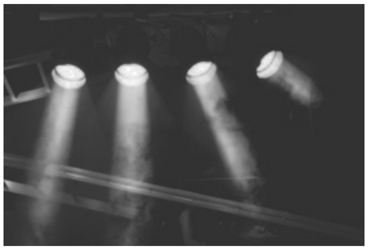
Name ______________________ Class ____________ Date ________

16.1 Segment Length and Midpoints

Essential Question: How do you draw a segment and measure its length?

Resource Locker

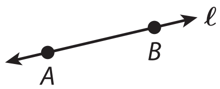
Explore Exploring Basic Geometric Terms



In geometry, some of the names of figures and other terms will already be familiar from everyday life. For example, a *ray* like a beam of light from a spotlight is both a familiar word and a geometric figure with a mathematical definition.

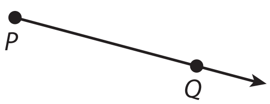
The most basic figures in geometry are *undefined terms*, which cannot be defined using other figures. The terms *point*, *line*, and *plane* are undefined terms. Although they do not have formal definitions, they can be described as shown in the table.

Undefined Terms

Term	Geometric Figure	Ways to Name the Figure
A **point** is a specific location. It has no dimension and is represented by a dot.		point P
A **line** is a connected straight path. It has no thickness and it continues forever in both directions.		line ℓ, line AB, line BA, $\overleftrightarrow{AB}$, or $\overleftrightarrow{BA}$
A **plane** is a flat surface. It has no thickness and it extends forever in all directions.		plane $\mathcal{R}$ or plane XYZ

In geometry, the word *between* is another undefined term, but its meaning is understood from its use in everyday language. You can use undefined terms as building blocks to write definitions for defined terms, as shown in the table.

Defined Terms

Term	Geometric Figure	Ways to Name the Figure
A **line segment** (or *segment*) is a portion of a line consisting of two points (called **endpoints**) and all points between them.		segment CD, segment DC, $\overline{CD}$, or $\overline{DC}$
A **ray** is a portion of a line that starts at a point (the *endpoint*) and continues forever in one direction.		ray PQ or $\overrightarrow{PQ}$

You can use points to sketch lines, segments, rays, and planes.

(A) Draw two points J and K. Then draw a line through them. (Remember that a line shows arrows at both ends.)

(B) Draw two points J and K again. This time, draw the line segment with endpoints J and K.

(C) Draw a point K again and draw a ray from endpoint K. Plot a point J along the ray.

(D) Draw three points J, K, and M so that they are not all on the same line. Then draw the plane that contains the three points. (You might also put a script letter such as $\mathcal{B}$ on your plane.)

(E) Give a name for each of the figures you drew. Then use a circle to choose whether the type of figure is an undefined term or a defined term.

Point	________________	undefined term/defined term
Line	________________	undefined term/defined term
Segment	________________	undefined term/defined term
Ray	________________	undefined term/defined term
Plane	________________	undefined term/defined term

Reflect

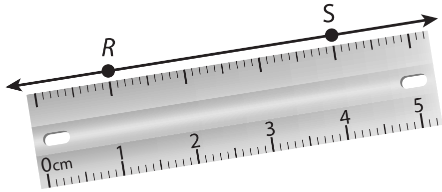
1. In Step C, would $\overrightarrow{JK}$ be the same ray as $\overrightarrow{KJ}$? Why or why not?

2. In Step D, when you name a plane using 3 letters, does the order of the letters matter?

3. **Discussion** If $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ are different names for the same line, what must be true about points P, Q, R, and S?

Explain 1 Constructing a Copy of a Line Segment

The distance along a line is undefined until a unit distance, such as 1 inch or 1 centimeter, is chosen. You can use a ruler to find the distance between two points on a line. The distance is the absolute value of the difference of the numbers on the ruler that correspond to the two points. This distance is the length of the segment determined by the points.



In the figure, the length of $\overline{RS}$, written RS (or SR), is the distance between R and S.

$$RS = |4 - 1| = |3| = 3 \text{ cm} \qquad \text{or} \qquad SR = |1 - 4| = |-3| = 3 \text{ cm}$$

Points that lie in the same plane are **coplanar**. Lines that lie in the same plane but do not intersect are **parallel**. Points that lie on the same line are **collinear**. The *Segment Addition Postulate* is a statement about collinear points. A **postulate** is a statement that is accepted as true without proof. Like undefined terms, postulates are building blocks of geometry.

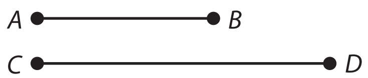
Postulate 1: Segment Addition Postulate

Let A, B, and C be collinear points. If B is between A and C, then $AB + BC = AC$.

A *construction* is a geometric drawing that produces an accurate representation without using numbers or measures. One type of construction uses only a compass and straightedge. You can construct a line segment whose length is equal to that of a given segment using these tools along with the Segment Addition Postulate.

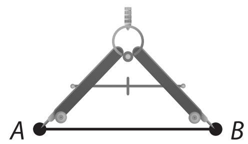
Example 1 **Use a compass and straightedge to construct a segment whose length is $AB + CD$.**

(A)



Step 1 Use the straightedge to draw a long line segment. Label an endpoint X. (See the art drawn in Step 4.)

Step 2 To copy segment AB, open the compass to the distance AB.



Step 3 Place the compass point on X, and draw an arc. Label the point Y where the arc and the segment intersect.

Step 4 To copy segment CD, open the compass to the distance CD. Place the compass point on Y, and draw an arc. Label the point Z where this second arc and the segment intersect.

$\overline{XZ}$ is the required segment.

(B)

Step 1 Use the straightedge to draw a long line segment. Label an endpoint *X*.

Step 2 To copy segment *AB*, open the compass to the distance *AB*.

Step 3 Place the compass point on *X*, and draw an arc. Label the point *Y* where the arc and the segment intersect.

Step 4 To copy segment *CD*, open the compass to the distance *CD*. Place the compass point on Y, and draw an arc. Label the point *Z* where this second arc and the segment intersect.

Reflect

4. **Discussion** Look at the line and ruler above Example 1. Why does it not matter whether you find the distance from *R* to *S* or the distance from *S* to *R*?

5. In Part B, how can you check that the length of $\overline{YZ}$ is the same as the length of $\overline{CD}$?

Your Turn

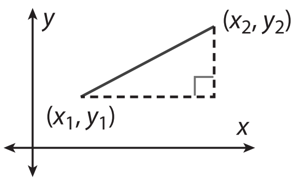
6. Use a ruler to draw a segment *PQ* that is 2 inches long. Then use your compass and straightedge to construct a segment *MN* with the same length as $\overline{PQ}$.

Explain 2 Using the Distance Formula on the Coordinate Plane

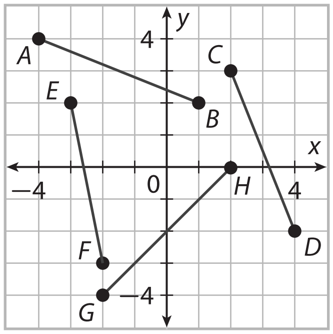
The Pythagorean Theorem states that $a^2 + b^2 = c^2$, where *a* and *b* are the lengths of the legs of a right triangle and *c* is the length of the hypotenuse. You can use the Distance Formula to apply the Pythagorean Theorem to find the distance between points on the coordinate plane.

The Distance Formula

The distance between two points (x_1, y_1) and (x_2, y_2) on the coordinate plane is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



Example 2 **Determine whether the given segments have the same length. Justify your answer.**



A $\overline{AB}$ and $\overline{CD}$

Write the coordinates of the endpoints. $A(-4, 4)$, $B(1, 2)$, $C(2, 3)$, $D(4, -2)$

Find the length of $\overline{AB}$. $AB = \sqrt{(1 - (-4))^2 + (2 - 4)^2}$

Simplify the expression. $= \sqrt{5^2 + (-2)^2} = \sqrt{29}$

Find the length of $\overline{CD}$. $CD = \sqrt{(4 - 2)^2 + (-2 - 3)^2}$

Simplify the expression. $= \sqrt{2^2 + (-5)^2} = \sqrt{29}$

So, $AB = CD = \sqrt{29}$. Therefore, $\overline{AB}$ and $\overline{CD}$ have the same length.

B $\overline{EF}$ and $\overline{GH}$

Write the coordinates of the endpoints. $E(-3, 2)$, $F(\square, \square)$, $G(-2, -4)$, $H(\square, \square)$

Find the length of $\overline{EF}$. $EF = \sqrt{(\square - (-3))^2 + (\square - 2)^2}$

Simplify the expression. $= \sqrt{(\square)^2 + (\square)^2} = \sqrt{\square}$

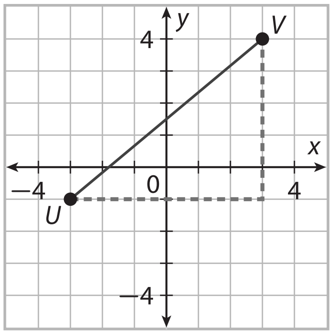
Find the length of $\overline{GH}$. $GH = \sqrt{(\square - (-2))^2 + (\square - (-4))^2}$

Simplify the expression. $= \sqrt{(\square)^2 + (\square)^2} = \sqrt{\square}$

So, ________. Therefore, ________________________________

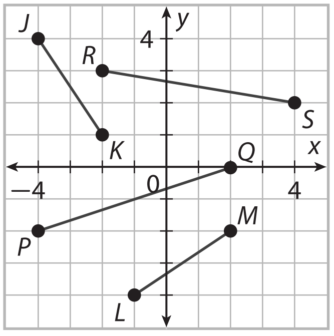
Reflect

7. Consider how the Distance Formula is related to the Pythagorean Theorem. To use the Distance Formula to find the distance from $U(-3, -1)$ to $V(3, 4)$, you write $UV = \sqrt{\left(3 - (-3)\right)^2 + \left(4 - (-1)\right)^2}$. Explain how $\left(3 - (-3)\right)$ in the Distance Formula is related to a in the Pythagorean Theorem and how $\left(4 - (-1)\right)$ in the Distance Formula is related to b in the Pythagorean Theorem.



Your Turn

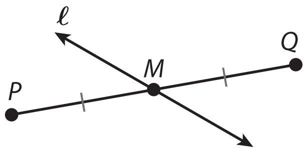
8. Determine whether $\overline{JK}$ and $\overline{LM}$ have the same length. Justify your answer.



Explain 3 Finding a Midpoint

The **midpoint** of a line segment is the point that divides the segment into two segments that have the same length. A line, ray, or other figure that passes through the midpoint of a segment is a **segment bisector**.

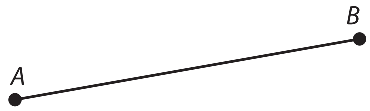
In the figure, the tick marks show that $PM = MQ$. Therefore, M is the midpoint of $\overline{PQ}$ and line ℓ bisects $\overline{PQ}$.



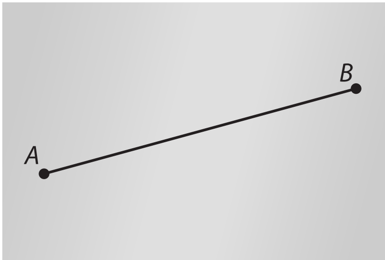
You can use paper folding as a method to construct a bisector of a given segment and locate the midpoint of the segment.

Example 3 **Use paper folding to construct a bisector of each segment.**

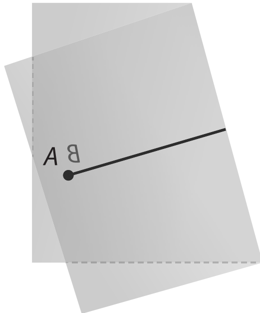
(A)



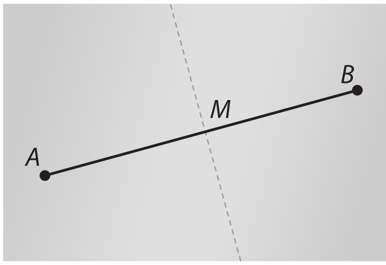
Step 1 Use a compass and straightedge to copy $\overline{AB}$ on a piece of paper.



Step 2 Fold the paper so that point B is on top of point A.

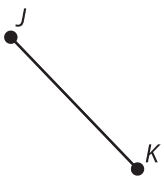


Step 3 Open the paper. Label the point where the crease intersects the segment as point M.



Point M is the midpoint of $\overline{AB}$ and the crease is a bisector of $\overline{AB}$.

(B) **Step 1** Use a compass and straightedge to copy $\overline{JK}$ on a piece of paper.



Step 2 Fold the paper so that point K is on top of point ____________.

Step 3 Open the paper. Label the point where the crease intersects the segment as point N.

Point N is the ____________ of $\overline{JK}$ and the crease is a ____________ of $\overline{JK}$.

Step 4 Make a sketch of your paper folding construction or attach your folded piece of paper.

Reflect

9. Explain how you could use paper folding to divide a line segment into four segments of equal length.

Your Turn

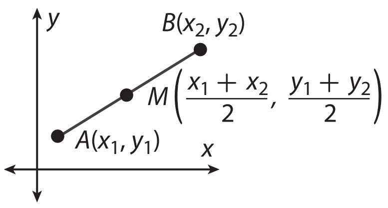
10. Explain how to use a ruler to check your construction in Part B.

Explain 4 Finding Midpoints on the Coordinate Plane

You can use the *Midpoint Formula* to find the midpoint of a segment on the coordinate plane.

The Midpoint Formula

The midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $M\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.



Example 4 **Show that each statement is true.**

(A) If $\overline{PQ}$ has endpoints $P(-4, 1)$ and $Q(2, -3)$, then the midpoint M of $\overline{PQ}$ lies in Quadrant III.

Use the Midpoint Formula to find the midpoint of $\overline{PQ}$. Substitute the coordinates, then simplify.

$$M\left(\frac{-4 + 2}{2}, \frac{1 + (-3)}{2}\right) = M(-1, -1)$$

So M lies in Quadrant III, since the x- and y-coordinates are both negative.

(B) If $\overline{RS}$ has endpoints $R(3, 5)$ and $S(-3, -1)$, then the midpoint M of $\overline{RS}$ lies on the y-axis.

Use the Midpoint Formula to find the midpoint of $\overline{RS}$. Substitute the coordinates, then simplify.

$$M\left(\frac{3 + \boxed{}}{2}, \frac{5 + \boxed{}}{2}\right) = M\left(\boxed{}, \boxed{}\right)$$

So M lies on the y-axis, since ______________.

Your Turn

Show that each statement is true.

11. If $\overline{AB}$ has endpoints $A(6, -3)$ and $B(-6, 3)$, then the midpoint M of $\overline{AB}$ is the origin.

12. If $\overline{JK}$ has endpoints $J(7, 0)$ and $K(-5, -4)$, then the midpoint M of $\overline{JK}$ lies in Quadrant IV.

Elaborate

13. Explain why the Distance Formula is not needed to find the distance between two points that lie on a horizontal or vertical line.

14. When you use the Distance Formula, does the order in which you subtract the x- and y-coordinates matter? Explain.

15. When you use the Midpoint Formula, can you take either point as (x_1, y_1) or (x_2, y_2)? Why or why not?

16. **Essential Question Check-In** What is the difference between finding the length of a segment that is drawn on a sheet of blank paper and a segment that is drawn on a coordinate plane?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

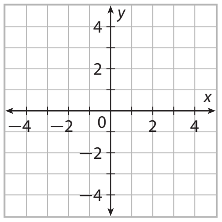
Write the term that is suggested by each figure or description. Then state whether the term is an undefined term or a defined term.

1.

2.

3.

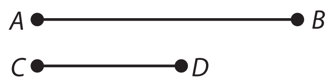
4.



Use a compass and straightedge to construct a segment whose length is $AB + CD$.

5.

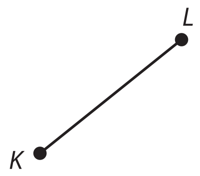
6.



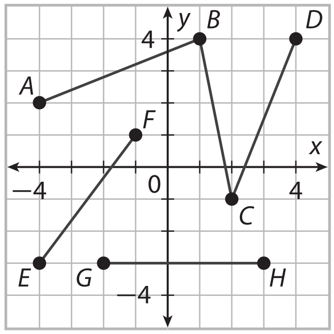
Copy each segment onto a sheet of paper. Then use paper folding to construct a bisector of the segment.

7.

8.



Determine whether the given segments have the same length. Justify your answer.



9. $\overline{AB}$ and $\overline{BC}$

10. $\overline{EF}$ and $\overline{GH}$

11. $\overline{AB}$ and $\overline{CD}$

12. $\overline{BC}$ and $\overline{EF}$

Show that each statement is true.

13. If $\overline{DE}$ has endpoints $D(-1, 6)$ and $E(3, -2)$, then the midpoint M of $\overline{DE}$ lies in Quadrant I.

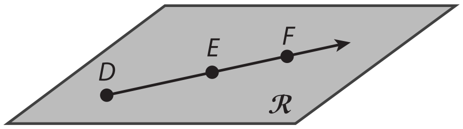
14. If $\overline{ST}$ has endpoints $S(-6, -1)$ and $T(0, 1)$, then the midpoint M of $\overline{ST}$ lies in on the x-axis.

Show that each statement is true.

15. If $\overline{JK}$ has endpoints $J(-2, 3)$ and $K(6, 5)$, and $\overline{LN}$ has endpoints $L(0, 7)$ and $N(4, 1)$, then $\overline{JK}$ and $\overline{LN}$ have the same midpoint.

16. If $\overline{GH}$ has endpoints $G(-8, 1)$ and $H(4, 5)$, then the midpoint M of $\overline{GH}$ lies on the line $y = -x + 1$.

Use the figure for Exercises 17 and 18.



17. Name two different rays in the figure.

18. Name three different segments in the figure.

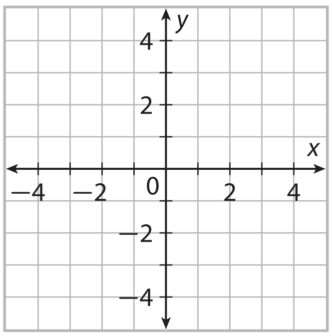
Sketch each figure.

19. two rays that form a straight line and that intersect at point P

20. two line segments that both have a midpoint at point M

21. Draw and label a line segment, $\overline{JK}$, that is 3 inches long. Use a ruler to draw and label the midpoint M of the segment.

22. Draw the segment PQ with endpoints $P(-2, -1)$ and $Q(2, 4)$ on the coordinate plane. Then find the length and midpoint of $\overline{PQ}$.



23. **Multi-Step** The sign shows distances from a rest stop to the exits for different towns along a straight section of highway. The state department of transportation is planning to build a new exit to Freestone at the midpoint of the exits for Roseville and Edgewood. When the new exit is built, what will be the distance from the exit for Midtown to the exit for Freestone?

24. On a town map, each unit of the coordinate plane represents 1 mile. Three branches of a bank are located at $A(-3, 1)$, $B(2, 3)$, and $C(4, -1)$. A bank employee drives from Branch A to Branch B and then drives halfway to Branch C before getting stuck in traffic. What is the minimum total distance the employee may have driven before getting stuck in traffic? Round to the nearest tenth of a mile.

25. A city planner designs a park that is a quadrilateral with vertices at $J(-3, 1)$, $K(1, 3)$, $L(5, -1)$, and $M(-1, -3)$. There is an entrance to the park at the midpoint of each side of the park. A straight path connects each entrance to the entrance on the opposite side. Assuming each unit of the coordinate plane represents 10 meters, what is the total length of the paths to the nearest meter?

26. **Communicate Mathematical Ideas** A video game designer places an anthill at the origin of a coordinate plane. A red ant leaves the anthill and moves along a straight line to $(1, 1)$, while a black ant leaves the anthill and moves along a straight line to $(-1, -1)$. Next, the red ant moves to $(2, 2)$, while the black ant moves to $(-2, -2)$. Then the red ant moves to $(3, 3)$, while the black ant moves to $(-3, -3)$, and so on. Explain why the red ant and the black ant are always the same distance from the anthill.

27. Which of the following points are more than 5 units from the point $P(-2, -2)$? Select all that apply.

A. $A(1, 2)$

B. $B(3, -1)$

C. $C(2, -4)$

D. $D(-6, -6)$

E. $E(-5, 1)$

H.O.T. Focus on Higher Order Thinking

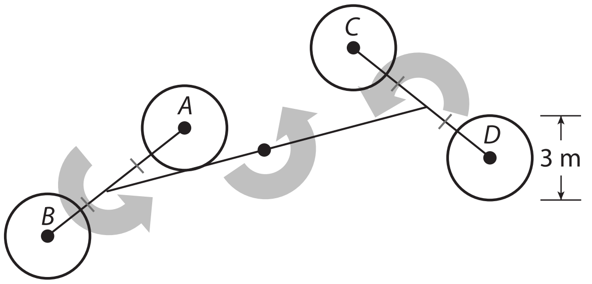
28. Analyze Relationships Use a compass and straightedge to construct a segment whose length is $AB - CD$. Use a ruler to check your construction.

29. Critical Thinking Point M is the midpoint of $\overline{AB}$. The coordinates of point A are $(-8, 3)$ and the coordinates of M are $(-2, 1)$. What are the coordinates of point B?

30. Make a Conjecture Use a compass and straightedge to copy $\overline{AB}$ so that one endpoint of the copy is at point X. Then repeat the process three more times, making three different copies of $\overline{AB}$ that have an endpoint at point X. Make a conjecture about the set of all possible copies of $\overline{AB}$ that have an endpoint at point X.

Lesson Performance Task

A carnival ride consists of four circular cars—A, B, C, and D—each of which spins about a point at its center. The center points of cars A and B are attached by a straight beam, as are the center points of cars C and D. The two beams are attached at their midpoints by a rotating arm. The figure shows how the beams and arm can rotate.



A plan for the ride uses a coordinate plane in which each unit represents one meter. In the plan, the center of car A is $(-6, -1)$, the center of car B is $(-2, -3)$, the center of car C is $(3, 4)$, and the center of car D is $(5, 0)$. Each car has a diameter of 3 meters.

The manager of the carnival wants to place a fence around the ride. Describe the shape and dimensions of a fence that will be appropriate to enclose the ride. Justify your answer.

Name ________________ Class ________ Date ______

16.2 Angle Measures and Angle Bisectors

Essential Question: How is measuring an angle similar to and different from measuring a line segment?

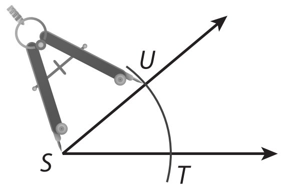
Resource Locker

Explore Constructing a Copy of an Angle

Start with a point X and use a compass and straightedge to construct a copy of $\angle S$.

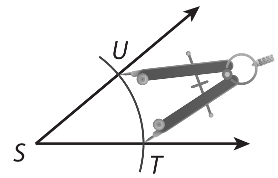
(A) Use a straightedge to draw a ray with endpoint X.

(B) Place the point of your compass on S and draw an arc that intersects both sides of the angle. Label the points of intersection T and U.



(C) Without adjusting the compass, place the point of the compass on X and draw an arc that intersects the ray. Label the intersection Y.

(D) Place the point of the compass on T and open it to the distance TU.



(E) Without adjusting the compass, place the point of the compass on Y and draw an arc. Label the intersection with the first arc Z.

(F) Use a straightedge to draw $\overrightarrow{XZ}$.
$\angle X$ is a copy of $\angle S$.

Reflect

1. If you could place the angle you drew on top of $\angle S$ so that $\overrightarrow{XY}$ coincides with $\overrightarrow{ST}$, what would be true about $\overrightarrow{XZ}$? Explain.

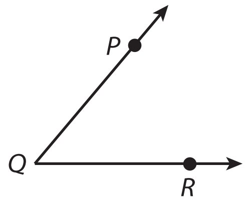
2. **Discussion** Is it possible to do the construction with a compass that is stuck open to a fixed distance? Why or why not?

Explain 1 Naming Angles and Parts of an Angle

An **angle** is a figure formed by two rays with the same endpoint.
The common endpoint is the **vertex** of the angle.
The rays are the **sides** of the angle.

Example 1 **Draw or name the given angle.**

Ⓐ $\angle PQR$

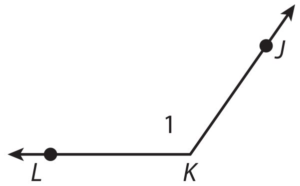


When an angle is named with three letters, the middle letter is the vertex. So, the vertex of angle $\angle PQR$ is point Q.

The sides of the angle are two rays with common endpoint Q. So, the sides of the angle are $\overrightarrow{QP}$ and $\overrightarrow{QR}$.

Draw and label the angle as shown.

Ⓑ



The vertex of the angle shown is point ___. A name for the angle is $\angle$___.

The vertex must be in the middle, so two more names for the angle are $\angle$___ ___ ___ and $\angle$___ ___ ___.

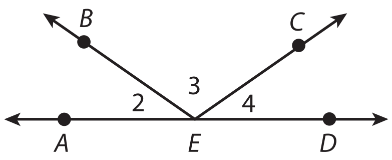
The angle is numbered, so another name is $\angle$___.

Reflect

3. Without seeing a figure, is it possible to give another name for $\angle MKG$? If so, what is it? If not, why not?

Your Turn

Use the figure for 4–5.



4. Name $\angle 2$ in as many different ways as possible.

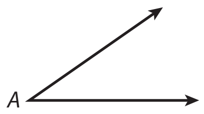
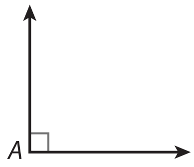
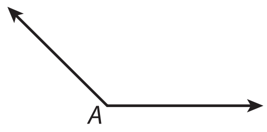
5. Use a compass and straightedge to copy $\angle BEC$.

Explain 2 Measuring Angles

The distance around a circular arc is undefined until a measurement unit is chosen. **Degrees** (°) are a common measurement unit for circular arcs. There are 360° in a circle, so an angle that measures 1° is $\frac{1}{360}$ of a circle. The measure of an angle is written m∠*A* or m∠*PQR*.

You can classify angles by their measures.

Classifying Angles

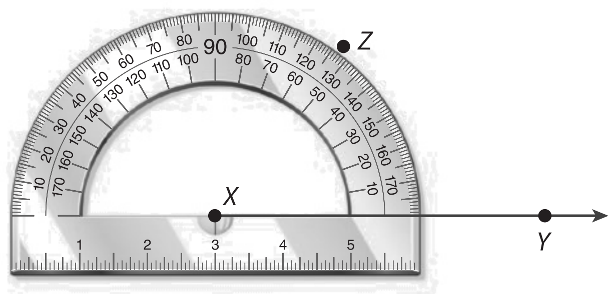
Acute Angle	Right Angle	Obtuse Angle	Straight Angle
			
$0° < m\angle A < 90°$	$m\angle A = 90°$	$90° < m\angle A < 180°$	$m\angle A = 180°$

Example 2 **Use a protractor to draw an angle with the given measure.**

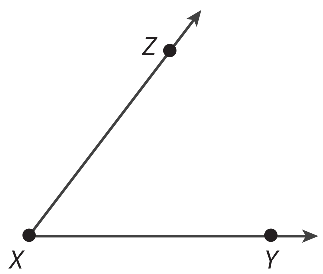
(A) 53°

Step 1 Use a straightedge to draw a ray, $\overrightarrow{XY}$.

Step 2 Place your protractor on point *X* as shown. Locate the point along the edge of the protractor that corresponds to 53°. Make a mark at this location and label it point *Z*.



Step 3 Draw $\overrightarrow{XZ}$. $m\angle ZXY = 53°$.



Ⓑ 138°

Step 1 Use a straightedge to draw a ray, $\overrightarrow{AB}$.

Step 2 Place your protractor on point A so that $\overrightarrow{AB}$ is at zero.

Step 3 Locate the point along the edge of the protractor that corresponds to 138°. Make a mark at this location and label it point C.

Step 4 Draw $\overrightarrow{AC}$. $m\angle CAB = 138°$.

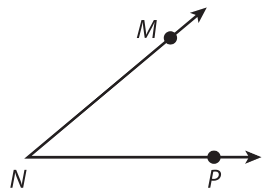
Reflect

6. Explain how you can use a protractor to check that the angle you constructed in the Explore is a copy of the given angle.

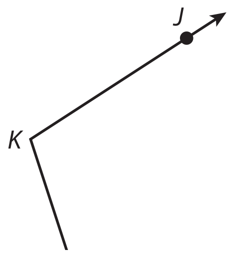
Your Turn

Each angle can be found in the rigid frame of the bicycle. Use a protractor to find each measure.

7.

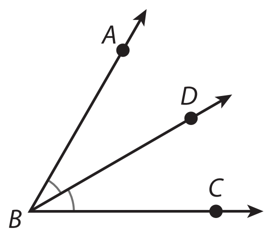


8.



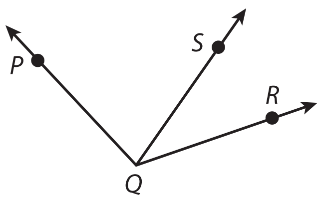
Explain 3 Constructing an Angle Bisector

An **angle bisector** is a ray that divides an angle into two angles that both have the same measure. In the figure, $\overrightarrow{BD}$ bisects $\angle ABC$, so $m\angle ABD = m\angle CBD$. The arcs in the figure show equal angle measures.



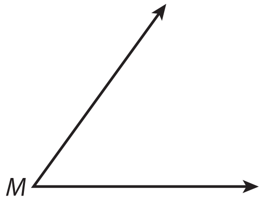
Postulate 2: Angle Addition Postulate

If S is in the interior of $\angle PQR$, then $m\angle PQR = m\angle PQS + m\angle SQR$.

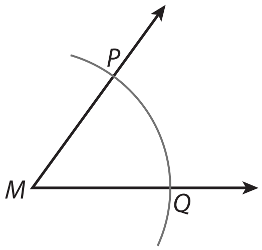


Example 3 **Use a compass and straightedge to construct the bisector of the given angle. Check that the measure of each of the new angles is one-half the measure of the given angle.**

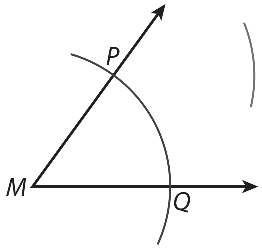
Ⓐ



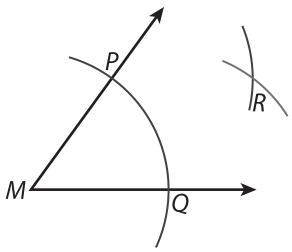
Step 1 Place the point of your compass on point M. Draw an arc that intersects both sides of the angle. Label the points of intersection P and Q.



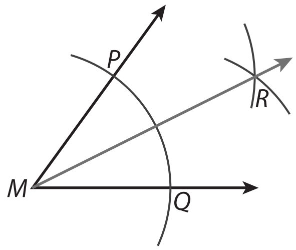
Step 2 Place the point of the compass on P and draw an arc in the interior of the angle.



Step 3 Without adjusting the compass, place the point of the compass on Q and draw an arc that intersects the last arc you drew. Label the intersection of the arcs R.

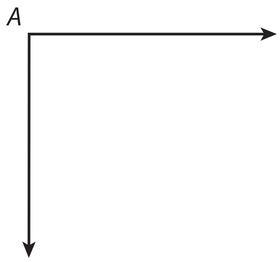


Step 4 Use a straightedge to draw $\overrightarrow{MR}$.



Step 5 Measure with a protractor to confirm that $m\angle PMR = m\angle QMR = \frac{1}{2}m\angle PMQ$.
$27° = 27° = \frac{1}{2}(54°)$ ✓

Ⓑ



Step 1 Draw an arc centered at A that intersects both sides of the angle. Label the points of intersection B and C.

Step 2 Draw an arc centered at B in the interior of the angle.

Step 3 Without adjusting the compass, draw an arc centered at C that intersects the last arc you drew. Label the intersection of the arcs D.

Step 4 Draw $\overrightarrow{AD}$.

Step 5 Check that $m\angle BAD = m\angle CAD = \frac{1}{2}m\angle BAC$.

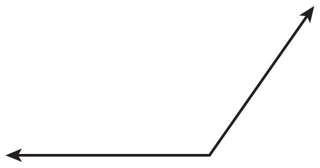
Reflect

9. **Discussion** Explain how you could use paper folding to construct the bisector of an angle.

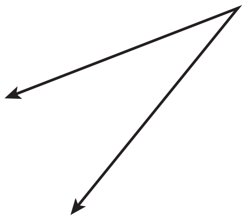
Your Turn

Use a compass and straightedge to construct the bisector of the given angle. Check that the measure of each of the new angles is one-half the measure of the given angle.

10.



11.



Elaborate

12. What is the relationship between a segment bisector and an angle bisector?

13. When you copy an angle, do the lengths of the segments you draw to represent the two rays affect whether the angles have the same measure? Explain.

14. **Essential Question Check-In** Many protractors have two sets of degree measures around the edge. When you measure an angle, how do you know which of the two measures to use?

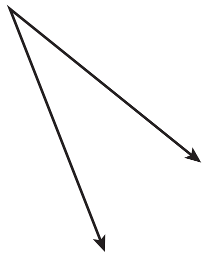
Evaluate: Homework and Practice

Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

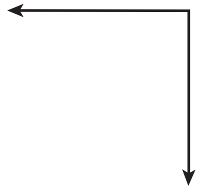
Use a compass and straightedge to construct a copy of each angle.

1.

2.



3.



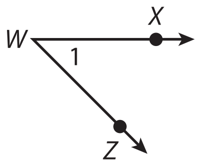
Draw an angle with the given name.

4. $\angle JWT$

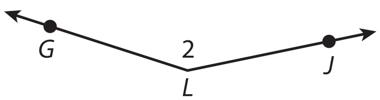
5. $\angle NBQ$

Name each angle in as many different ways as possible.

6.



7.



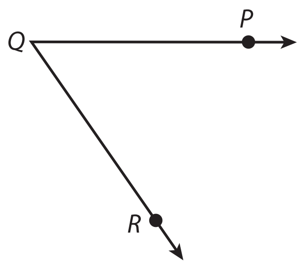
Use a protractor to draw an angle with the given measure.

8. 19°

9. 100°

Use a protractor to find the measure of each angle.

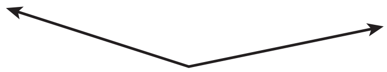
10.



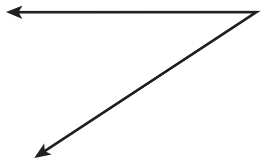
11.

Use a compass and straightedge to construct the bisector of the given angle. Check that the measure of each of the new angles is one-half the measure of the given angle.

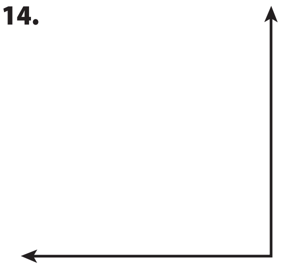
12.



13.



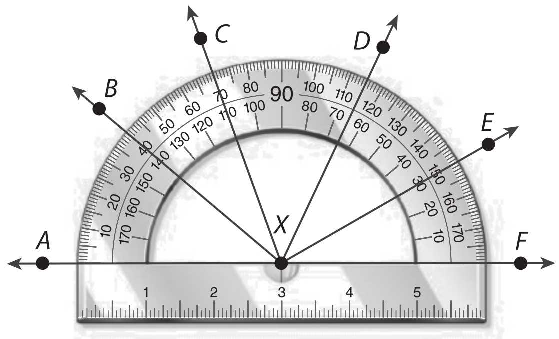
14.



Use the Angle Addition Postulate to find the measure of each angle.

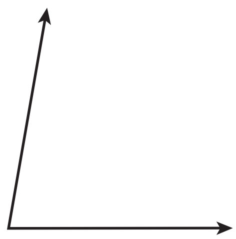
15. $\angle BXC$

16. $\angle BXE$

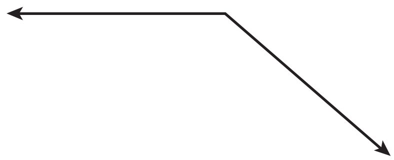


Use a compass and straightedge to copy each angle onto a separate piece of paper. Then use paper folding to construct the angle bisector.

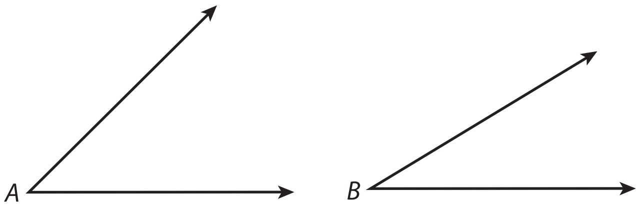
17.



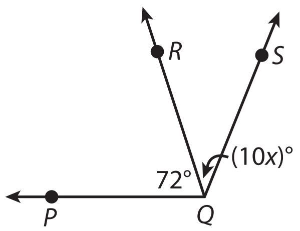
18.



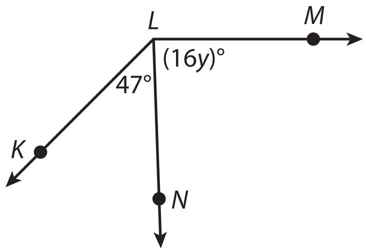
19. Use a compass and straightedge to construct an angle whose measure is $m\angle A + m\angle B$. Use a protractor to check your construction.



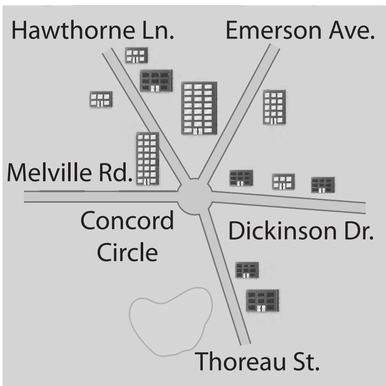
20. Find the value of x, given that $m\angle PQS = 112°$.



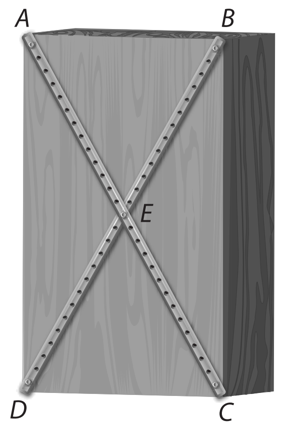
21. Find the value of y, given that $m\angle KLM = 135°$.



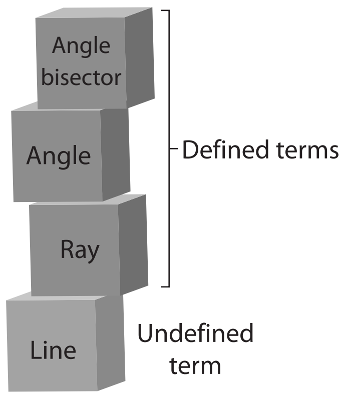
22. **Multi-Step** The figure shows a map of five streets that meet at Concord Circle. The measure of the angle formed by Melville Road and Emerson Avenue is 118°. The measure of the angle formed by Emerson Avenue and Thoreau Street is 134°. Hawthorne Lane bisects the angle formed by Melville Road and Emerson Avenue. Dickinson Drive bisects the angle formed by Emerson Avenue and Thoreau Street. What is the measure of the angle formed by Hawthorne Lane and Dickinson Drive? Explain your reasoning.



23. **Represent Real-World Problems** A carpenter is building a rectangular bookcase with diagonal braces across the back, as shown. The carpenter knows that $\angle ADC$ is a right angle and that m$\angle BDC$ is 32° greater than m$\angle ADB$. Write and solve an equation to find m$\angle BDC$ and m$\angle ADB$.

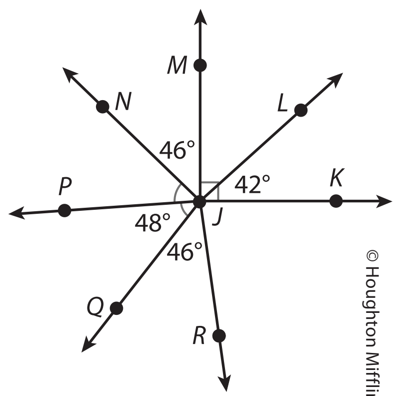


24. Describe the relationships among the four terms.

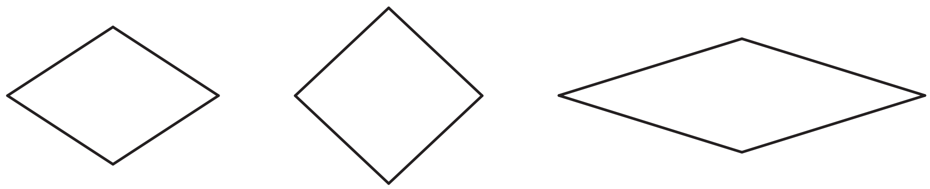


25. Determine whether each of the following pairs of angles have equal measures. Select the correct answer for each lettered part.

A. $\angle KJL$ and $\angle LJM$	◯ Yes	◯ No
B. $\angle MJP$ and $\angle PJR$	◯ Yes	◯ No
C. $\angle LJP$ and $\angle NJR$	◯ Yes	◯ No
D. $\angle MJK$ and $\angle PJR$	◯ Yes	◯ No
E. $\angle KJR$ and $\angle MJP$	◯ Yes	◯ No



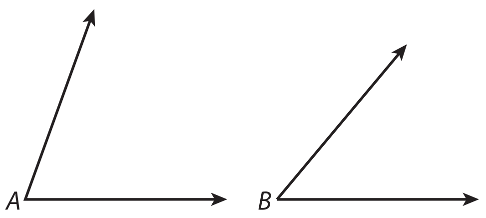
26. **Make a Conjecture** A rhombus is a quadrilateral with four sides of equal length. Use a compass and straightedge to bisect one of the angles in each of the rhombuses shown. Then use your results to state a conjecture.



H.O.T. Focus on Higher Order Thinking

27. **What If?** What happens if you perform the steps for constructing an angle bisector when the given angle is a straight angle? Does the construction still work? If so, explain why and show a sample construction. If not, explain why not.

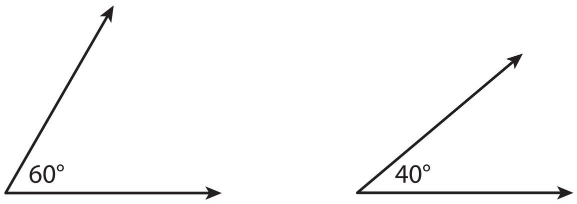
28. **Critical Thinking** Use a compass and straightedge to construct an angle whose measure is $m\angle A - m\angle B$. Use a protractor to check your construction.



29. **Communicate Mathematical Ideas** Explain the steps for using a compass and straightedge to construct an angle with $\frac{1}{4}$ the measure of a given angle. Then draw an angle and show the construction.

Lesson Performance Task

A store sells custom-made stands for tablet computers. When an order comes in, the customer specifies the angle at which the stand should hold the tablet. Then an employee bends a piece of aluminum to the correct angle to make the stand. The figure shows the templates that the employee uses to make a 60° stand and a 40° stand.



The store receives an order for a 50° stand. The employee does not have a template for a 50° stand and does not have a protractor. Can the employee use the existing templates and a compass and straightedge to make a template for a 50° stand? If so, explain how and show the steps the employee should use. If not, explain why not.

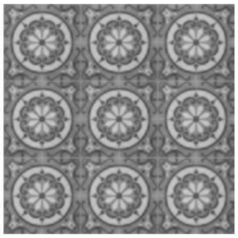
Name ______________________ Class ______________ Date ________

16.3 Representing and Describing Transformations

Essential Question: **How can you describe transformations in the coordinate plane using algebraic representations and using words?**

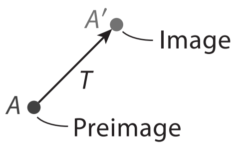
Resource Locker

Explore — Performing Transformations Using Coordinate Notation



A **transformation** is a function that changes the position, shape, and/or size of a figure. The inputs of the function are points in the plane; the outputs are other points in the plane. A figure that is used as the input of a transformation is the **preimage**. The output is the **image**. Translations, reflections, and rotations are three types of transformations. The decorative tiles shown illustrate all three types of transformations.

You can use *prime notation* to name the image of a point. In the diagram, the transformation T moves point A to point A' (read "A prime"). You can use function notation to write $T(A) = A'$. Note that a transformation is sometimes called a *mapping*. Transformation T *maps* A to A'.

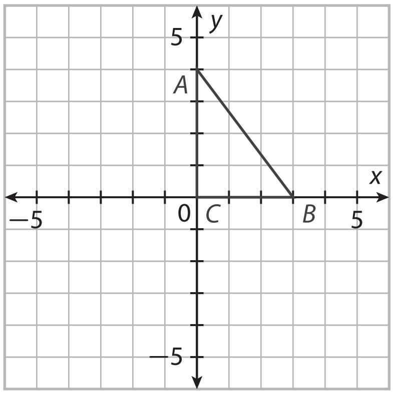


Coordinate notation is one way to write a rule for a transformation on a coordinate plane. The notation uses an arrow to show how the transformation changes the coordinates of a general point, (x, y).

Find the unknown coordinates for each transformation and draw the image. Then complete the description of the transformation and compare the image to its preimage.

Ⓐ $(x, y) \rightarrow (x - 4, y - 3)$

Preimage (x, y)		Rule $(x, y) \rightarrow (x - 4, y - 3)$		Image $(x - 4, y - 3)$
$A(0, 4)$	$\rightarrow$	$A'(0 - 4, 4 - 3)$	$=$	$A'(-4, 1)$
$B(3, 0)$	$\rightarrow$	$B'(3 - 4, 0 - 3)$	$=$	$B'(\square, \square)$
$C(0, 0)$	$\rightarrow$	$C'(0 - 4, 0 - 3)$	$=$	$C'(\square, \square)$



The transformation is a translation 4 units (left/right)

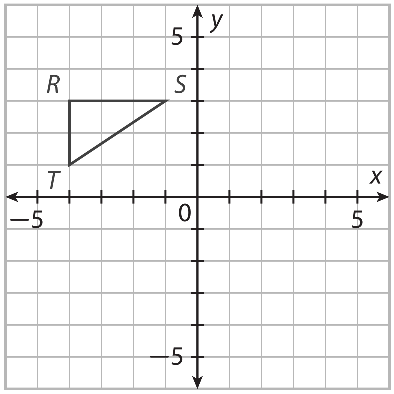
and 3 units (up/down).

A comparison of the image to its preimage shows that

____.

Ⓑ $(x, y) \rightarrow (-x, y)$

Preimage (x, y)		**Rule** $(x, y) \rightarrow (-x, y)$		**Image** $(-x, y)$
$R(-4, 3)$	$\rightarrow$	$R'(-(-4), 3)$	$=$	$R'(\square, \square)$
$S(-1, 3)$	$\rightarrow$	$S'(-(-1), 3)$	$=$	$S'(\square, \square)$
$T(-4, 1)$	$\rightarrow$	$T'(-(-4), 1)$	$=$	$T'(\square, \square)$



The transformation is a reflection across the (x-axis/y-axis).

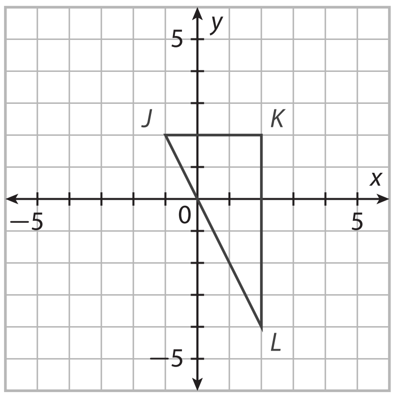
A comparison of the image to its preimage shows that

____

____.

Ⓒ $(x, y) \rightarrow (2x, y)$

Preimage (x, y)		**Rule** $(x, y) \rightarrow (2x, y)$		**Image** $(2x, y)$
$J(\square, \square)$	$\rightarrow$	$J'(2 \cdot \square, \square)$	$=$	$J'(\square, \square)$
$K(\square, \square)$	$\rightarrow$	$K'(2 \cdot \square, \square)$	$=$	$K'(\square, \square)$
$L(\square, \square)$	$\rightarrow$	$L'(2 \cdot \square, \square)$	$=$	$L'(\square, \square)$



The transformation is a (horizontal/vertical) stretch by a factor of _______.

A comparison of the image to its preimage shows that

____

____.

Reflect

1. Discussion How are the transformations in Steps A and B different from the transformation in Step C?

____

____

2. For each transformation, what rule could you use to map the image back to the preimage?

____

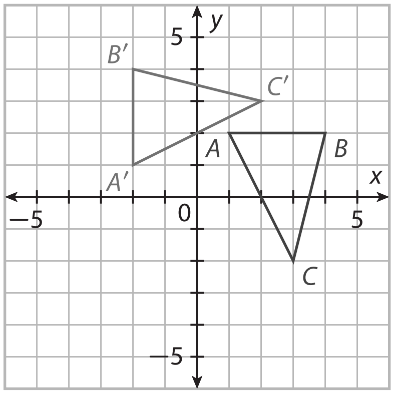
Explain 1 Describing Rigid Motions Using Coordinate Notation

Some transformations preserve length and angle measure, and some do not. A **rigid motion** (or *isometry*) is a transformation that changes the position of a figure without changing the size or shape of the figure. Translations, reflections, and rotations are rigid motions.

Properties of Rigid Motions	
• Rigid motions preserve distance. • Rigid motions preserve angle measure. • Rigid motions preserve betweenness.	• Rigid motions preserve collinearity. • Rigid motions preserve parallelism.

If a figure is determined by certain points, then its image after a rigid motion is determined by the images of those points. This is true because of the betweenness and collinearity properties of rigid motions. Rotations and translations also preserve *orientation*. This means that the order of the vertices of the preimage and image are the same, either clockwise or counterclockwise. Reflections do not preserve orientation.

Example 1 **Use coordinate notation to write the rule that maps each preimage to its image. Then identify the transformation and confirm that it preserves length and angle measure.**



Ⓐ

Preimage		**Image**
$A(1, 2)$	$\rightarrow$	$A'(-2, 1)$
$B(4, 2)$	$\rightarrow$	$B'(-2, 4)$
$C(3, -2)$	$\rightarrow$	$C'(2, 3)$

Look for a pattern in the coordinates.

The x-coordinate of each image point is the opposite of the y-coordinate of its preimage.

The y-coordinate of each image point equals the x-coordinate of its preimage.

The transformation is a rotation of 90° counterclockwise around the origin given by the rule $(x, y) \longrightarrow (-y, x)$.

Find the length of each side of $\triangle ABC$ and $\triangle A'B'C'$. Use the Distance Formula as needed.

$AB = 3$ $\qquad$ $A'B' = 3$

$BC = \sqrt{(3-4)^2 + (-2-2)^2} = \sqrt{17}$ $\qquad$ $B'C' = \sqrt{\left(2-(-2)\right)^2 + (3-4)^2} = \sqrt{17}$

$AC = \sqrt{(3-1)^2 + (-2-2)^2} = \sqrt{20}$ $\qquad$ $A'C' = \sqrt{\left(2-(-2)\right)^2 + (3-1)^2} = \sqrt{20}$

Since $AB = A'B'$, $BC = B'C'$, and $AC = A'C'$, the transformation preserves length.

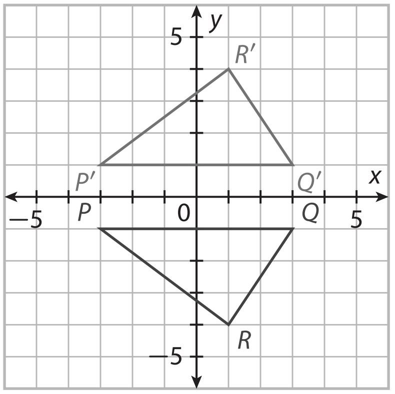
Find the measure of each angle of $\triangle ABC$ and $\triangle A'B'C'$. Use a protractor.

$m\angle A = 63°$, $m\angle B = 76°$, $m\angle C = 41°$ $\qquad$ $m\angle A' = 63°$, $m\angle B' = 76°$, $m\angle C' = 41°$

Since $m\angle A = m\angle A'$, $m\angle B = m\angle B'$, and $m\angle C = m\angle C'$, the transformation preserves angle measure.

(B) **Preimage** → **Image**

$P(-3, -1) \rightarrow P'(-3, 1)$
$Q(3, -1) \rightarrow Q'(3, 1)$
$R(1, -4) \rightarrow R'(1, 4)$



Look for a pattern in the coordinates.

The x-coordinate of each image point ______ the x-coordinate of its preimage.

The y-coordinate of each image point ______ the y-coordinate of its preimage.

The transformation is a ______ given by the rule ______.

Find the length of each side of $\triangle PQR$ and $\triangle P'Q'R'$.

$PQ = \square$

$P'Q' = \square$

$QR = \sqrt{\left(1 - \square\right)^2 + \left(-4 - \square\right)^2}$
$= \sqrt{\square}$

$Q'R' = \sqrt{\left(1 - \square\right)^2 + \left(4 - \square\right)^2}$
$= \sqrt{\square}$

$PR = \sqrt{\left(1 - \square\right)^2 + \left(-4 - \square\right)^2}$
$= \sqrt{\square} = \square$

$P'R' = \sqrt{\left(1 - \square\right)^2 + \left(4 - \square\right)^2}$
$= \sqrt{\square} = \square$

Since ______, the transformation preserves length.

Find the measure of each angle of $\triangle PQR$ and $\triangle P'Q'R'$. Use a protractor.

$m\angle P = \square$, $m\angle Q = \square$, $m\angle R = \square$ $m\angle P' = \square$, $m\angle Q' = \square$, $m\angle R' = \square$

Since ______, the transformation preserves angle measure.

Reflect

3. How could you use a compass to test whether corresponding lengths in a preimage and image are the same?

______

4. Look back at the transformations in the Explore. Classify each transformation as a rigid motion or not a rigid motion.

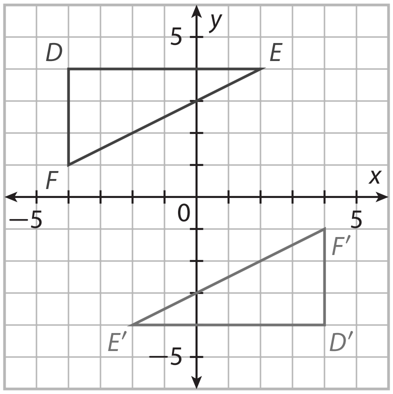
______

Your Turn

Use coordinate notation to write the rule that maps each preimage to its image. Then identify the transformation and confirm that it preserves length and angle measure.

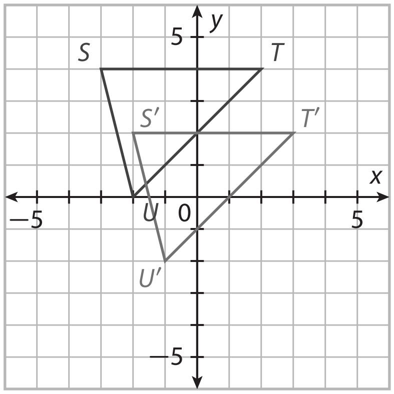
5.

Preimage		Image
$D(-4, 4)$	$\rightarrow$	$D'(4, -4)$
$E(2, 4)$	$\rightarrow$	$E'(-2, -4)$
$F(-4, 1)$	$\rightarrow$	$F'(4, -1)$



6.

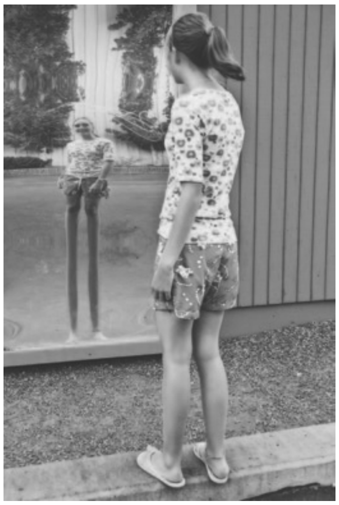
Preimage		Image
$S(-3, 4)$	$\rightarrow$	$S'(-2, 2)$
$T(2, 4)$	$\rightarrow$	$T'(3, 2)$
$U(-2, 0)$	$\rightarrow$	$U'(-1, -2)$



Explain 2 Describing Nonrigid Motions Using Coordinate Notation

Transformations that stretch or compress figures are not rigid motions because they do not preserve distance.

The view in the fun house mirror is an example of a vertical stretch.



Example 2 **Use coordinate notation to write the rule that maps each preimage to its image. Then confirm that the transformation is not a rigid motion.**

(A) $\triangle JKL$ maps to triangle $\triangle J'K'L'$.

Preimage		**Image**
$J(4, 1)$	$\rightarrow$	$J'(4, 3)$
$K(-2, -1)$	$\rightarrow$	$K'(-2, -3)$
$L(0, -3)$	$\rightarrow$	$L'(0, -9)$

Look for a pattern in the coordinates.

The x-coordinate of each image point equals the x-coordinate of its preimage.
The y-coordinate of each image point is 3 times the y-coordinate of its preimage.
The transformation is given by the rule $(x, y) \rightarrow (x, 3y)$.

Compare the length of a segment of the preimage to the length of the corresponding segment of the image.

$$JK = \sqrt{(-2-4)^2 + (-1-1)^2} \qquad J'K' = \sqrt{(-2-4)^2 + (-3-3)^2}$$

$$= \sqrt{40} \qquad = \sqrt{72}$$

Since $JK \neq J'K'$, the transformation is not a rigid motion.

(B) $\triangle MNP$ maps to triangle $\triangle M'N'P'$.

Preimage		**Image**
$M(-2, 2)$	$\rightarrow$	$M'(-4, 1)$
$N(4, 0)$	$\rightarrow$	$N'(8, 0)$
$P(-2, -2)$	$\rightarrow$	$P'(-4, -1)$

The x-coordinate of each image point is ______ the x-coordinate of its preimage.

The y-coordinate of each image point is ______ the y-coordinate of its preimage.

The transformation is given by the rule ____________.

Compare the length of a segment of the preimage to the length of the corresponding segment of the image.

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad M'N' = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{\left(4 - \square\right)^2 + \left(0 - \square\right)^2} \qquad = \sqrt{\left(\square - \square\right)^2 + \left(\square - \square\right)^2}$$

$$= \sqrt{\square^2 + \square^2} \qquad = \sqrt{\square^2 + \square^2}$$

$$= \sqrt{\square} \qquad = \sqrt{\square}$$

Since __________, the transformation is not a rigid motion.

Reflect

7. How could you confirm that a transformation is not a rigid motion by using a protractor?

Your Turn

Use coordinate notation to write the rule that maps each preimage to its image. Then confirm that the transformation is not a rigid motion.

8. $\triangle ABC$ maps to triangle $\triangle A'B'C'$.

Preimage		Image
$A(2, 2)$	$\rightarrow$	$A'(3, 3)$
$B(4, 2)$	$\rightarrow$	$B'(6, 3)$
$C(2, -4)$	$\rightarrow$	$C'(3, -6)$

9. $\triangle RST$ maps to triangle $\triangle R'S'T'$.

Preimage		Image
$R(-2, 1)$	$\rightarrow$	$R'(-1, 3)$
$S(4, 2)$	$\rightarrow$	$S'(2, 6)$
$T(2, -2)$	$\rightarrow$	$T'(1, -6)$

Elaborate

10. **Critical Thinking** To confirm that a transformation is not a rigid motion, do you have to check the length of every segment of the preimage and the length of every segment of the image? Why or why not?

11. **Make a Conjecture** A polygon is transformed by a rigid motion. How are the perimeters of the preimage polygon and the image polygon related? Explain.

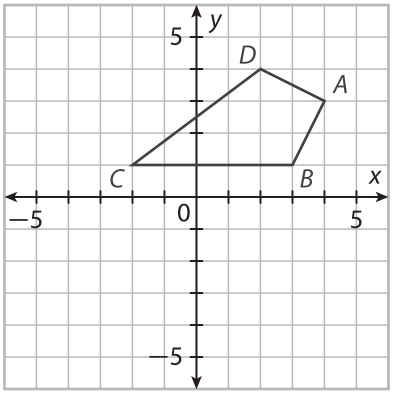
12. **Essential Question Check-In** How is coordinate notation for a transformation, such as $(x, y) \rightarrow (x + 1, y - 1)$, similar to and different from algebraic function notation, such as $f(x) = 2x + 1$?

Evaluate: Homework and Practice

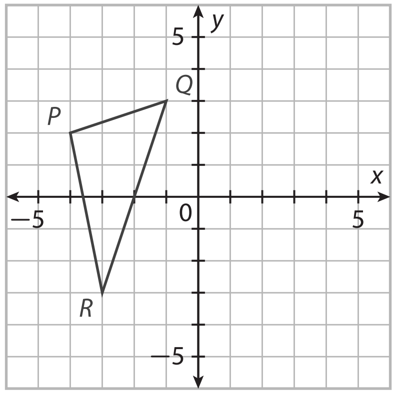
• Online Homework
• Hints and Help
• Extra Practice

Draw the image of each figure under the given transformation. Then describe the transformation in words.

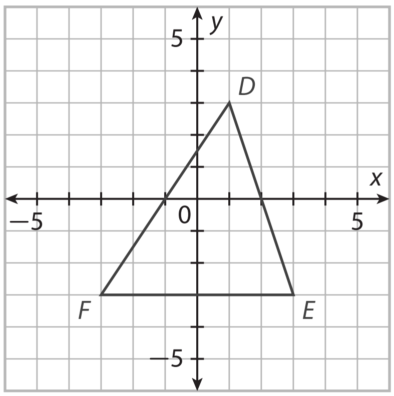
1. $(x, y) \rightarrow (-x, -y)$



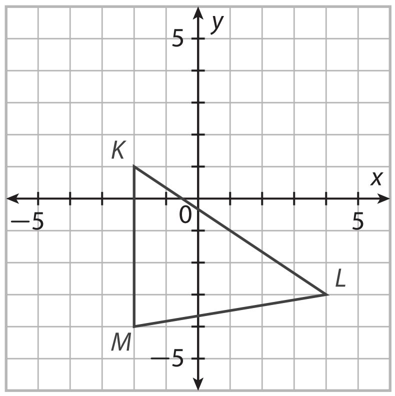
2. $(x, y) \rightarrow (x + 5, y)$



3. $(x, y) \rightarrow \left(x, \frac{1}{3}y\right)$



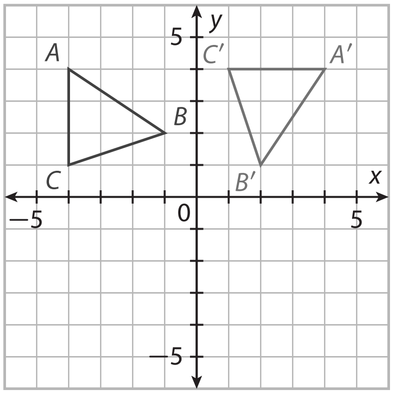
4. $(x, y) \rightarrow (y, x)$



Use coordinate notation to write the rule that maps each preimage to its image. Then identify the transformation and confirm that it preserves length and angle measure.

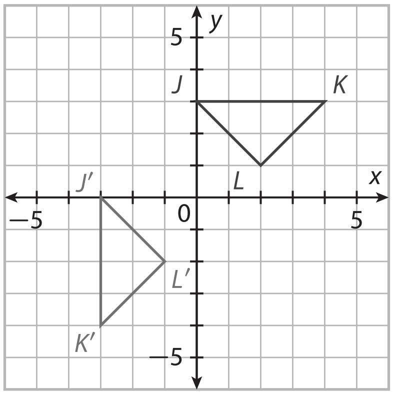
5.

Preimage		**Image**
$A(-4, 4)$	$\rightarrow$	$A'(4, 4)$
$B(-1, 2)$	$\rightarrow$	$B'(2, 1)$
$C(-4, 1)$	$\rightarrow$	$C'(1, 4)$



6.

Preimage		**Image**
$J(0, 3)$	$\rightarrow$	$J'(-3, 0)$
$K(4, 3)$	$\rightarrow$	$K'(-3, -4)$
$L(2, 1)$	$\rightarrow$	$L'(-1, -2)$



Use coordinate notation to write the rule that maps each preimage to its image. Then confirm that the transformation is not a rigid motion.

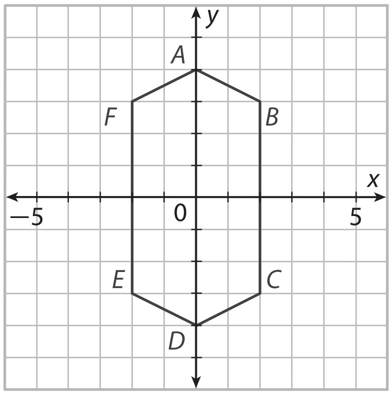
7. $\triangle ABC$ maps to triangle $\triangle A'B'C'$.

Preimage		**Image**
$A(6, 6)$	$\rightarrow$	$A'(3, 3)$
$B(4, -2)$	$\rightarrow$	$B'(2, -1)$
$C(0, 0)$	$\rightarrow$	$C'(0, 0)$

8. $\triangle FGH$ maps to triangle $\triangle F'G'H'$.

Preimage		**Image**
$F(-1, 1)$	$\rightarrow$	$F'(-2, 1)$
$G(1, -1)$	$\rightarrow$	$G'(2, -1)$
$H(-2, -2)$	$\rightarrow$	$H'(-4, -2)$

9. **Analyze Relationships** A mineralogist is studying a quartz crystal. She uses a computer program to draw a side view of the crystal, as shown. She decides to make the drawing 50% wider, but to keep the same height. Draw the transformed view of the crystal. Then write a rule for the transformation using coordinate notation. Check your rule using the original coordinates.



10. Use the points $A(2, 3)$ and $B(2, -3)$.

a. Describe segment AB and find its length.

b. Describe the image of segment AB under the transformation $(x, y) \rightarrow (x, 2y)$.

c. Describe the image of segment AB under the transformation $(x, y) \rightarrow (x + 2, y)$.

d. Compare the two transformations.

11. Use the points $H(-4, 1)$ and $K(4, 1)$.

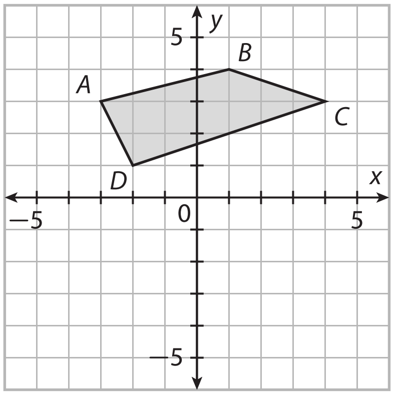
a. Describe segment HK and find its length.

b. Describe the image of segment HK under the transformation $(x, y) \rightarrow (-y, x)$.

c. Describe the image of segment HK under the transformation $(x, y) \rightarrow (2x, y)$.

d. Compare the two transformations.

12. **Make a Prediction** A landscape architect designs a flower bed that is a quadrilateral, as shown in the figure. The plans call for a light to be placed at the midpoint of the longest side of the flower bed. The architect decides to change the location of the flower bed using the transformation $(x, y) \rightarrow (x, -y)$. Describe the location of the light in the transformed flower bed. Then make the required calculations to show that your prediction is correct.



13. **Multiple Representations** If a transformation moves points only up or down, how do the coordinates of the point change? What can you conclude about the coordinate notation for the transformation?

14. Match each transformation with the correct description.

A. $(x, y) \to (3x, y)$ ________ dilation with scale factor 3

B. $(x, y) \to (x + 3, y)$ ________ translation 3 units up

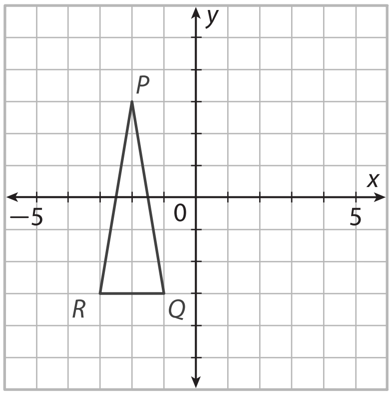
C. $(x, y) \to (x, 3y)$ ________ translation 3 units right

D. $(x, y) \to (x, y + 3)$ ________ horizontal stretch by a factor of 3

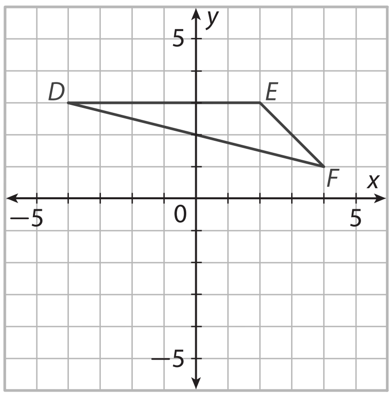
E. $(x, y) \to (3x, 3y)$ ________ vertical stretch by a factor of 3

Draw the image of each figure under the given transformation. Then describe the transformation as a rigid motion or not a rigid motion. Justify your answer.

15. $(x, y) \to (2x + 4, y)$



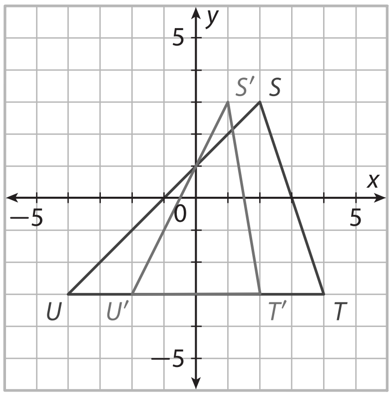
16. $(x, y) \to (0.5x, y - 4)$



H.O.T. Focus on Higher Order Thinking

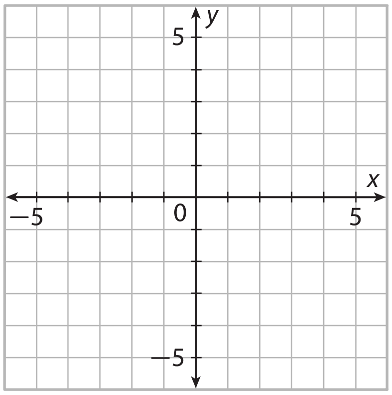
17. Explain the Error A student claimed that the transformation $(x, y) \to (3x, y)$ is a rigid motion because the segment joining $(5, 0)$ to $(5, 2)$ is transformed to the segment joining $(15, 0)$ to $(15, 2)$, and both of these segments have the same length. Explain the student's error.

18. Critical Thinking Write a rule for a transformation that maps $\triangle STU$ to $\triangle S'T'U'$.



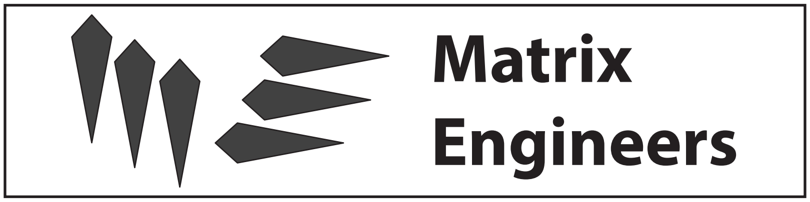
19. Justify Reasoning Consider the transformation given by the rule $(x, y) \to (0, 0)$. Describe the transformation in words. Then explain whether or not the transformation is a rigid motion and justify your reasoning.

20. Communicate Mathematical Ideas One of the properties of rigid motions states that rigid motions preserve parallelism. Explain what this means, and give an example using a specific figure and a specific rigid motion. Include a graph of the preimage and image.



Lesson Performance Task

A Web designer has created the logo shown here for Matrix Engineers.



The logo is 100 pixels wide and 24 pixels high. Images placed in Web pages can be stretched horizontally and vertically by changing the dimensions in the code for the Web page.

The Web designer would like to change the dimensions of the logo so that lengths are increased or decreased but angle measures are preserved.

a. Find three different possible sets of dimensions for the width and height so that lengths are changed but angle measures are preserved. The dimensions must be whole numbers of pixels. Justify your choices.

b. Explain how the Web designer can use transformations to find additional possible dimensions for the logo.

Name ____________________ Class ____________ Date ________

16.4 Reasoning and Proof

Essential Question: How do you go about proving a statement?

Resource Locker

Explore Exploring Inductive and Deductive Reasoning

A **conjecture** is a statement that is believed to be true. You can use *inductive* or *deductive* reasoning to show, or *prove*, that a conjecture is true. **Inductive reasoning** is the process of reasoning that a rule or statement is true because specific cases are true. **Deductive reasoning** is the process of using logic to draw conclusions.

Complete the steps to make a conjecture about the sum of three consecutive counting numbers.

(A) Write a sum to represent the first three consecutive counting numbers, starting with 1. ____________________

(B) Is the sum divisible by 3? ____________________

(C) Write the sum of the next three consecutive counting numbers, starting with 2. ____________________

(D) Is the sum divisible by 3? ____________________

(E) Complete the conjecture:

The __________ of three consecutive counting numbers is divisible by __________.

Recall that postulates are statements you accept are true. A **theorem** is a statement that you can prove is true using a series of logical steps. The steps of deductive reasoning involve using appropriate undefined words, defined words, mathematical relationships, postulates, or other previously-proven theorems to prove that the theorem is true.

Use deductive reasoning to prove that the sum of three consecutive counting numbers is divisible by 3.

(F) Let the three consecutive counting numbers be represented by n, $n + 1$, and $\boxed{}$.

(G) The sum of the three consecutive counting numbers can be written as $3n + \boxed{}$.

(H) The expression $3n + 3$ can be factored as $3(\boxed{})$.

(I) The expression $3(n + 1)$ is divisible by $\boxed{}$ for all values of n.

(J) Recall the conjecture in Step E: The sum of three consecutive counting numbers is divisible by 3.

Look at the steps in your deductive reasoning. Is the conjecture true or false? ______________

Reflect

1. **Discussion** A **counterexample** is an example that shows a conjecture to be false. Do you think that counterexamples are used mainly in inductive reasoning or in deductive reasoning?

2. Suppose you use deductive reasoning to show that an angle is not acute. Can you conclude that the angle is obtuse? Explain.

Explain 1 Introducing Proofs

A **conditional statement** is a statement that can be written in the form "If p, then q" where p is the *hypothesis* and q is the *conclusion*. For example, in the conditional statement "If $3x - 5 = 13$, then $x = 6$," the hypothesis is "$3x - 5 = 13$" and the conclusion is "$x = 6$."

Most of the Properties of Equality can be written as conditional statements. You can use these properties to solve an equation like "$3x - 5 = 13$" to prove that "$x = 6$."

Properties of Equality	
Addition Property of Equality	If $a = b$, then $a + c = b + c$.
Subtraction Property of Equality	If $a = b$, then $a - c = b - c$.
Multiplication Property of Equality	If $a = b$, then $ac = bc$.
Division Property of Equality	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property of Equality	$a = a$
Symmetric Property of Equality	If $a = b$, then $b = a$.
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$.
Substitution Property of Equality	If $a = b$, then b can be substituted for a in any expression.

Example 1 **Use deductive reasoning to solve the equation. Use the Properties of Equality to justify each step.**

(A) $\mathbf{14 = 3x - 4}$

$14 = 3x - 4$	
$18 = 3x$	Addition Property of Equality
$6 = x$	Division Property of Equality
$x = 6$	Symmetric Property of Equality

(B) $\mathbf{9 = 17 - 4x}$

$9 = 17 - 4x$	
$\boxed{} = -4x$	________ Property of Equality
$\boxed{} = -4x$	
$\boxed{} = x$	________ Property of Equality
$x = \boxed{}$	________ Property of Equality

Your Turn

Write each statement as a conditional.

3. All zebras belong to the genus *Equus*.

4. The bill will pass if it gets two-thirds of the vote in the Senate.

5. Use deductive reasoning to solve the equation $3 - 4x = -5$.

6. Identify the Property of Equality that is used in each statement.

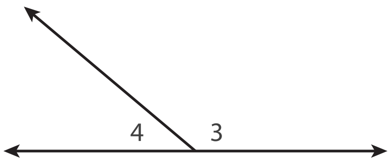
If $x = 2$, then $2x = 4$.	
$5 = 3a$; therefore, $3a = 5$.	
If $T = 4$, then $5T + 7$ equals 27.	
If $9 = 4x$ and $4x = m$, then $9 = m$.	

Explain 2 Using Postulates about Segments and Angles

Recall that two angles whose measures add up to 180° are called *supplementary angles*. The following theorem shows one type of supplementary angle pair, called a *linear pair*. A **linear pair** is a pair of adjacent angles whose non-common sides are opposite rays. You will prove this theorem in an exercise in this lesson.

The Linear Pair Theorem

If two angles form a linear pair, then they are supplementary.

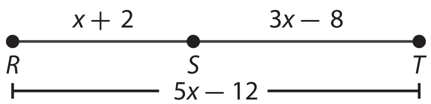


$m\angle 3 + m\angle 4 = 180°$

You can use the Linear Pair Theorem, as well as the Segment Addition Postulate and Angle Addition Postulate, to find missing values in expressions for segment lengths and angle measures.

Example 2 **Use a postulate or theorem to find the value of *x* in each figure.**

(A) Given: $RT = 5x - 12$



Use the Segment Addition Postulate.

$$RS + ST = RT$$

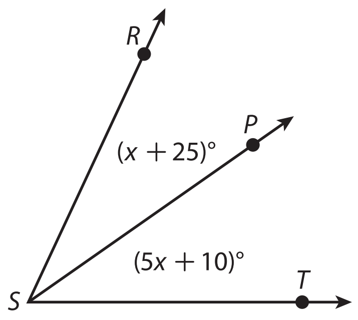
$$(x + 2) + (3x - 8) = 5x - 12$$

$$4x - 6 = 5x - 12$$

$$6 = x$$

$$x = 6$$

(B) Given: $m\angle RST = (15x - 10)°$



Use the ______________ Postulate.

$m\angle RST = m\angle$ $\boxed{}$ $+ m\angle$ $\boxed{}$

$(15x - 10)° =$ $\boxed{}$ $° +$ $\boxed{}$ $°$

$15x - 10 =$ $\boxed{}$

$\boxed{}$ $x =$ $\boxed{}$

$x =$ $\boxed{}$

Reflect

7. **Discussion** The Linear Pair Theorem uses the terms *opposite rays* as well as *adjacent angles*. Write a definition for each of these terms. Compare your definitions with your classmates.

____

____

____

Your Turn

8. Two angles *LMN* and *NMP* form a linear pair. The measure of $\angle LMN$ is twice the measure of $\angle NMP$. Find $m\angle LMN$.

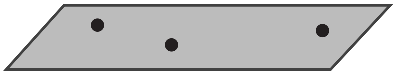
Explain 3 Using Postulates about Lines and Planes

Postulates about points, lines, and planes help describe geometric figures.

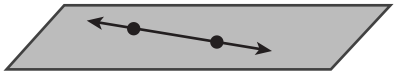
Postulates about Points, Lines, and Planes

Through any two points, there is exactly one line.

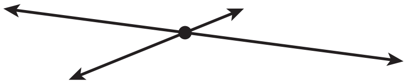
Through any three noncollinear points, there is exactly one plane containing them.



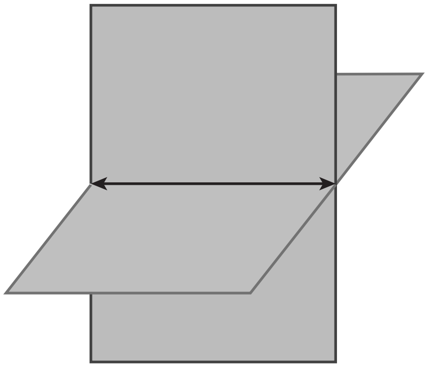
If two points lie in a plane, then the line containing those points lies in the plane.



If two lines intersect, then they intersect in exactly one point.

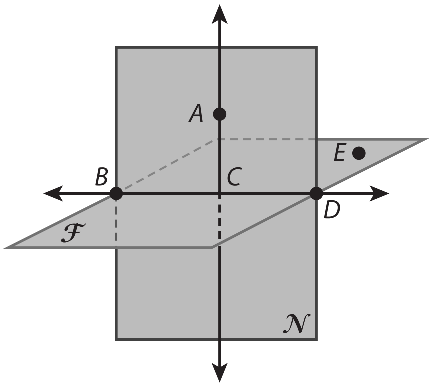


If two planes intersect, then they intersect in exactly one line.



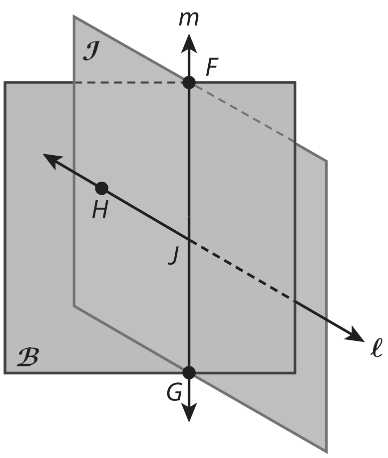
Example 3 **Use each figure to name the results described.**

(A)



Description	Example from the figure
the line of intersection of two planes	Possible answer: The two planes intersect in line *BD*.
the point of intersection of two lines	The line through point *A* and the line through point *B* intersect at point *C*.
three coplanar points	Possible answer: The points *B*, *D*, and *E* are coplanar.
three collinear points	The points *B*, *C*, and *D* are collinear.

(B)



Description	Example from the figure
the line of intersection of two planes	
the point of intersection of two lines	
three coplanar points	
three collinear points	

Reflect

9. Find examples in your classroom that illustrate the postulates of lines, planes, and points.

10. Draw a diagram of a plane with three collinear points and three points that are noncollinear.

Elaborate

11. What is the difference between a postulate and a definition? Give an example of each.

12. Give an example of a diagram illustrating the Segment Addition Postulate. Write the Segment Addition Postulate as a conditional statement.

13. Explain why photographers often use a tripod when taking pictures.

14. **Essential Question Check-In** What are some of the reasons you can give in proving a statement using deductive reasoning?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

Explain why the given conclusion uses inductive reasoning.

1. Find the next term in the pattern: 3, 6, 9.
The next term is 12 because the previous terms are multiples of 3.

2. $3 + 5 = 8$ and $13 + 5 = 18$, therefore the sum of two odd numbers is an even number.

3. My neighbor has two cats and both cats have yellow eyes.
Therefore when two cats live together, they will both have yellow eyes.

4. It always seems to rain the day after July 4th.

Give a counterexample for each conclusion.

5. If x is a prime number, then $x + 1$ is not a prime number.

6. The difference between two even numbers is positive.

7. Points A, B, and C are noncollinear, so therefore they are noncoplanar.

8. The square of a number is always greater than the number.

In Exercises 9–12 use deductive reasoning to write a conclusion.

9. If a number is divisible by 2, then it is even.
The number 14 is divisible by 2.

Use deductive reasoning to write a conclusion.

10. If two planes intersect, then they intersect in exactly one line.
Planes $\mathfrak{R}$ and $\mathfrak{I}$ intersect.

11. Through any three noncollinear points, there is exactly one plane containing them.
Points W, X, and Y are noncollinear.

12. If the sum of the digits of an integer is divisible by 3, then the number is divisible by 3.
The sum of the digits of 46,125 is 18, which is divisible by 3.

Identify the hypothesis and conclusion of each statement.

13. If the ball is red, then it will bounce higher.

14. If a plane contains two lines, then they are coplanar.

15. If the light does not come on, then the circuit is broken.

16. You must wear your jacket if it is cold outside.

Use a definition, postulate, or theorem to find the value of *x* in the figure described.

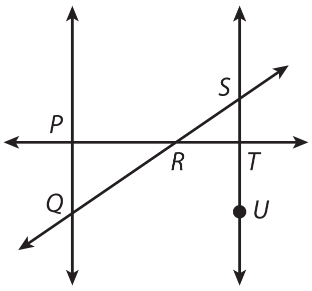
17. Point E is between points D and F. If $DE = x - 4$, $EF = 2x + 5$, and $DF = 4x - 8$, find x.

18. Y is the midpoint of $\overline{XZ}$. If $XZ = 8x - 2$ and $YZ = 2x + 1$, find x.

19. $\overrightarrow{SV}$ is an angle bisector of $\angle RST$. If $m\angle RSV = (3x + 5)°$ and $m\angle RST = (8x - 14)°$, find x.

20. $\angle ABC$ and $\angle CBD$ are a linear pair. If $m\angle ABC = m\angle CBD = 3x - 6$, find x.

Use the figure for Exercises 21 and 22.



21. Name three collinear points.

22. Name two linear pairs.

Explain the error in each statement.

23. Two planes can intersect in a single point.

24. Three points have to be collinear.

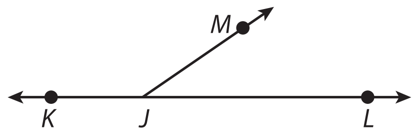
25. A line is contained in exactly one plane

26. If $x^2 = 25$, then $x = 5$.

H.O.T. Focus on Higher Order Thinking

27. **Analyze Relationships** What is the greatest number of intersection points 4 coplanar lines can have? What is the greatest number of planes determined by 4 noncollinear points? Draw diagrams to illustrate your answers.

28. **Justify Reasoning** Prove the Linear Pair Theorem.
Given: $\angle MJK$ and $\angle MJL$ are a linear pair of angles.
Prove: $\angle MJK$ and $\angle MJL$ are supplementary.



Complete the proof by writing the missing reasons. Choose from the following reasons.

Angle Addition Postulate

Definition of linear pair

Substitution Property of Equality

Given

Statements	Reasons
1. $\angle MJK$ and $\angle MJL$ are a linear pair.	**1.**
2. $\overrightarrow{JL}$ and $\overrightarrow{JK}$ are opposite rays.	**2.**
3. $\overrightarrow{JL}$ and $\overrightarrow{JK}$ form a straight line.	**3.** Definition of opposite rays
4. $m\angle LJK = 180°$	**4.** Definition of straight angle
5. $m\angle MJK + m\angle MJL = m\angle LJK$	**5.**
6. $m\angle MJK + m\angle MJL = 180°$	**6.**
7. $\angle MJK$ and $\angle MJL$ are supplementary.	**7.** Definition of supplementary angles

Lesson Performance Task

If two planes intersect, then they intersect in exactly one line.

Find a real-world example that illustrates the postulate above. Then formulate a conjecture by completing the following statement:

If three planes intersect, then ______________________________.

Justify your conjecture with real-world examples or a drawing.