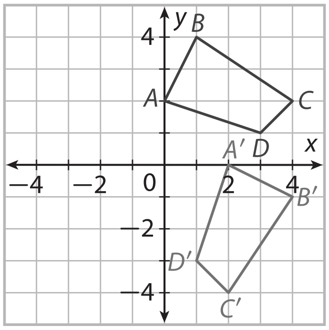
$C(4, 2) \rightarrow C'(2, -4)$

$D(3, 1) \rightarrow D'(1, -3)$



Predict the quadrant in which the image will lie. Since quadrilateral $ABCD$ lies in Quadrant I and the quadrilateral is rotated counterclockwise by 270°, the image will lie in Quadrant IV.

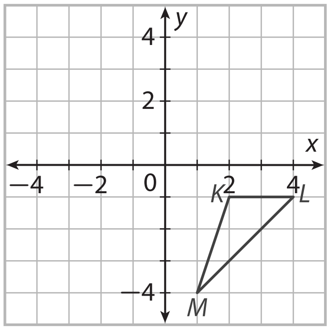
Plot A', B', C', and D' to graph the image.



Ⓑ $\triangle KLM$; 180°

The rotation image of (x, y) is $\left(\square, \square\right)$.

Find the coordinates of the vertices of the image.



$K(2, -1) \rightarrow K'\left(\square, \square\right)$

$L(4, -1) \rightarrow L'\left(\square, \square\right)$

$M(1, -4) \rightarrow M'\left(\square, \square\right)$

Predict the quadrant in which the image will lie. Since $\triangle KLM$ lies in Quadrant _____ and the triangle is rotated by 180°, the image will lie in Quadrant _____.

Plot K', L', and M' to graph the image.

Reflect

6. Discussion Suppose you rotate quadrilateral $ABCD$ in Part A by 810°. In which quadrant will the image lie? Explain.

____

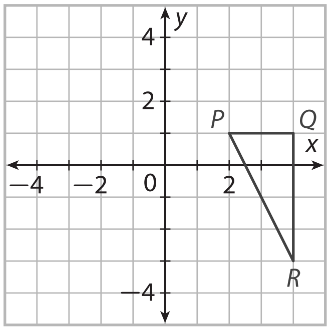
____

____

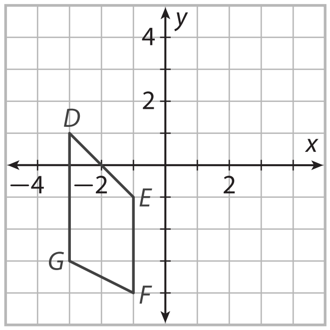
Your Turn

Draw the image of the figure under the given rotation.

7. $\triangle PQR$; 90°



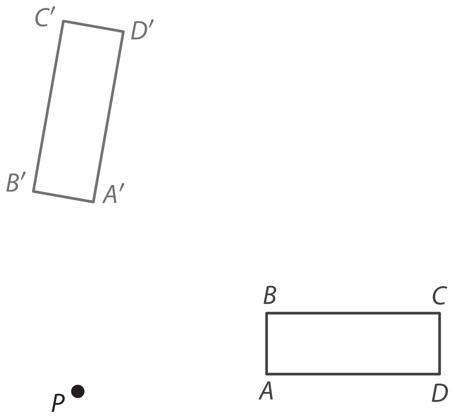
8. Quadrilateral $DEFG$; 270°



Explain 3 Specifying Rotation Angles

Example 3 **Find the angle of rotation and direction of rotation in the given figure. Point P is the center of rotation.**

(A)

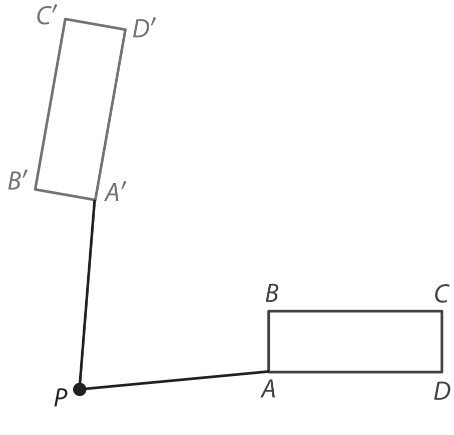


Draw segments from the center of rotation to a vertex and to the image of the vertex.

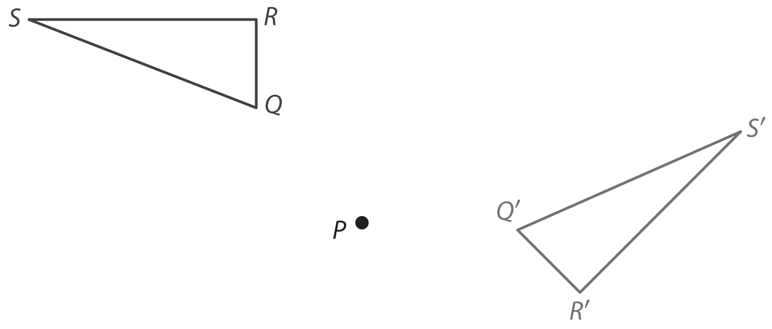
Measure the angle formed by the segments. The angle measure is 80°.

Compare the locations of the preimage and image to find the direction of the rotation.

The rotation is 80° counterclockwise.



Ⓑ



Draw segments from the center of rotation to a vertex and to the image of the vertex.

Measure the angle formed by the segments.

The angle measure is ☐°.

The rotation is ☐° (clockwise/counterclockwise).

Reflect

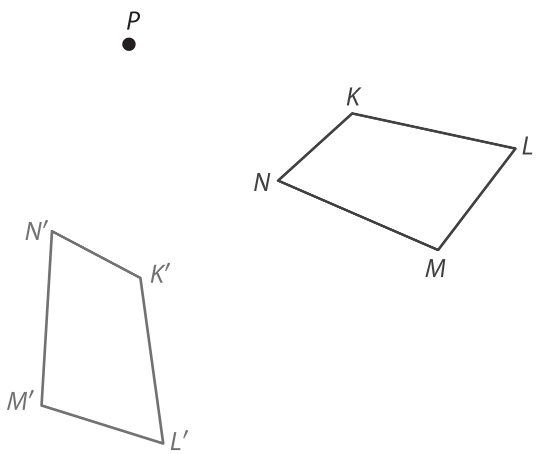
9. Discussion Does it matter which points you choose when you draw segments from the center of rotation to points of the preimage and image? Explain.

10. In Part A, is a different angle of rotation and direction possible? Explain.

Your Turn

Find the angle of rotation and direction of rotation in the given figure. Point *P* is the center of rotation.

11.



Elaborate

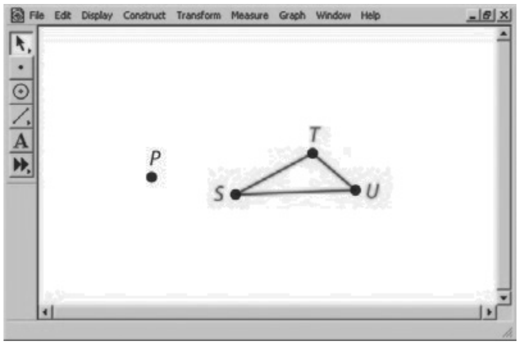
12. If you are given a figure, a center of rotation, and an angle of rotation, what steps can you use to draw the image of the figure under the rotation?

13. Suppose you are given $\triangle DEF$, $\triangle D'E'F'$, and point P. What are two different ways to prove that a rotation around point P cannot be used to map $\triangle DEF$ to $\triangle D'E'F'$?

14. **Essential Question Check-In** How do you draw the image of a figure under a counterclockwise rotation of 90° around the origin?

Evaluate: Homework and Practice

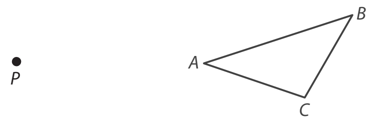
1. Alberto uses geometry software to draw $\triangle STU$ and point P, as shown. He marks P as a center and uses the software to rotate $\triangle STU$ 115° around point P. He labels the image of $\triangle STU$ as $\triangle S'T'U'$.



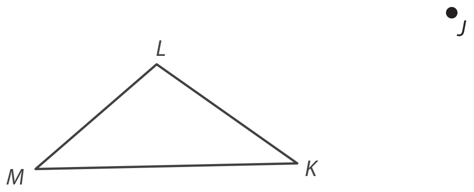
Which three angles must have the same measure? What is the measure of these angles?

Draw the image of the triangle after the given rotation.

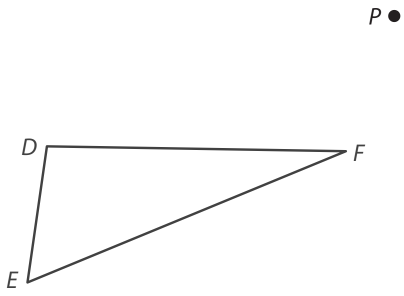
2. Counterclockwise rotation of 30° around point P



3. Clockwise rotation of 55° around point J

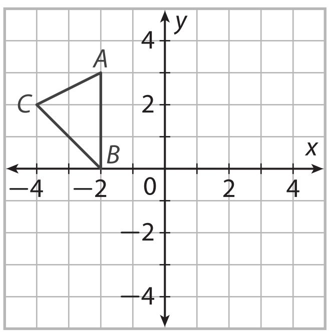


4. Counterclockwise rotation of 90° around point P

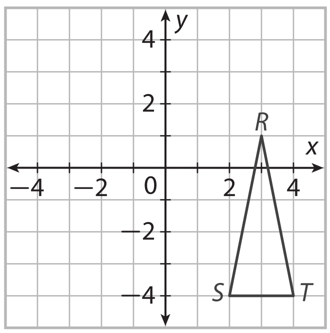


Draw the image of the figure under the given rotation.

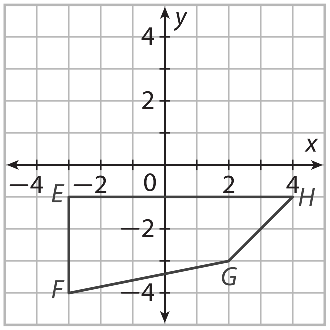
5. $\triangle ABC$; 270°



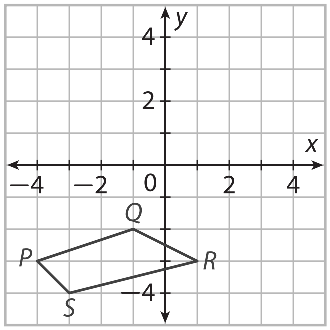
6. $\triangle RST$; 90°



7. Quadrilateral *EFGH*; 180°

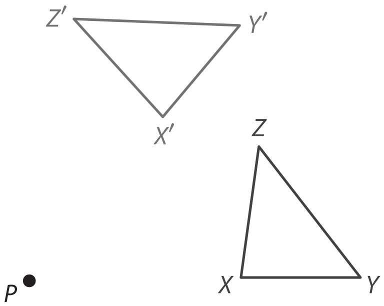


8. Quadrilateral *PQRS*; 270°

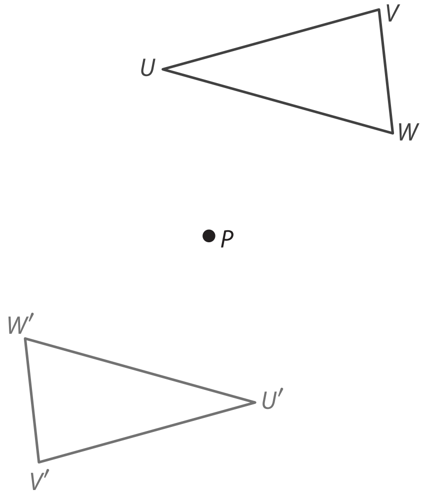


Find the angle of rotation and direction of rotation in the given figure. Point *P* is the center of rotation.

9.

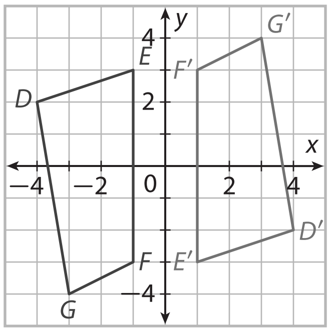


10.

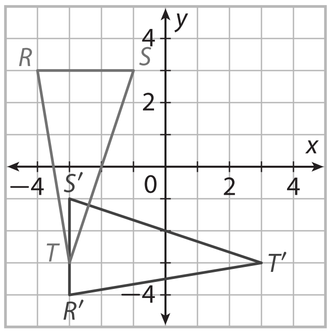


Write an algebraic rule for the rotation shown. Then describe the transformation in words.

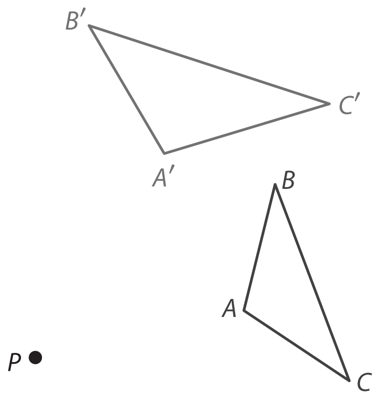
11.



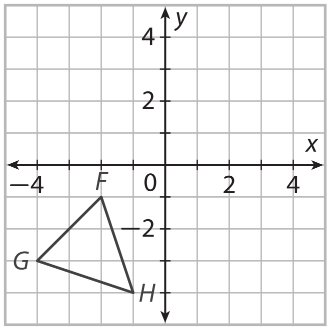
12.



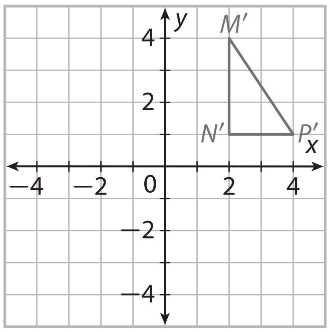
13. Vanessa used geometry software to apply a transformation to $\triangle ABC$, as shown. According to the software, $m\angle APA' = m\angle BPB' = m\angle CPC'$. Vanessa said this means the transformation must be a rotation. Do you agree? Explain.



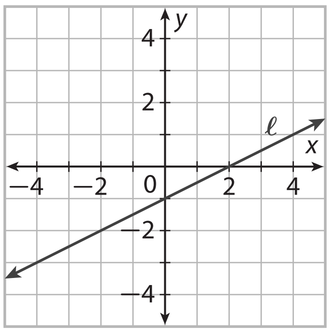
14. **Make a Prediction** In which quadrant will the image of $\triangle FGH$ lie after a counterclockwise rotation of 1980°? Explain how you made your prediction.



15. **Critical Thinking** The figure shows the image of $\triangle MNP$ after a counterclockwise rotation of 270°. Draw and label $\triangle MNP$.



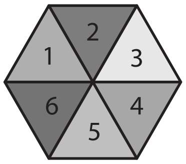
16. **Multi-Step** Write the equation of the image of line ℓ after a clockwise rotation of 90°. (*Hint*: To find the image of line ℓ, choose two or more points on the line and find the images of the points.)



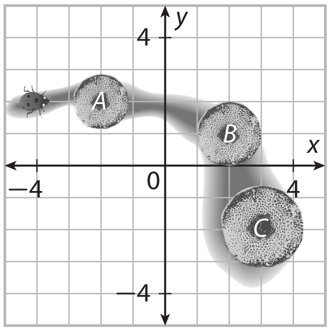
17. A Ferris wheel has 20 cars that are equally spaced around the circumference of the wheel. The wheel rotates so that the car at the bottom of the ride is replaced by the next car. By how many degrees does the wheel rotate?

18. The Skylon Tower, in Niagara Falls, Canada, has a revolving restaurant 775 feet above the falls. The restaurant makes a complete revolution once every hour. While a visitor was at the tower, the restaurant rotated through 135°. How long was the visitor at the tower?

19. Amani plans to use drawing software to make the design shown here. She starts by drawing Triangle 1. Explain how she can finish the design using rotations.



20. An animator is drawing a scene in which a ladybug moves around three mushrooms. The figure shows the starting position of the ladybug. The animator rotates the ladybug 180° around mushroom A, then 180° around mushroom B, and finally 180° around mushroom C. What are the final coordinates of the ladybug?



21. Determine whether each statement about the rotation $(x, y) \rightarrow (y, -x)$ is true or false. Select the correct answer for each lettered part.

a. Every point in Quadrant I is mapped to a point in Quadrant II. ◯ True ◯ False

b. Points on the x-axis are mapped to points on the y-axis. ◯ True ◯ False

c. The origin is a fixed point under the rotation. ◯ True ◯ False

d. The rotation has the same effect as a 90° clockwise rotation. ◯ True ◯ False

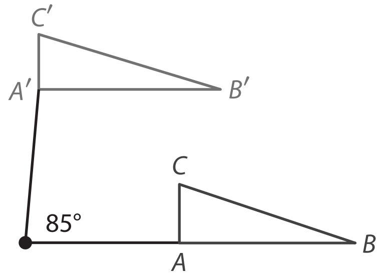
e. The angle of rotation is 180°. ◯ True ◯ False

f. A point on the line $y = x$ is mapped to another point on the line $y = x$. ◯ True ◯ False

H.O.T. Focus on Higher Order Thinking

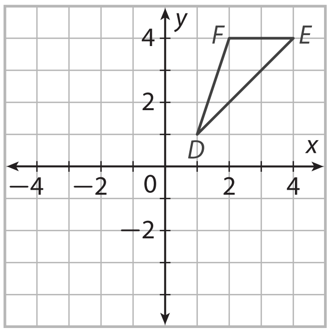
22. Communicate Mathematical Ideas Suppose you are given a figure and a center of rotation *P*. Describe two different ways you can use a ruler and protractor to draw the image of the figure after a 210° counterclockwise rotation around *P*.

23. Explain the Error Kevin drew the image of △*ABC* after a rotation of 85° around point *P*. Explain how you can tell from the figure that he made an error. Describe the error.



24. Critique Reasoning Isabella said that all points turn around the center of rotation by
the same angle, so all points move the same distance under a rotation. Do you agree with Isabella's statement? Explain.

25. Look for a Pattern Isaiah uses software to draw △*DEF* as shown. Each time he presses the left arrow key, the software rotates the figure on the screen 90° counterclockwise. Explain how Isaiah can determine which quadrant the triangle will lie in if he presses the left arrow key *n* times.



Lesson Performance Task

A tourist in London looks up at the clock in Big Ben tower and finds that it is exactly 8:00. When she looks up at the clock later, it is exactly 8:10.

a. Through what angle of rotation did the minute hand turn? Through what angle of rotation did the hour hand turn?

b. Make a table that shows different amounts of time, from 5 minutes to 60 minutes, in 5-minute increments. For each number of minutes, provide the angle of rotation for the minute hand of a clock and the angle of rotation for the hour hand of a clock.

Name ______________________ Class ____________ Date ________

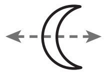
17.4 Investigating Symmetry

Essential Question: How do you determine whether a figure has line symmetry or rotational symmetry?

Resource Locker

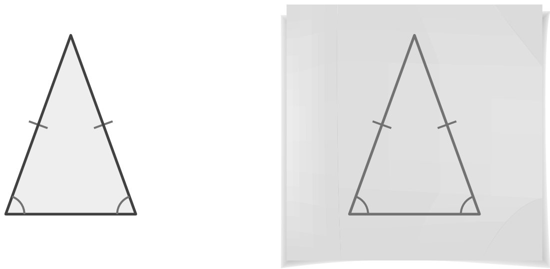
Explore 1 Identifying Line Symmetry

A figure has **symmetry** if a rigid motion exists that maps the figure onto itself. A figure has **line symmetry** (or *reflectional symmetry*) if a reflection maps the figure onto itself. Each of these lines of reflection is called a **line of symmetry**.

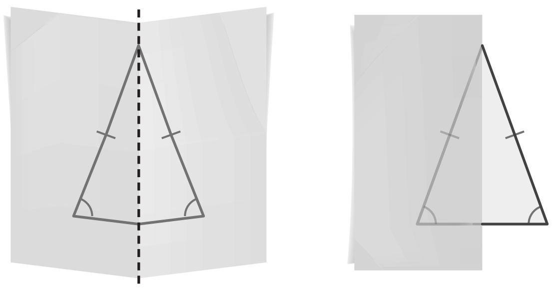


You can use paper folding to determine whether a figure has line symmetry.

(A) Trace the figure on a piece of tracing paper.

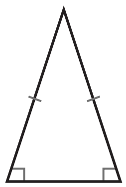


(B) If the figure can be folded along a straight line so that one half of the figure exactly matches the other half, the figure has line symmetry. The crease is the line of symmetry. Place your shape against the original figure to check that each crease is a line of symmetry.

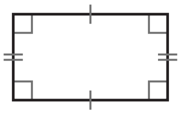
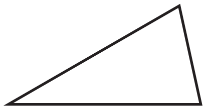
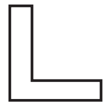


(C) Sketch any lines of symmetry on the figure.

The figure has ________ line of symmetry.



(D) Draw the lines of symmetry, if any, on each figure and tell the total number of lines of symmetry each figure has.

Figure			
How many lines of symmetry?			

Reflect

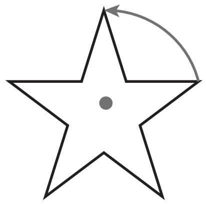
1. What do you have to know about any segments and angles in a figure to decide whether the figure has line symmetry?

2. What figure has an infinite number of lines of symmetry? ______

3. **Discussion** A figure undergoes a rigid motion, such as a rotation. If the figure has line symmetry, does the image of the figure have line symmetry as well? Give an example.

Explore 2 Identifying Rotational Symmetry

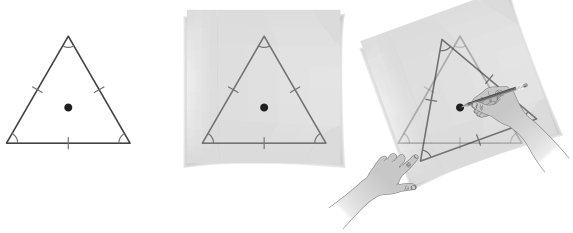
A figure has **rotational symmetry** if a rotation maps the figure onto itself. The **angle of rotational symmetry**, which is greater than 0° but less than or equal to 180°, is the smallest angle of rotation that maps a figure onto itself.



Angle of rotational symmetry: 72°

An angle of rotational symmetry is a fractional part of 360°. Notice that every time the 5-pointed star rotates $\frac{360°}{5} = 72°$, the star coincides with itself. The angles of rotation for the star are 72°, 144°, 216°, and 288°. If a copy of the figure rotates to exactly match the original, the figure has rotational symmetry.

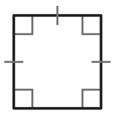
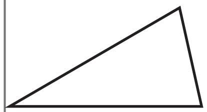
(A) Trace the figure onto tracing paper. Hold the center of the traced figure against the original figure with your pencil. Rotate the traced figure counterclockwise until it coincides again with the original figure beneath.



By how many degrees did you rotate the figure? ______

What are all the angles of rotation? ______

(B) Determine whether each figure has rotational symmetry. If so, identify all the angles of rotation less than 360°.

Figure			
Angles of rotation less than 360°			

Reflect

4. What figure is mapped onto itself by a rotation of any angle? ______________________

5. **Discussion** A figure is formed by line *l* and line *m*, which intersect at an angle of 60°. Does the figure have an angle of rotational symmetry of 60°? If not, what is the angle of rotational symmetry?

____

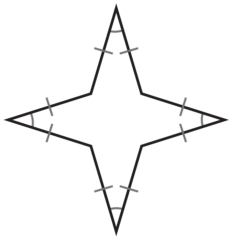
____

Explain 1 Describing Symmetries

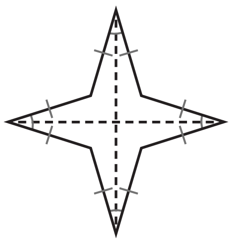
A figure may have line symmetry, rotational symmetry, both types of symmetry, or no symmetry.

Example 1 **Describe the symmetry of each figure. Draw the lines of symmetry, name the angles of rotation, or both if the figure has both.**

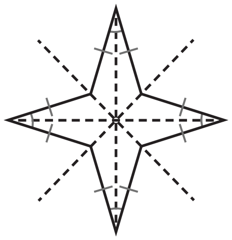
(A)



Step 1 Begin by finding the line symmetry of the figure. Look for matching halves of the figure. For example, you could fold the left half over the right half, and fold the top half over the bottom half. Draw one line of symmetry for each fold. Notice that the lines intersect at the center of the figure.

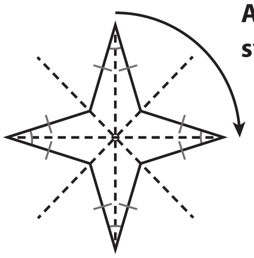


Step 2 Now look for other lines of symmetry. The two diagonals also describe matching halves. The figure has a total of 4 lines of symmetry.



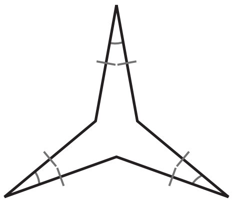
Step 3 Next, look for rotational symmetry. Think of the figure rotated about its center until it matches its original position. The angle of rotational symmetry of this figure is $\frac{1}{4}$ of 360°, or 90°.

The other angles of rotation for the figure are the multiples of 90° that are less than 360°. So the angles of rotation are 90°, 180°, and 270°.



Number of lines of symmetry: 4 Angles of rotation: 90°, 180°, 270° __________

B



Step 1 Look for lines of symmetry. One line divides the figure into left and right halves. Draw this line on the figure. Then draw similar lines that begin at the other vertices of the figure.

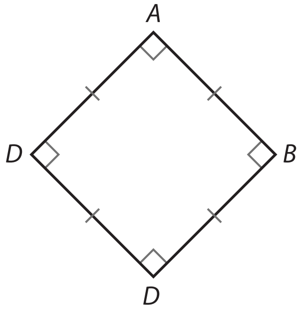
Step 2 Now look for rotational symmetry. Think of the figure rotating about its center until it matches the original figure. It rotates around the circle by a fraction of __________. Multiply by 360° to find the angle of rotation, which is __________. Find multiples of this angle to find other angles of rotation.

Number of lines of symmetry: __________ Angles of rotation: __________

Your Turn

Describe the type of symmetry for each figure. Draw the lines of symmetry, name the angles of rotation, or both if the figure has both.

6. Figure *ABCD*

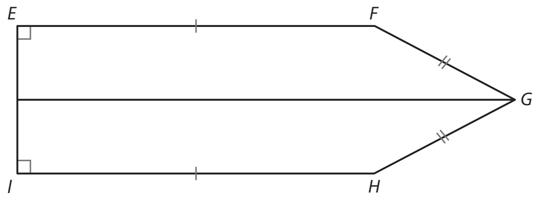


Types of symmetry: ____________________

Number of lines of symmetry: ____________

Angles of rotation: ____________________

7. Figure *EFGHI*

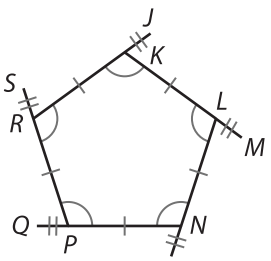


Types of symmetry: ____________________

Number of lines of symmetry: ____________

Angles of rotation: ____________________

8. Figure *KLNPR*

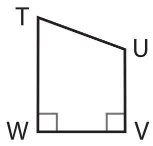


Types of symmetry: ____________________

Number of lines of symmetry: ____________

Angles of rotation: ____________________

9. Figure *TUVW*



Types of symmetry: ____________________

Number of lines of symmetry: ____________

Angles of rotation: ____________________

Elaborate

10. How are the two types of symmetry alike? How are they different?

____

____

____

11. **Essential Question Check-In** How do you determine whether a figure has line symmetry or rotational symmetry?

____

____

____

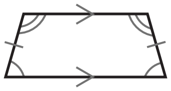
____

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

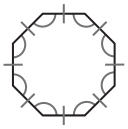
Draw all the lines of symmetry for the figure, and give the number of lines of symmetry. If the figure has no line symmetry, write zero.

1.



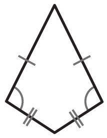
Lines of symmetry: ________

2.



Lines of symmetry: ________

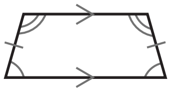
3.



Lines of symmetry: ________

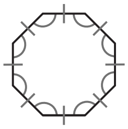
For the figures that have rotational symmetry, list the angles of rotation less than 360°. For figures without rotational symmetry, write "no rotational symmetry."

4.



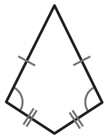
Angles of rotation: ________________

5.



Angles of rotation: ________________

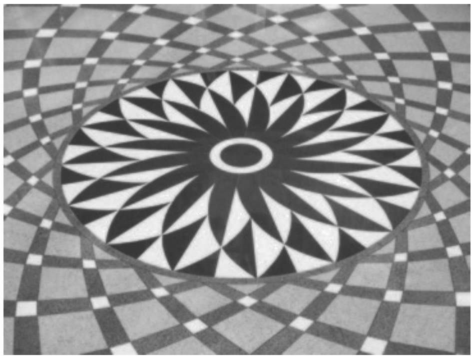
6.



Angles of rotation: ________________

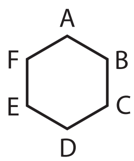
In the tile design shown, identify whether the pattern has line symmetry, rotational symmetry, both line and rotational symmetry, or no symmetry.

7.



8.

For figure *ABCDEF* shown here, identify the image after each transformation described. For example, a reflection across $\overline{AD}$ has an image of figure *AFEDCB*. In the figure, all the sides are the same length and all the angles are the same measure.



9. Reflection across $\overline{CF}$

Figure ________

10. rotation of 240° clockwise, or 120° counterclockwise

Figure ________

11. reflection across the line that connects the midpoint of $\overline{BC}$ and the midpoint of $\overline{EF}$

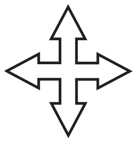
Figure ________

In the space provided, sketch an example of a figure with the given characteristics.

12. no line symmetry; angle of rotational symmetry: 180°

13. one line of symmetry; no rotational symmetry

14. Describe the line and rotational symmetry in this figure.

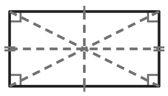


H.O.T. Focus on Higher Order Thinking

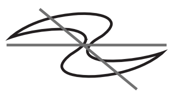
15. **Communicate Mathematical Ideas** How is a rectangle similar to an ellipse? Use concepts of symmetry in your answer.

16. **Explain the Error** A student was asked to draw all of the lines of symmetry on each figure shown. Identify the student's work as correct or incorrect. If incorrect, explain why.

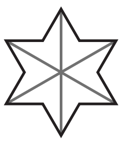
a.



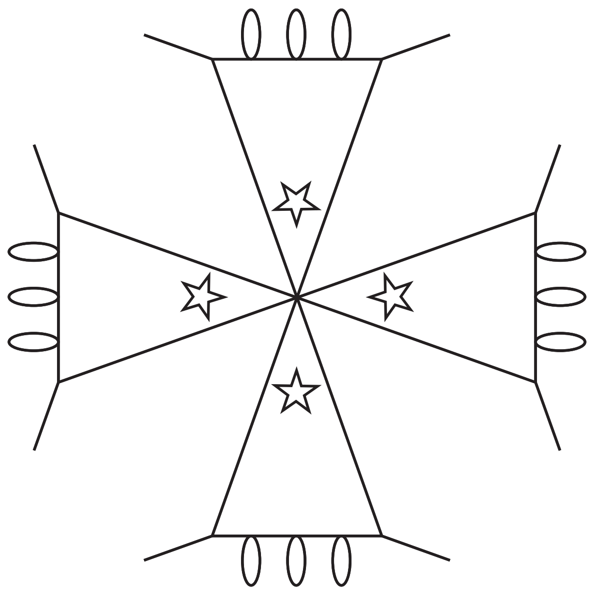
b.



c.



Lesson Performance Task



Use symmetry to design a work of art. Begin by drawing one simple geometric figure, such as a triangle, square, or rectangle, on a piece of construction paper. Then add other lines or two-dimensional shapes to the figure. Next, make identical copies of the figure, and then arrange them in a symmetric pattern.

Evaluate the symmetry of the work of art you created. Rotate it to identify an angle of rotational symmetry. Compare the line symmetry of the original figure with the line symmetry of the finished work.