Name ______________________ Class ______________ Date __________

13.1 Understanding Piecewise-Defined Functions

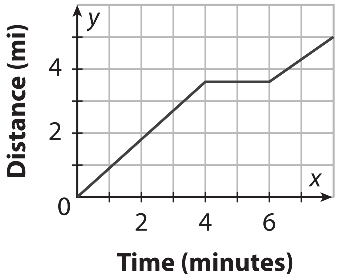
Essential Question: How are piecewise-defined functions different from other functions?

Resource Locker

Explore Exploring Piecewise-Defined Function Models

A **piecewise function** has different rules for different parts of its domain. The following situation can be modeled by a piecewise function.

Armando drives from his home to the grocery store at a speed of 0.9 mile per minute for 4 minutes, stops for 2 minutes to buy snacks, and then drives to the soccer field at a speed of 0.7 mile per minute for 3 minutes. The graph shows Armando's distance from home.



The three different sections in the graph show that there are three different parts to the function. The function for this graph will have three function rules, each for a different range of values for *x*.

The following steps can be used to build the piecewise function to model this situation.

(A) Determine the function rule for the first 4 minutes for the domain, $0 \le x \le 4$. The rate of change is Armando's speed in miles per minute.

$$m = \frac{\square \text{ mi}}{\square \text{ min}} = \square$$

The y-intercept is ____.

The function for the first 4 minutes is $y = \square$.

(B) Determine the function rule for the next 2 minutes for the domain, $4 < x \le 6$.

Because the distance is not changing, the rate of change is $m = \frac{\square \text{ mi}}{\square \text{ min}} = \square$.

The function for the next 2 minutes is a constant function. The constant y-value is Armando's distance from home at the end of the first 4 minutes.

$y = 0.9x$

$y = 0.9(4) = \square$

(C) Determine the function rule for the last 3 minutes.
The rate of change is Armando's speed in miles per minute.

$m = \frac{\square \text{ mi}}{\square \text{ min}} = \square$

The point at which the function begins is $(\square, \square)$.

Use the point and slope to construct the function rule for the last 3 minutes.

$y - \square = \square(x - \square)$

$y - \square = \square x - \square$

$y = \square x - \square$

(D) Use all three parts to build the piecewise function that represents this situation.

$$f(x) = \begin{cases} \square & \text{if } 0 \leq x \leq 4 \\ \square & \text{if } 4 < x \leq 6 \\ \square & \text{if } 6 < x \leq 9 \end{cases}$$

Reflect

1. **Discussion** Describe how the domain is constructed so that the piecewise function is a function with no more than one dependent variable for any independent variable.

Explain 1 Evaluating Piecewise-Defined Functions

The **greatest integer function** is a piecewise function whose rule is denoted by $\lfloor x \rfloor$, which represents the greatest integer less than or equal to x. The greatest integer function is an example of a **step function**, a piecewise function in which each function rule is a constant function. To evaluate a piecewise function for a given value of x, substitute the value of x into the rule for the part of the domain that includes x.

Example 1 **Evaluate each piecewise function for the given values.**

(A) Find $f(-3)$, $f(-2.9)$, $f(0.7)$, and $f(1.06)$ for $f(x) = \lfloor x \rfloor$.

The greatest integer function $f(x) = \lfloor x \rfloor$ can also be written in the form below.

$$f(x) = \begin{cases} \vdots \\ -3 & \text{if } -3 \leq x < -2 \\ -2 & \text{if } -2 \leq x < -1 \\ -1 & \text{if } -1 \leq x < 0 \\ 0 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } 1 \leq x < 2 \\ 2 & \text{if } 2 \leq x < 3 \\ \vdots \end{cases}$$

-3 is in the interval $-3 \leq x < -2$, so $f(-3) = -3$.

-2.9 is in the interval $-3 \leq x < -2$, so $f(-2.9) = -3$.

0.7 is in the interval $0 \leq x < 1$, so $f(0.7) = 0$.

1.06 is in the interval $1 \leq x < 2$, so $f(1.06) = 1$.

(B) Find $f(-3)$, $f(-0.2)$, $f(0)$, and $f(2)$ for $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}$

$-3 < 0$, so $f(-3) = -(-3) = \square$

$0 \geq 0$, so $f(0) = \square + 1 = \square$

$-0.2 < 0$, so $f(-0.2) = \square = \square$

$2 \geq 0$, so $f(2) = \square + \square = \square$

Reflect

2. For positive numbers, how is applying the greatest integer function different from the method of rounding to the nearest whole number?

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Your Turn

3. Find $f(-2)$, $f(-0.4)$, $f(3.7)$, and $f(5)$ for $f(x) = \begin{cases} -x & \text{if } x < 2 \\ 2x + 3 & \text{if } 2 \leq x < 4. \\ x^2 & \text{if } x \geq 4 \end{cases}$

Explain 2 Graphing Piecewise-Defined Functions

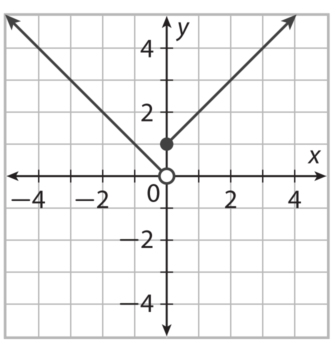
You can graph piecewise-defined functions to illustrate their behavior.

Example 2 **Graph each function.**

(A) $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x + 1 & \text{if } x \geq 0 \end{cases}$

Make a table of values.

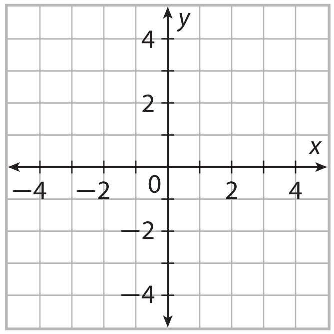
x	−3	−2	−1	0	1	2
$f(x)$	3	2	1	1	2	3



(B) $f(x) = \lfloor x \rfloor$

Make a table of values.

x	−3	−2.9	−2.1	−2	−1.5	−1	0	1	1.5	2
$f(x)$	−3			−2			0			



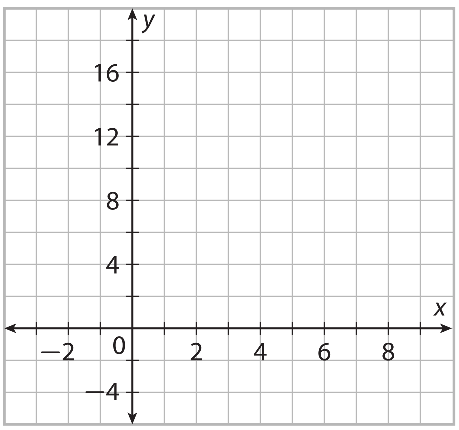
Reflect

4. Why does the graph in Example 2A use rays and not lines?

5. Use the graph of the greatest integer function from Example 2B to explain why this function is called a step function.

Your Turn

6. $f(x) = \begin{cases} x & \text{if} \quad x < 2 \\ 2x + 3 & \text{if} \quad 2 \leq x < 4 \\ x^2 & \text{if} \quad x \geq 4 \end{cases}$



Explain 3 Modeling with Piecewise-Defined Functions

Some real-world situations can be described by piecewise functions.

Example 3 **Write a piecewise function for each situation. Then graph the function.**

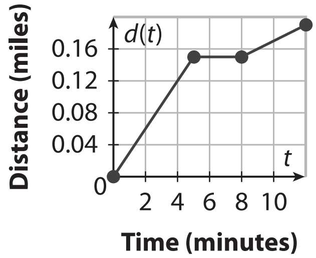
Ⓐ **Travel** On her way to a concert, Maisee walks at a speed of 0.03 mile per minute from her car for 5 minutes, waits in line for a ticket for 3 minutes, and then walks to her seat for 4 minutes at a speed of 0.01 mile per minute.

Express the Maisee's distance traveled d (in miles) as a function of time t (in minutes).

For $0 \leq t \leq 5,\ m = 0.03$ and $b = 0$, so $d(t) = 0.03t$.

For $5 < t \leq 8,\ m = 0$ and $b = 0.15$, so $d(t) = 0.15$.

For $8 < t \leq 12$, $m = 0.01$ beginning at $(8, 0.15)$, so $d(t) = 0.01t + 0.07$.



$$d(t) = \begin{cases} 0.03t & \text{if } 0 \leq t \leq 5 \\ 0.15 & \text{if } 5 < t \leq 8 \\ 0.01t + 0.07 & \text{if } 8 < t \leq 12 \end{cases}$$

Ⓑ **Travel** On his way to class from his dorm room, a college student walks at a speed of 0.05 mile per minute for 3 minutes, stops and talks to a friend for 1 minute, and then to avoid being late for class, runs at a speed of 0.10 mile per minute for 2 minutes.

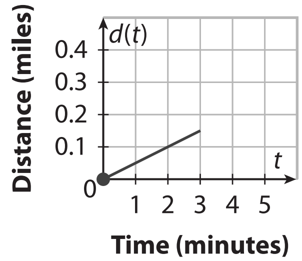
Express the student's distance traveled d (in miles) as a function of time t (in minutes).

For $0 \leq t \leq 3$, $m =$ $\square$ and $b = 0$, so $d(t) =$ $\square$ t.

For $3 < t \leq$ $\square$, $m = 0$ and $b = 0.15$, so $d(t) = 0.15$.

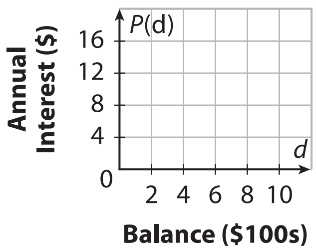
For $4 < t \leq 6$, $m =$ $\square$ beginning at $\square$, so $y =$ $\square$ $x -$ $\square$.

$$d(t) = \begin{cases} \square\, t & \text{if } 0 \leq t \leq 3 \\ 0.15 & \text{if } 3 < t \leq \square \\ \square\, t - \square & \text{if } 4 < t \leq 6 \end{cases}$$



Your Turn

7. **Finance** A savings account earns 1.4% simple interest annually for balances of \$100 or less, 2.4% simple interest for balances greater than \$100 and up to \$500, and 3.4% simple interest for balances greater than \$500. Write a function rule for the interest paid by the account and graph the function.

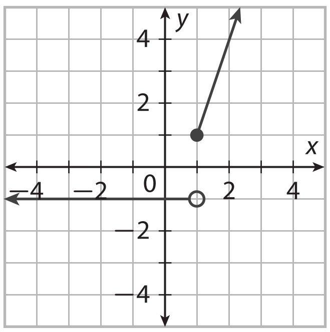


Explain 4 Building Piecewise-Defined Functions from Graphs

You can find the function rules for a piecewise function when you are given the graph of the function.

Example 4 **Write an equation for each graph.**

Ⓐ



Find the equation of the ray on the right.

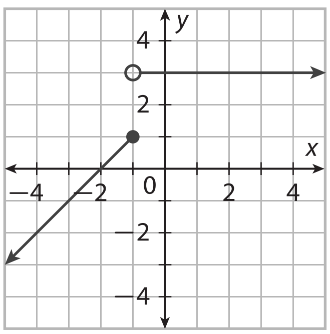
$m = \dfrac{1-4}{1-2} = \dfrac{-3}{-1} = 3$

Because the point (1, 1) is on the ray, $y - 1 = 3(x - 1)$, so $y = 3x - 2$

The equation of the line that contains the horizontal ray is $y = -1$.

The equation for the function is $y = \begin{cases} -1 & \text{if } x < 1. \\ 3x - 2 & \text{if } x \geq 1 \end{cases}$

Ⓑ



Find the equation for the ray on the left.

$m = \dfrac{1 - \square}{-1 - \square} = \dfrac{\square}{\square} = \square$

Because the point (-1, 1) is on the ray, $y \square = \square (x \square)$,

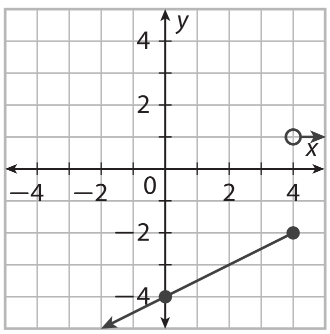
so $y = \square$

The equation of the horizontal ray is $y = \square$.

The equation for the function is $y = \begin{cases} \square x + \square & \text{if } x \leq -1 \\ \square & \text{if } x > -1 \end{cases}$

Your Turn

8.



Elaborate

9. How are the greatest integer function and $f(x) = 2\lfloor x \rfloor$ related?

10. **Essential Question Check-In** How many function rules do functions that are not piecewise-defined have?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

Evaluate each piecewise function for the given values.

1. Find $f(-4)$, $f(-3.1)$, $f(1.2)$, and $f(2.8)$ for $f(x) = \lfloor x \rfloor$.

2. Find $f(-3)$, $f(-2.1)$, $f(0.6)$, and $f(3.3)$ for $f(x) = \begin{cases} -1 & \text{if } x \leq 0 \\ 3x & \text{if } 0 < x < 1. \\ x + 3 & \text{if } x \geq 1 \end{cases}$

3. Find $f(-4)$, $f(-2.9)$, and $f(1.9)$ for $f(x) = \begin{cases} -5 & \text{if } x \leq -3 \\ x + 2 & \text{if } -3 < x \leq 0. \\ x^2 + 7 & \text{if } x \geq 0 \end{cases}$

4. Find $f(-6)$, $f(-2.2)$, $f(1.4)$ and $f(3.6)$ for $f(x) = -2\lfloor x \rfloor$.

5. Find $f(-3)$, $f(-1)$, and $f(1)$ for $f(x) = \begin{cases} \frac{2}{x} & \text{if } x \leq -2 \\ x & \text{if } -2 < x \leq 0 \\ 1 & \text{if } x \geq 0 \end{cases}$

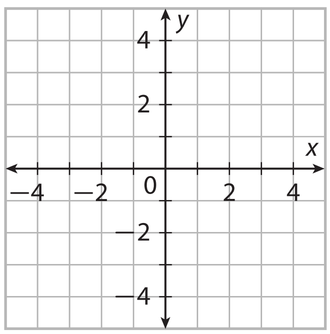
6. Find $f(-2)$, $f(-1)$, $f(0)$, $f(4)$, and $f(9)$ for $f(x) = \begin{cases} -x^2 & \text{if } x \leq -2 \\ 2x & \text{if } -2 < x < 2 \\ x + 6 & \text{if } 2 \leq x \leq 4 \\ \sqrt{x} + 8 & \text{if } x > 4. \end{cases}$

7. Find $f(-2.8)$, $f(-1.2)$, $f(0.4)$, and $f(1.6)$ for $f(x) = \lfloor x \rfloor^2$

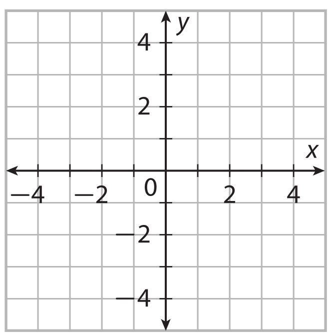
8. Find $f(0)$, $f(2)$, and $f(4)$ for $f(x) = \begin{cases} 8 & \text{if } x \leq 0 \\ 0 & \text{if } x > 0 \end{cases}.$

Graph each piecewise function.

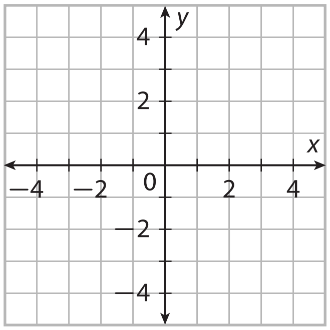
9. $f(x) = \begin{cases} -x + 1 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$



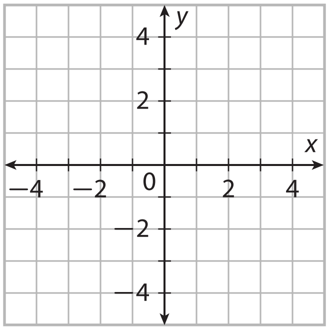
10. $f(x) = \begin{cases} -1 & \text{if } x < 1 \\ 2x - 2 & \text{if } x \geq 1 \end{cases}$



11. $f(x) = \lfloor x \rfloor + 1$

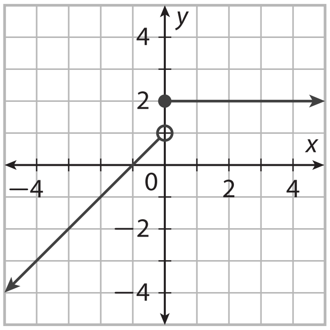


12. $f(x) = 2\lfloor x \rfloor - 2$



Write an equation for each graph.

13.



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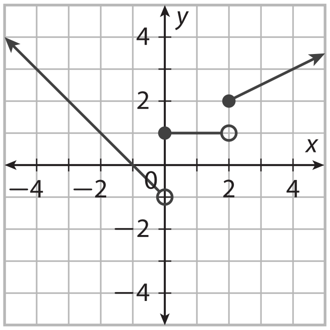

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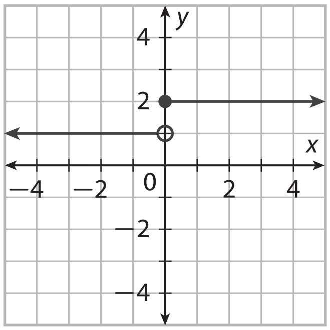

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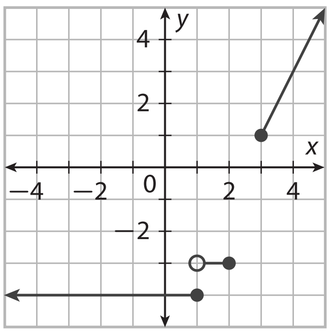


Write a piecewise function for each situation. Then complete the table and the graph.

17. **Finance** A garage charges the following rates for parking (with an 8 hour limit):

$4 per hour for the first 2 hours

$2 per hour for the next 4 hours

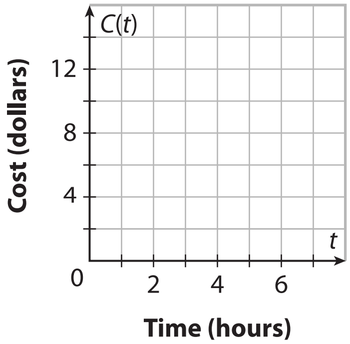
No additional charge for the next 2 hours

Express the cost C (in dollars) as a function of the time t (in hours) that a car is parked in the garage.

$C(t) =$

t	0	1	2	3	4
***C*(*t*)**					

t	5	6	7	8
***C*(*t*)**				

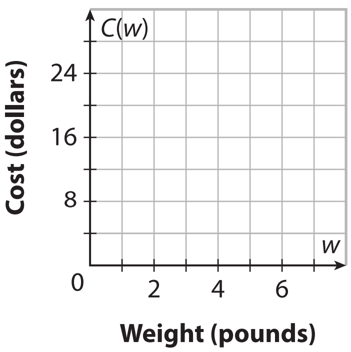


18. **Cost Analysis** The cost to send a package between two cities is $8.00 for any weight less than 1 pound. The cost increases by $4.00 when the weight reaches 1 pound and again each time the weight reaches a whole number of pounds after that.

Express the shipping cost C (in dollars) as a function of the weight (in pounds). Express your answer in terms of the greatest integer function $\lfloor w \rfloor$.

$C(w) =$

w	0.5	1	1.5	2	2.5
***C*(*w*)**					

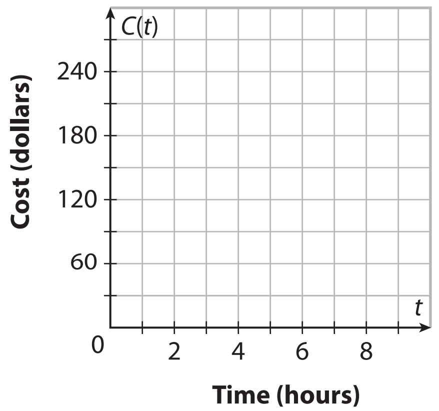


19. **Golfing** A local golf course charges members $30 an hour for the first three hours, $35 an hour for the next five hours, and nothing for the last 2 hours, for a maximum of 10 hours.

Express the cost C (in dollars) as a function of the time t (in hours) that a member plays golf at this golf course.

$C(t) =$

t	2	4	6	8	10
$C(t)$					

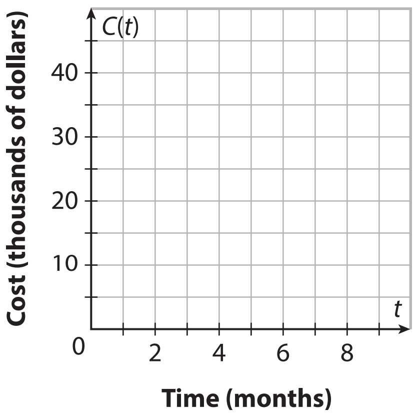


20. **Construction** A construction company is building a new parking garage and is charging the following rates: $5000 a month for the first 2 months; $8000 a month for the next 4 months; $6000 in total for the last 4 months, when the garage will be completed. This amount will be paid in a lump sum at the end of the 6th month.

Express the cost C (in thousands of dollars) as a function of the time t (in months) that the construction company works on the parking garage.

$C(t) =$

t	2	4	6	8	10
$C(t)$					



21. State the domain and range of each piecewise function.

A. $y = \begin{cases} 5 & \text{if } 0 < x \leq 1 \\ 0 & \text{if } x > 1 \end{cases}$

B. $y = \begin{cases} -x & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$

C. $y = \begin{cases} x^2 & \text{if } x \leq 4 \\ x^3 & \text{if } x > 4 \end{cases}$

D. $y = \begin{cases} x^2 + 1 & \text{if } x \leq -2 \\ x^3 & \text{if } -2 < x < 4 \\ x^x & \text{if } x \geq 4 \end{cases}$

E. $y = \begin{cases} 1 & \text{if } x \leq -3 \\ 1 & \text{if } -3 < x < 8 \\ 1 & \text{if } x \geq 8 \end{cases}$

H.O.T. Focus on Higher Order Thinking

22. **Critical Thinking** Rewrite the piecewise function into a function of the greatest integer function.

$$f(x) = \begin{cases} -6 & \text{if } -2 \leq x < -1 \\ -3 & \text{if } -1 \leq x < 0 \\ 0 & \text{if } 0 \leq x < 1 \\ 3 & \text{if } 1 \leq x < 2 \\ 6 & \text{if } 2 \leq x < 3 \end{cases}$$

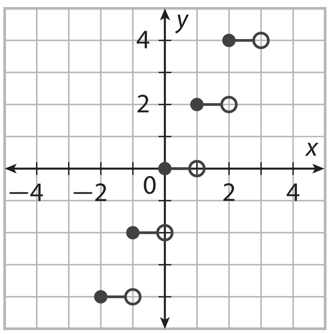
23. **Explain the Error** Clara was given the following situation and told to write a piecewise function to describe it.

While exercising, a person loses weight in the following manner:
0.5 pound per hour for the first hour
0.7 pound per hour for the next three hours
0.1 pound per hour until the workout is finished

Clara produced the following result. What did she do wrong and what is the correct answer?

$$W(t) = \begin{cases} 0.5t & \text{if } 0 \leq t \leq 1 \\ 0.7t & \text{if } 1 \leq t < 4 \\ 0.1t & \text{if } t \geq 4 \end{cases}$$

24. **Critical Thinking** Write an equation for the shown graph. Express the answer in terms of $\lfloor x \rfloor$.



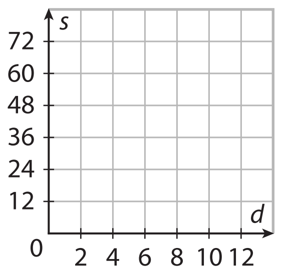
25. **Communicate Mathematical Ideas** Is a piecewise function still a function if it contains a vertical line? Explain why or why not.

Lesson Performance Task

Suppose someone is traveling from New York City to Miami, Florida. The following table describes the average speeds at various intervals on this 1200-mile trip.

Distance Traveled (hundreds of miles)	Average Speed (mi/h)
$0 < d \leq 2$	37.7
$2 < d \leq 4$	46.6
$4 < d \leq 6$	63.3
$6 < d \leq 8$	45.5
$8 < d \leq 10$	64.4
$10 < d \leq 12$	49.9

A. Graph the distance function. Make sure to use appropriate labels.



B. Write the piecewise function that is given by the table.

C. Suppose the destination was changed from Miami, Florida to Minneapolis, Minnesota instead. Explain why it is not okay to use the piecewise function created for the trip from New York to Miami when traveling to Minneapolis, even though the distance is comparable.

Name ______________________ Class ______________ Date ________

13.2 Absolute Value Functions and Transformations

Resource Locker

Essential Question: What are the effects of parameter changes on the graph of $y = a|x - h| + k$?

Explore Understanding the Parent Absolute Value Function

The most basic **absolute value function** is a piecewise function given by the following rule.

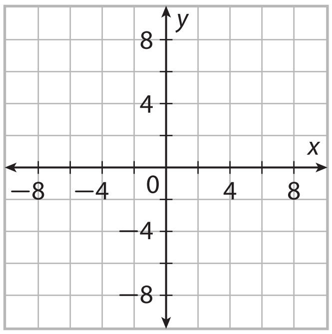
$$f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

This function is sometimes called the parent absolute value function. Complete each step to graph this function.

(A) Complete the table of values.

x	$f(x) = \lvert x \rvert$
−3	3
−2	
−1	
0	
1	
2	
3	3

(B) Plot these points on a graph and using two rays, connect them to display the absolute value function.



(C) The vertex of an absolute value function is the single point that both rays have in common. Identify the vertex of the parent absolute value function.

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Reflect

1. What is the domain of $f(x) = |x|$? What is the range?

2. For what values of x is the function $f(x) = |x|$ increasing? decreasing?

Explain 1 Graphing Translations of Absolute Value Functions

You can compare the graphs of absolute value functions in the form $g(x) = |x - h| + k$, where h and k are real numbers, with the graph of the parent function $f(x) = |x|$ to see how h and k affect the parent function.

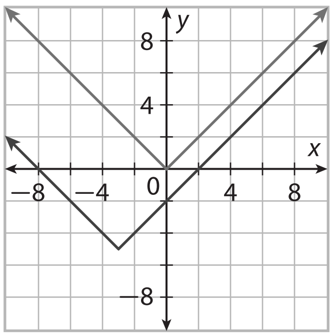
Example 1 **Graph each absolute value function with respect to the parent function $f(x) = |x|$.**

(A) $g(x) = |x + 3| - 5$

First, create a table of values for x and $g(x)$.

x	$g(x) = \lvert x + 3 \rvert - 5$
−6	−2
−3	−5
−1	−3
0	−2
1	−1
3	1
6	4

Now graph the function along with the parent function.

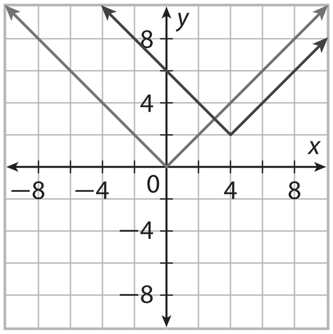


(B) $g(x) = |x - 4| + 2$

First, create a table of values for x and $g(x)$.

x	$g(x) = \|x - 4\| + 2$
−5	
−3	
−1	
0	
1	
3	
5	

Now graph the function along with the parent function.



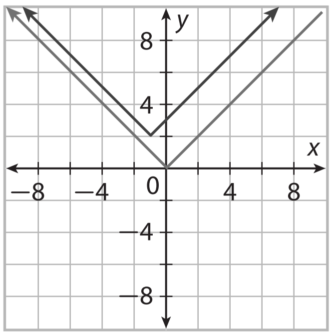
Reflect

3. How is the graph of $g(x) = |x - 4| + 2$ related to the graph of the parent function $f(x) = |x|$?

4. In general, how is the graph of $g(x) = |x - h| + k$ related to the graph of $f(x) = |x|$?

Your Turn

5. Graph the absolute value function $g(x) = |x + 1| + 2$ along with the parent function $f(x) = |x|$.

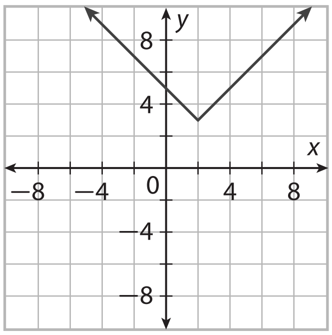


Explain 2 Constructing Functions for Given Graphs of Absolute Value Functions

You can write an absolute value function from a graph of the function.

Example 2 **Write an equation for each absolute value function whose graph is shown.**

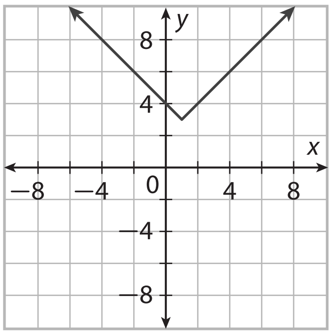
(A)



- h is the number of units that the parent function is translated horizontally. For a translation to the right, h is positive; for a translation to the left, h is negative. In this situation, $h = 2$.
- k is the number of units that the parent function is translated vertically. For a translation up, k is positive; for a translation down, k is negative. In this situation, $k = 3$.

The function is $g(x) = |x - 2| + 3$.

(B)



- h is the number of units that the parent function is translated horizontally. For a translation to the right, h is positive; for a translation to the left, h is negative. In this situation, $h =$ ___.
- k is the number of units that the parent function is translated vertically. For a translation up, k is positive; for a translation down, k is negative. In this situation, $k =$ ___.

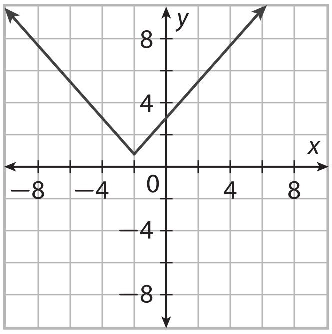
The function is $g(x) = |x - \_\_\_| + \_\_\_$.

Reflect

6. If the graph of an absolute value function is a translation of the graph of the parent function, explain how you can use the vertex of the translated graph to help you determine the equation for the function.

YourTurn

7.

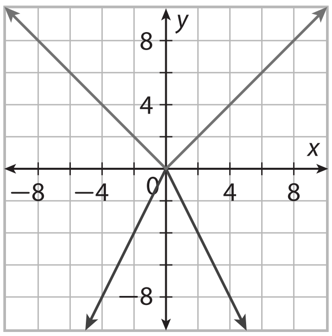


Explain 3 Graphing Stretches and Compressions of Absolute Value Functions

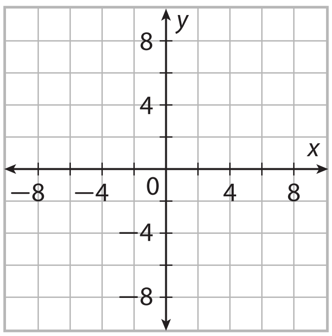
You can compare the graphs of absolute value functions in the form $g(x) = a|x|$, where a is a real number, with the graph of the parent function $f(x) = |x|$ to see how a affects the absolute value function.

Example 3 **Graph each absolute value function.**

Ⓐ $g(x) = -2|x|$



Ⓑ $g(x) = \frac{1}{4}|x|$



Reflect

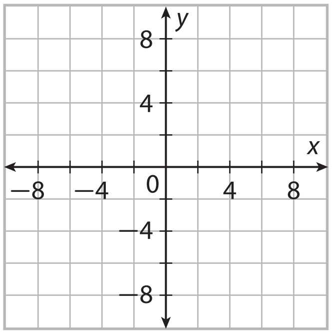
8. Describe how the graphs of $g(x) = \frac{1}{4}|x|$ and $h(x) = -2|x|$ compare with the graph of $f(x) = |x|$. Use either the word *stretch* or *shrink*, and include the directions of movement.

9. What other transformation occurs when the value of a in $g(x) = a|x|$ is negative?

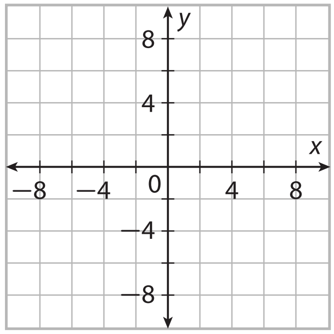
Your Turn

Graph each absolute value function.

10. $g(x) = -\frac{1}{2}|x|$



11. $g(x) = 4|x|$



Elaborate

12. Why is it important to note both the direction and the distance that a point has been translated either vertically or horizontally?

13. How does knowing a point in the graph other than the vertex help you find the value of a?

14. When graphing an absolute value function, how are $g(x) = a|x|$ and $h(x) = -a|x|$ related?

15. **Essential Question Check-In** How would the graph of the parent function $f(x) = |x|$ be affected if $h > 0$, $k < 0$ and $a > 1$?

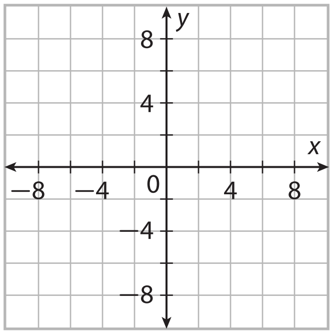
Evaluate: Homework and Practice

Personal Math Trainer

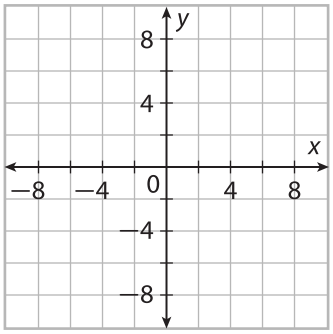
- Online Homework
- Hints and Help
- Extra Practice

Graph each absolute value function.

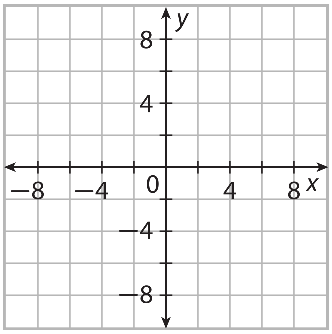
1. $g(x) = |x + 1| + 1$



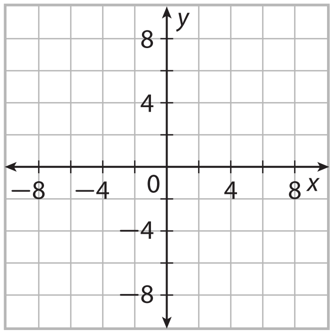
2. $g(x) = |x - 4| + 2$



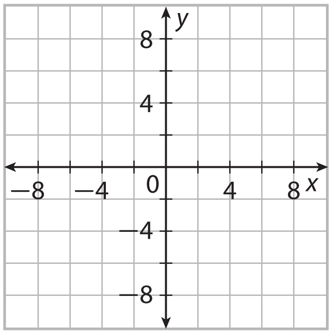
3. $g(x) = |x - 3| - 5$



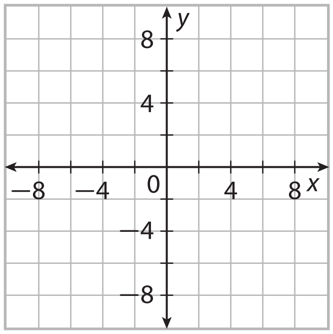
4. $g(x) = |x + 7| - 1$



5. $g(x) = |x + 3| - 1$

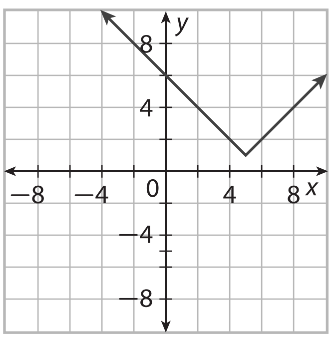


6. $g(x) = |x + 5| - 3$



Write an equation for each absolute value function whose graph is shown.

7.



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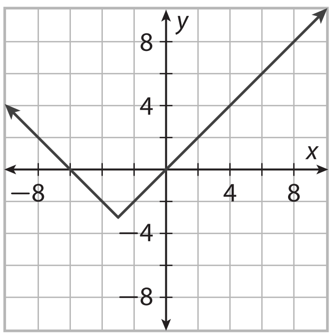

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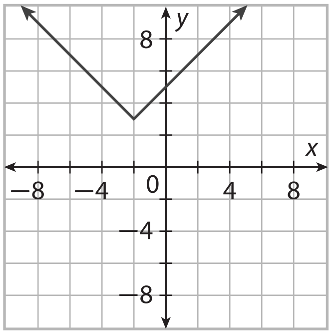

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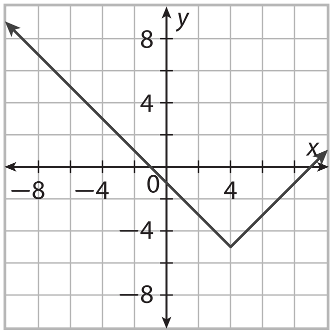

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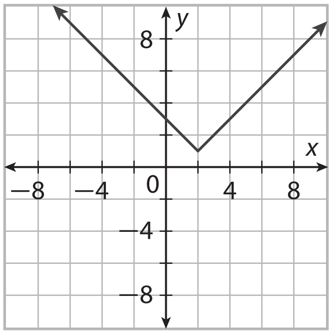
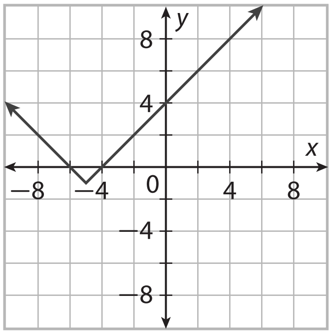


12.



Determine the domain and range of each function.

13. $g(x) = |x + 3| - 1$

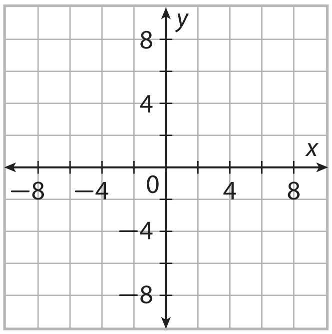
14. $g(x) = |x + 2| + 2$

15. $g(x) = |x| + 1$

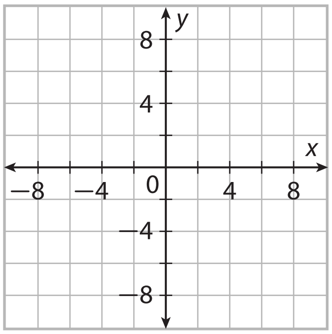
16. $g(x) = |x - 9| + 6$

Graph each absolute value function.

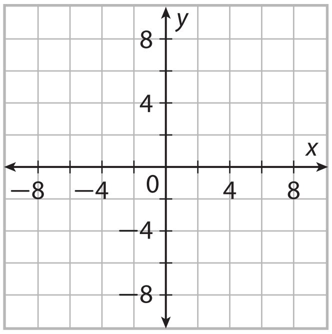
17. $g(x) = 3|x|$



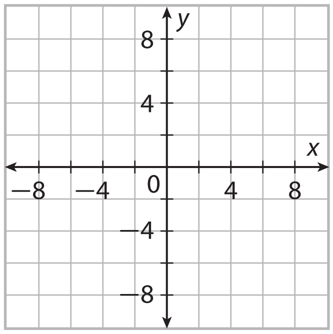
18. $g(x) = -2.5|x|$



19. $g(x) = \frac{1}{2}|x|$



20. $g(x) = -\frac{2}{3}|x|$



21. Identify the values of h, k, and a given each absolute value function.

A. $f(x) = 3|x - 2| + 2$

B. $f(x) = -0.2|x - 3| + 4$

C. $f(x) = -5|x + 6| - 1$

D. $f(x) = 0.5|x + 2| - 7$

E. $f(x) = 0.8|x| + 3$

22. **Body Temperature** The average body temperature of a human is generally accepted to be 98.6 °F. Complete the absolute value function below describing the difference $d(x)$ in degrees Fahrenheit of the temperature x of an individual human and the average temperature of a human. How is the graph of $d(x)$ related to the graph of the parent function $f(x) = |x|$?

$d(x) = |x - \square|$

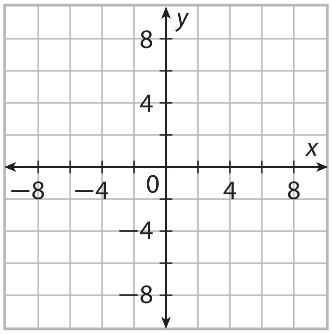
23. **Population Statistics** The average height of an American man is 69.3 inches. Complete the absolute value function below describing the difference $d(x)$ in inches of the height x of an individual American man and the average height of an American man. How is the graph of $d(x)$ related to the graph of the parent function $f(x) = |x|$?

$d(x) = |x - \square|$

H.O.T. Focus on Higher Order Thinking

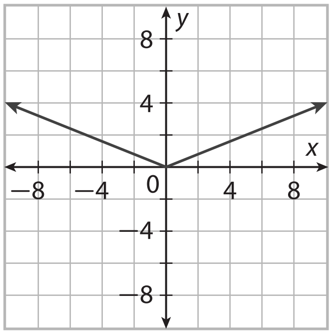
24. **Make a Prediction** Complete the table and graph all the functions on the same coordinate plane. How do the graphs of $f(x) = a|x|$ and $g(x) = |ax|$ compare?

x	-6	-3	0	3	6
$g(x) = \frac{1}{3}\lvert x\rvert$					
$g(x) = \left\lvert\frac{1}{3}x\right\rvert$					
$g(x) = -\frac{1}{3}\lvert x\rvert$					
$g(x) = \left\lvert-\frac{1}{3}x\right\rvert$					



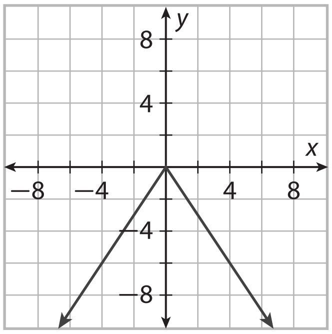
Multiple Representations **Write an equation for each absolute value function whose graph is shown.**

25.



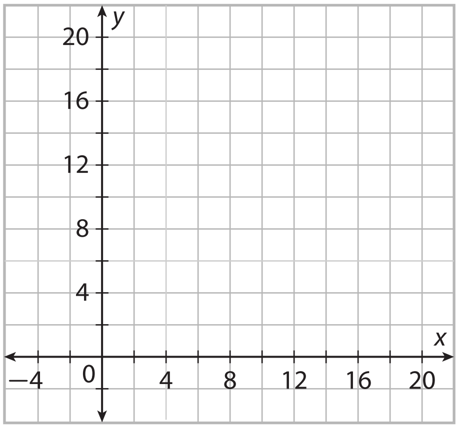
Note: One point on this graph is $(10, 4)$.

26.



Note: One point on this graph is $(6, -9)$.

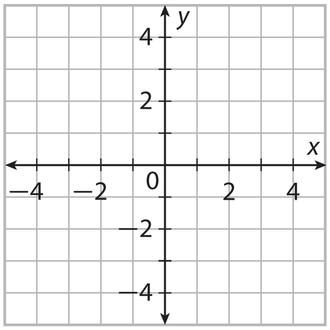
27. Represent Real-World Problems From his driveway at point $(4, 6)$, Kerry is adjusting his rearview mirror before backing out onto the street. The farthest object behind him to the left he can see is a neighbor's mailbox at $(-4, 12)$. The farthest object behind him to the right he can see is a telephone pole at $(20, 18)$. Create an absolute value function in the form $f(x) = a|x - h| + k$, with Kerry at the vertex, to represent the boundaries of Kerry's visual field in the rearview mirror. Graph the function, and label Kerry, the mailbox, and the telephone pole.



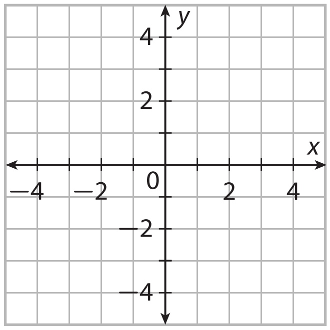
Lesson Performance Task

Geese are initially flying south in a V-shaped pattern that can be modeled by the absolute value function $f(x) = a|x - h| + k$, where a represents the growth or shrinkage of the distance between geese, k represents the height change of the flock, and h represents a left or right shift in the flock.

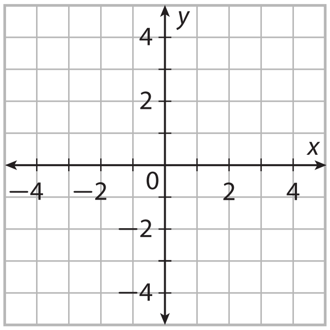
A. Graph the original flock function, $g(x) = |x|$. State the function's domain and range in words.



B. While flying south, the flock encounters a jet stream and is forced to drop 2 feet. Write the new equation and graph this function along with the original. State the new function's domain and range in words.



C. A short while after passing through the jet stream, the flock of geese encounters a rain storm and is forced to double the distance between each of its members in order to avoid colliding with one another. Write the new equation and graph all three functions. State the new function's domain and range in words.



Name ____________ Class ____________ Date ____________

13.3 Solving Absolute Value Equations

Essential Question: How can you solve an absolute value equation?

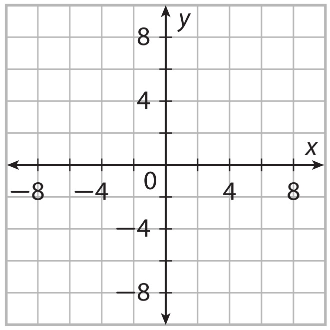
Resource Locker

Explore Solving Absolute Value Equations Graphically

Absolute value equations differ from linear equations in that they may have two solutions. This is indicated with a **disjunction**, a mathematical statement created by a connecting two other statements with the word "or." To see why there can be two solutions, you can solve an absolute value equation using graphs.

(A) Solve the equation $2|x - 5| - 4 = 2$.

Plot the function $f(x) = 2|x - 5| - 4$ on the grid. Then plot the function $g(x) = 2$ as a horizontal line on the same grid, and mark the points where the graphs intersect.



(B) Write the solution to this equation as a disjunction:

$x =$ ______ or $x =$ ______

Reflect

1. Why might you expect most absolute value equations to have two solutions? Why not three or four?

____

____

____

____

2. Is it possible for an absolute value equation to have no solutions? one solution? If so, what would each look like graphically?

____

____

____

____

Explain 1 Solving Absolute Value Equations Algebraically

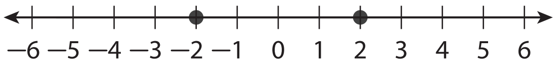
To solve absolute value equations algebraically, first isolate the absolute value expression on one side of the equation the same way you would isolate a variable. Then use the rule:

If $|x| = a$ (where a is a positive number), then $x = a$ OR $x = -a$.

Notice the use of a **disjunction** here in the rule for values of x. You cannot know from the original equation whether the expression inside the absolute value bars is positive or negative, so you must work through both possibilities to finish isolating x.

Example 1 **Solve each absolute value equation algebraically. Graph the solutions on a number line.**

Ⓐ $|3x| + 2 = 8$



Subtract 2 from both sides. $|3x| = 6$

Rewrite as two equations. $3x = 6$ or $3x = -6$

Solve for x. $x = 2$ or $x = -2$

Ⓑ $3|4x - 5| - 2 = 19$

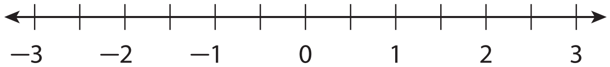
Add 2 to both sides. $3|4x - 5| = \square$

Divide both sides by 3. $|4x - 5| = \square$

Rewrite as two equations. $4x - 5 = \square$ or $4x - 5 = \square$

Add 5 to all four sides. $4x = \square$ or $4x = \square$

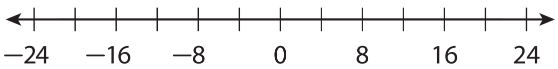
Solve for x. $x = \square$ or $x = \dfrac{\square}{\square}$



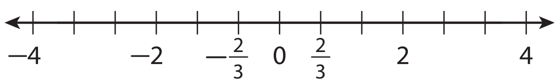
Your Turn

Solve each absolute value equation algebraically. Graph the solutions on a number line.

3. $\frac{1}{2}|x + 2| = 10$



4. $-2|3x - 6| + 5 = 1$



Explain 2 Absolute Value Equations with Fewer than Two Solutions

You have seen that absolute value equations have two solutions when the isolated absolute value expression is equal to a positive number. When the absolute value is equal to zero, there is a single solution because zero is its own opposite. When the absolute value expression is equal to a negative number, there is no solution because absolute value is never negative.

Example 2 **Isolate the absolute value expression in each equation to determine if the equation can be solved. If so, finish the solution. If not, write "no solution."**

(A) $-5|x + 1| + 2 = 12$

Subtract 2 from both sides.	$-5|x + 1| = 10$
Divide both sides by -5.	$|x + 1| = -2$
Absolute values are never negative.	No Solution

(B) $\frac{3}{5}|2x - 4| - 3 = -3$

Add 3 to both sides.	$\frac{3}{5}|2x - 4| =$ ☐
Multiply both sides by $\frac{5}{3}$.	$|2x - 4| =$ ☐
Rewrite as one equation.	$2x - 4 =$ ☐
Add 4 to both sides.	$2x =$ ☐
Divide both sides by 2.	$x =$ ☐

Your Turn

Isolate the absolute value expression in each equation to determine if the equation can be solved. If so, finish the solution. If not, write "no solution."

5. $3\left|\frac{1}{2}x + 5\right| + 7 = 5$

6. $9\left|\frac{4}{3}x - 2\right| + 7 = 7$

Elaborate

7. Why is important to solve both equations in the disjunction arising from an absolute value equation? Why not just pick one and solve it, knowing the solution for the variable will work when plugged backed into the equation?

8. **Discussion** Discuss how the range of the absolute value function differs from the range of a linear function. Graphically, how does this explain why a linear equation always has exactly one solution while an absolute value equation can have one, two, or no solutions?

9. **Essential Question Check-In** Describe, in your own words, the basic steps to solving absolute value equations and how many solutions to expect.

Evaluate: Homework and Practice

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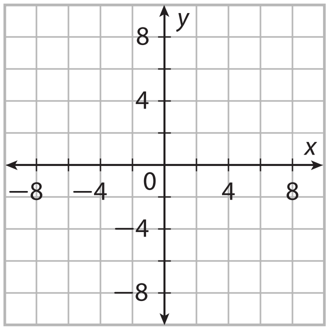
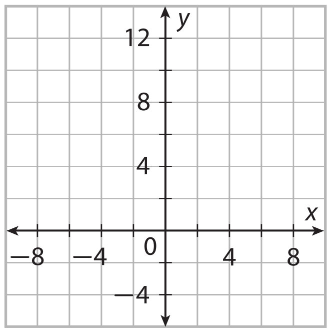
- Online Homework
- Hints and Help
- Extra Practice

Solve the following absolute value equations by graphing.

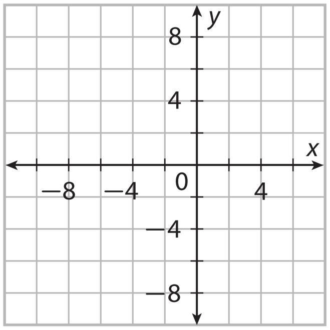
1. $|x - 3| + 2 = 5$



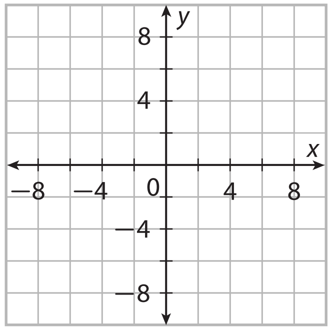
2. $2|x + 1| + 5 = 9$



3. $-2|x + 5| + 4 = 2$

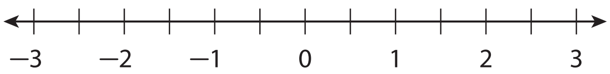


4. $\left|\frac{3}{2}(x - 2)\right| + 3 = 2$

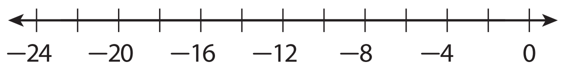


Solve each absolute value equation algebraically. Graph the solutions on a number line.

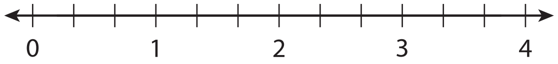
5. $|2x| = 3$



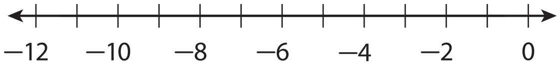
6. $\left|\frac{1}{3}x + 4\right| = 3$



7. $3|2x - 3| + 2 = 3$

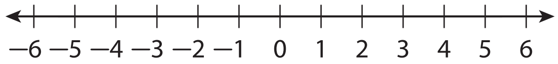


8. $-8|-x - 6| + 10 = 2$

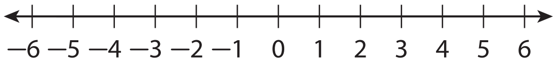


Isolate the absolute value expressions in the following equations to determine if they can be solved. If so, find and graph the solution(s). If not, write "no solution".

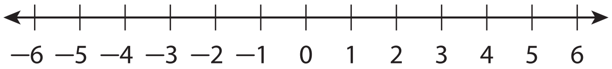
9. $\frac{1}{4}|x + 2| + 7 = 5$



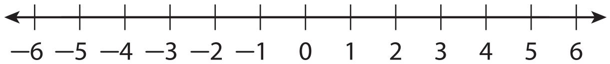
10. $-3|x - 3| + 3 = 6$



11. $2\left(|x + 4| + 3\right) = 6$

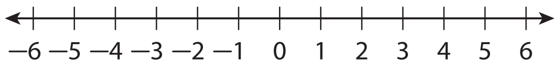


12. $5|2x + 4| - 3 = -3$

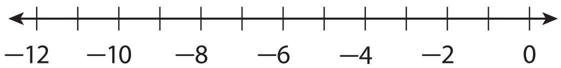


Solve the absolute value equations.

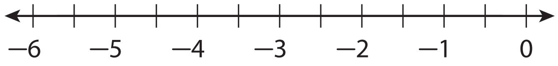
13. $|3x - 4| + 2 = 1$



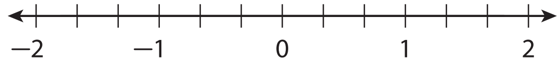
14. $7\left|\frac{1}{2}x + 3\frac{1}{2}\right| - 2 = 5$



15. $\left|2(x+5)-3\right|+2=6$

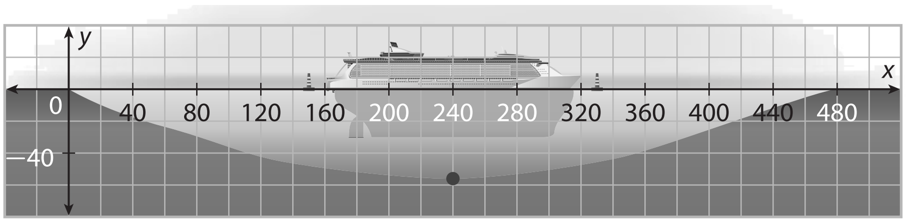


16. $-5\left|-3x+2\right|-2=-2$



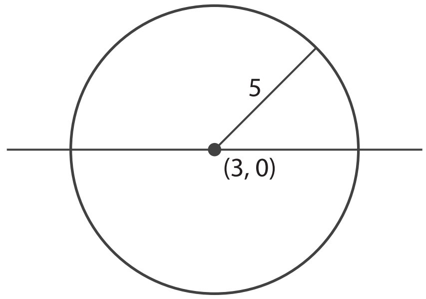
17. The bottom of a river makes a V-shape that can be modeled with the absolute value function, $d(h)=\frac{1}{5}\left|h-240\right|-48$, where d is the depth of the river bottom (in feet) and h is the horizontal distance to the left-hand shore (in feet).

A ship risks running aground if the bottom of its keel (its lowest point under the water) reaches down to the river bottom. Suppose you are the harbormaster and you want to place buoys where the river bottom is 30 feet below the surface. How far from the left-hand shore should you place the buoys?



18. A flock of geese is flying past a photographer in a V-formation that can be described using the absolute value function $b(d)=\frac{3}{2}\left|d-50\right|$, where $b(d)$ is the distance (in feet) of a goose behind the leader, and d is the distance from the photographer. If the flock reaches 27 feet behind the leader on both sides, find the distance of the nearest goose to the photographer.

19. **Geometry** Find the points where a circle centered at (3, 0) with a radius of 5 crosses the x-axis. Use an absolute value equation and the fact that all points on a circle are the same distance (the radius) from the center.



20. Select the value or values of x that satisfy the equation $-\frac{1}{2}|3x - 3| + 2 = 1$.

A. $x = \frac{5}{3}$ **B.** $x = -\frac{5}{3}$

C. $x = \frac{1}{3}$ **D.** $x = -\frac{1}{3}$

E. $x = 3$ **F.** $x = -3$

G. $x = 1$ **H.** $x = -1$

21. Terry is trying to place a satellite dish on the roof of his house at the recommended height of 30 feet. His house is 32 feet wide, and the height of the roof can be described by the function $h(x) = -\frac{3}{2}|x - 16| + 24$, where x is the distance along the width of the house. Where should Terry place the dish?

H.O.T. Focus on Higher Order Thinking

22. **Explain the Error** While attempting to solve the equation $-3|x - 4| - 4 = 3$, a student came up with the following results. Explain the error and find the correct solution:

$$-3|x - 4| - 4 = 3$$

$$-3|x - 4| = 7$$

$$|x - 4| = -\frac{7}{3}$$

$$x - 4 = -\frac{7}{3} \quad \text{or} \quad x - 4 = \frac{7}{3}$$

$$x = \frac{5}{3} \quad \text{or} \quad x = \frac{19}{3}$$

23. Communicate Mathematical Ideas Solve this absolute value equation and explain what algebraic properties make it possible to do so.

$3|x-2| = 5|x-2| - 7$

24. Justify Your Reasoning This absolute value equation has nested absolute values. Use your knowledge of solving absolute value equations to solve this equation. Justify the number of possible solutions.

$\left||2x+5|-3\right| = 10$

25. Check for Reasonableness For what type of real-world quantities would the negative answer for an absolute value equation not make sense?

Lesson Performance Task

A snowball comes apart as a child throws it north, resulting in two halves traveling away from the child. The child is standing 12 feet south and 6 feet east of the school door, along an east-west wall. One fragment flies off to the northeast, moving 2 feet east for every 5 feet north of travel, and the other moves 2 feet west for every 5 feet north of travel. Write an absolute value function that describes the northward position, $n(e)$, of both fragments as a function of how far east of the school door they are. How far apart are the fragments when they strike the wall?

Name ____________________ Class ____________ Date ________

13.4 Solving Absolute Value Inequalities

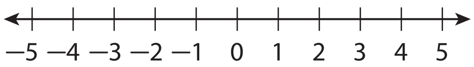
Resource Locker

Essential Question: What are two ways to solve an absolute value inequality?

Explore Visualizing the Solution Set of an Absolute Value Inequality

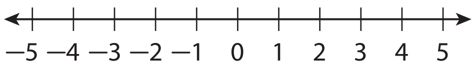
You know that when solving an absolute value equation, it's possible to get two solutions. Here, you will explore what happens when you solve absolute value inequalities.

(A) Determine whether each of the integers from −5 to 5 is a solution of the inequality $|x| + 2 < 5$. Write *yes* or *no* for each number in the table. If a number is a solution, plot it on the number line.



Number	Solution?
$x = -5$	
$x = -4$	
$x = -3$	
$x = -2$	
$x = -1$	
$x = 0$	
$x = 1$	
$x = 2$	
$x = 3$	
$x = 4$	
$x = 5$	

(B) Determine whether each of the integers from −5 to 5 is a solution of the inequality $|x| + 2 > 5$. Write *yes* or *no* for each number in the table. If a number is a solution, plot it on the number line.

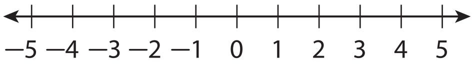


Number	Solution?
$x = -5$	
$x = -4$	
$x = -3$	
$x = -2$	
$x = -1$	
$x = 0$	
$x = 1$	
$x = 2$	
$x = 3$	
$x = 4$	
$x = 5$	

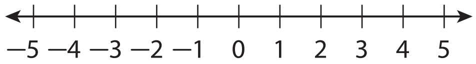
(C) State the solutions of the equation $|x| + 2 = 5$ and relate them to the solutions you found for the inequalities in Steps A and B.

(D) If x is any real number and not just an integer, graph the solutions of $|x| + 2 < 5$ and $|x| + 2 > 5$.

Graph of all real solutions of $|x| + 2 < 5$:



Graph of all real solutions of $|x| + 2 > 5$:



Reflect

1. It's possible to describe the solutions of $|x| + 2 < 5$ and $|x| + 2 > 5$ using inequalities that don't involve absolute value. For instance, you can write the solutions of $|x| + 2 < 5$ as $x > -3$ and $x < 3$. Notice that the word *and* is used because x must be both greater than −3 and less than 3. How would you write the solutions of $|x| + 2 > 5$? Explain.

2. Describe the solutions of $|x| + 2 \leq 5$ and $|x| + 2 \geq 5$ using inequalities that don't involve absolute value.

Explain 1 Solving Absolute Value Inequalities Graphically

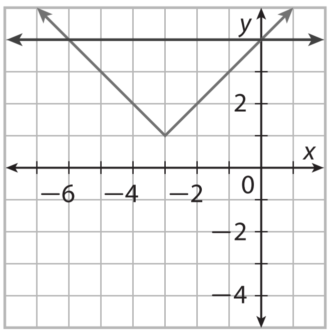
You can use a graph to solve an absolute value inequality of the form $f(x) > g(x)$ or $f(x) < g(x)$, where $f(x)$ is an absolute value function and $g(x)$ is a constant function. Graph each function separately on the same coordinate plane and determine the intervals on the x-axis where one graph lies above or below the other. For $f(x) > g(x)$, you want to find the x-values for which the graph $f(x)$ is above the graph of $g(x)$. For $f(x) < g(x)$, you want to find the x-values for which the graph of $f(x)$ is below the graph of $g(x)$.

Example 1 **Solve the inequality graphically.**

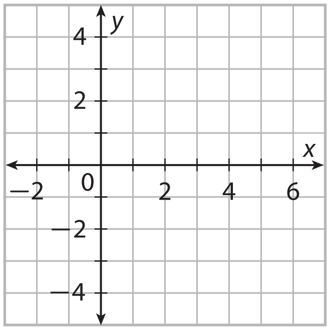
(A) $|x + 3| + 1 > 4$

The inequality is of the form $f(x) > g(x)$, so determine the intervals on the x-axis where the graph of $f(x) = |x + 3| + 1$ lies above the graph of $g(x) = 4$.

The graph of $f(x) = |x + 3| + 1$ lies above the graph of $g(x) = 4$ to the left of $x = -6$ and to the right of $x = 0$, so the solution of $|x + 3| + 1 > 4$ is $x < -6$ or $x > 0$.



(B) $|x-2|-3<1$



The inequality is of the form $f(x) < g(x)$, so determine the intervals on the x-axis where the graph of $f(x) = |x-2|-3$ lies ______ the graph of $g(x) = 1$.

The graph of $f(x) = |x-2|-3$ lies ______ the graph of $g(x) = 1$ between $x =$ ☐ and $x =$ ☐, so the solution of $|x-2|-3<1$ is $x >$ ☐ *and* $x <$ ☐.

Reflect

3. Suppose the inequality in Part A is $|x+3|+1 \geq 4$ instead of $|x+3|+1>4$. How does the solution change?

______

4. In Part B, what is another way to write the solution $x > -2$ and $x < 6$?

______

5. **Discussion** Suppose the graph of an absolute value function $f(x)$ lies entirely above the graph of the constant function $g(x)$. What is the solution of the inequality $f(x) > g(x)$? What is the solution of the inequality $f(x) < g(x)$?

______

______

______

______

Your Turn

6. Solve $|x+1|-4 \leq -2$ graphically.

Explain 2 Solving Absolute Value Inequalities Algebraically

To solve an absolute value inequality algebraically, start by isolating the absolute value expression. When the absolute value expression is by itself on one side of the inequality, apply one of the following rules to finish solving the inequality for the variable.

Solving Absolute Value Inequalities Algebraically
1. If $\lvert x \rvert > a$ where a is a positive number, then $x < -a$ or $x > a$.
2. If $\lvert x \rvert < a$ where a is a positive number, then $-a < x < a$.

Example 2 **Solve the inequality algebraically. Graph the solution on a number line.**

Ⓐ $\lvert 4 - x \rvert + 15 > 21$

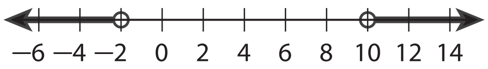
$\lvert 4 - x \rvert > 6$

$4 - x < -6$ or $4 - x > 6$

$-x < -10$ or $-x > 2$

$x > 10$ or $x < -2$

The solution is $x > 10$ or $x < -2$.



Ⓑ $\lvert x + 4 \rvert - 10 \leq -2$

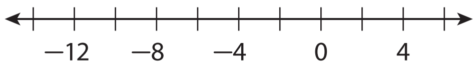
$\lvert x + 4 \rvert \leq \square$

$x + 4 \geq \square$ and $x + 4 \leq \square$

$x \geq \square$ and $x \leq \square$

The solution is $x \geq \square$ and $x \leq \square$,

or $\square \leq x \leq \square$.



Reflect

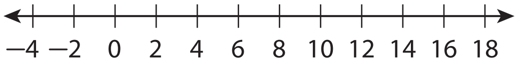
7. In Part A, suppose the inequality were $\lvert 4 - x \rvert + 15 > 14$ instead of $\lvert 4 - x \rvert + 15 > 21$. How would the solution change? Explain.

8. In Part B, suppose the inequality were $\lvert x + 4 \rvert - 10 \leq -11$ instead of $\lvert x + 4 \rvert - 10 \leq -2$. How would the solution change? Explain.

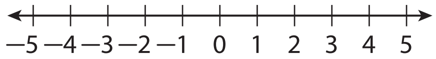
Your Turn

Solve the inequality algebraically. Graph the solution on a number line.

9. $3|x - 7| \geq 9$



10. $|2x + 3| < 5$



Explain 3 Solving a Real-World Problem with Absolute Value Inequalities

Absolute value inequalities are often used to model real-world situations involving a margin of error or *tolerance*. Tolerance is the allowable amount of variation in a quantity.

Example 3

A machine at a lumber mill cuts boards that are 3.25 meters long. It is acceptable for the length to differ from this value by at most 0.02 meters. Write and solve an absolute value inequality to find the range of acceptable lengths.

Analyze Information

Identify the important information.

- The boards being cut are $\square$ meters long.
- The length can differ by at most 0.02 meters.

Formulate a Plan

Let the length of a board be ℓ. Since the sign of the difference between ℓ and 3.25 doesn't matter, take the absolute value of the difference. Since the absolute value of the difference can be at most 0.02, the inequality that models the situation is

$\left|\ell - \square\right| \leq \square$.

Solve

$|\ell - 3.25| \leq 0.02$

$\ell - 3.25 \geq -0.02$ and $\ell - 3.25 \leq 0.02$

$\ell \geq \square$ and $\ell \leq \square$

So, the range of acceptable lengths is $\square \leq \ell \leq \square$.

Justify and Evaluate

The bounds of the range are positive and close to ☐, so this is a reasonable answer.

The answer is correct since ☐ $+ 0.02 = 3.25$ and ☐ $- 0.02 = 3.25$.

Your Turn

11. A box of cereal is supposed to weigh 13.8 oz, but it's acceptable for the weight to vary as much as 0.1 oz. Write and solve an absolute value inequality to find the range of acceptable weights.

Elaborate

12. Describe the values of x that satisfy the inequalities $|x| < a$ and $|x| > a$ where a is a positive constant.

13. How do you algebraically solve an absolute value inequality?

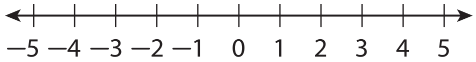
14. Explain why the solution of $|x| > a$ is all real numbers if a is a negative number.

15. **Essential Question Check-In** How do you solve an absolute value inequality graphically?

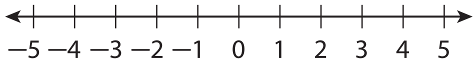
Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

1. Determine whether each of the integers from -5 to 5 is a solution of the inequality $|x-1|+3 \geq 5$. If a number is a solution, plot it on the number line.

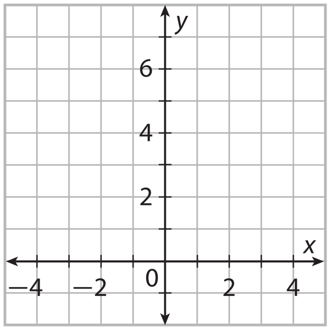


2. Determine whether each of the integers from -5 to 5 is a solution of the inequality $|x+1|-2 \leq 1$. If a number is a solution, plot it on the number line.

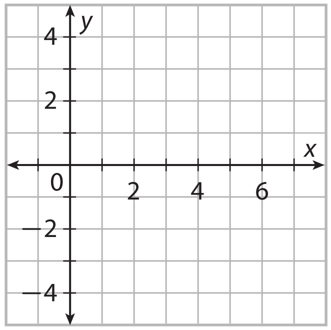


Solve each inequality graphically.

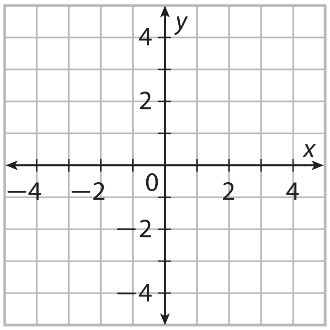
3. $2|x| \leq 6$



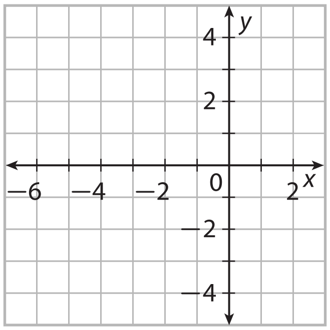
4. $|x-3|-2 > -1$



5. $\frac{1}{2}|x|+2 < 3$



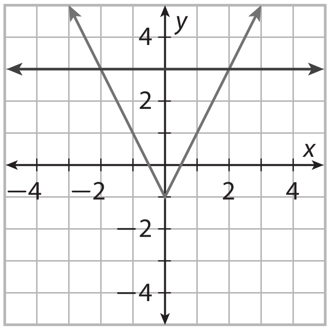
6. $|x+2|-4 \geq -2$



Match each graph with the corresponding absolute value inequality. Then give the solution of the inequality.

A. $2|x| + 1 > 3$ **B.** $2|x + 1| < 3$ **C.** $2|x| - 1 > 3$ **D.** $2|x - 1| < 3$

7.



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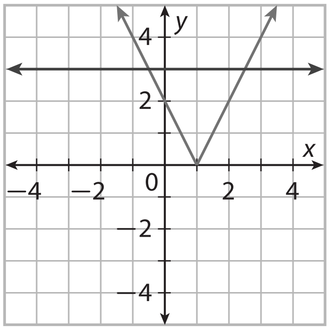

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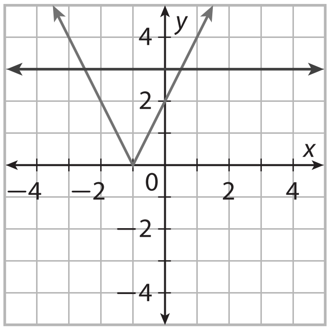

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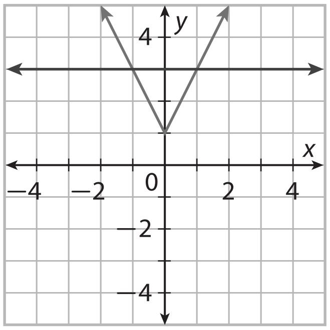
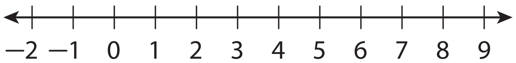


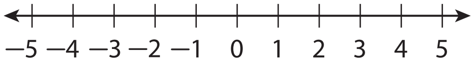
Solve each absolute value inequality algebraically. Graph the solution on a number line.

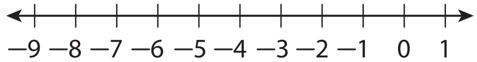
11. $2\left|x - \frac{7}{2}\right| + 3 > 4$



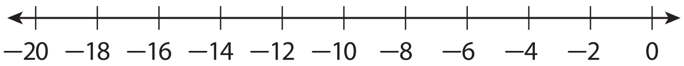
12. $|2x + 1| - 4 < 5$

13. $3|x + 4| + 2 \geq 5$

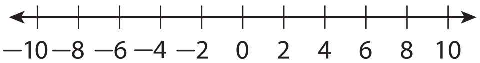




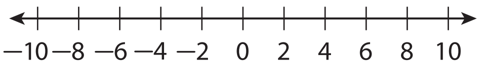
14. $|x + 11| - 8 \leq -3$



15. $-5|x - 3| - 5 < 15$



16. $8|x + 4| + 10 < 2$



Solve each problem using an absolute value inequality.

17. The thermostat for a house is set to 68 °F, but the actual temperature may vary by as much as 2 °F. What is the range of possible temperatures?

18. The balance of Jason's checking account is $320. The balance varies by as much as $80 each week. What are the possible balances of Jason's account?

19. On average, a squirrel lives to be 6.5 years old. The lifespan of a squirrel may vary by as much as 1.5 years. What is the range of ages that a squirrel lives?

20. You are playing a history quiz game where you must give the years of historical events. In order to score any points at all for a question about the year in which a man first stepped on the moon, your answer must be no more than 3 years away from the correct answer, 1969. What is the range of answers that allow you to score points?

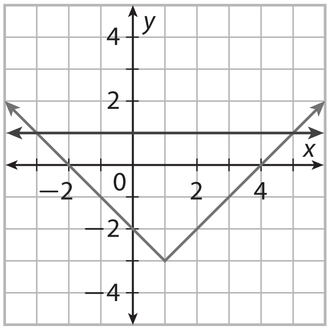
21. The speed limit on a road is 30 miles per hour. Drivers on this road typically vary their speed around the limit by as much as 5 miles per hour. What is the range of typical speeds on this road?

H.O.T. Focus on Higher Order Thinking

22. Represent Real-World Problems A poll of likely voters shows that the incumbent will get 51% of the vote in an upcoming election. Based on the number of voters polled, the results of the poll could be off by as much as 3 percentage points. What does this mean for the incumbent?

23. Explain the Error A student solved the inequality $|x - 1| - 3 > 1$ graphically. Identify and correct the student's error.

I graphed the functions $f(x) = |x - 1| - 3$ and $g(x) = 1$. Because the graph of $g(x)$ lies above the graph of $f(x)$ between $x = -3$ and $x = 5$, the solution of the inequality is $-3 < x < 5$.



24. Multi-Step Recall that a literal equation or inequality is one in which the constants have been replaced by letters.

a. Solve $|ax + b| > c$ for x. Write the solution in terms of a, b, and c. Assume that $a > 0$ and $c \geq 0$.

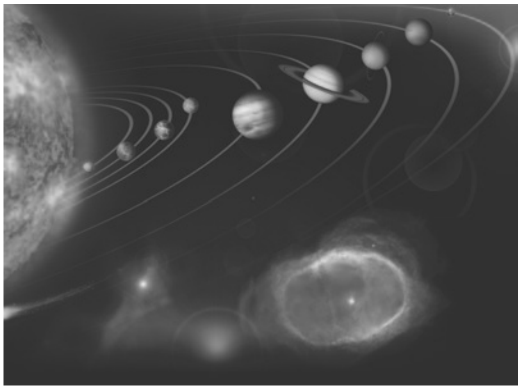
b. Use the solution of the literal inequality to find the solution of $|10x + 21| > 14$.

c. Explain why you must assume that $a > 0$ and $c \geq 0$ before you begin solving the literal inequality.

Lesson Performance Task

The distance between the Sun and each planet in our solar system varies because the planets travel in elliptical orbits around the Sun. Here is a table of the average distance and the variation in the distance for the five innermost planets in our solar system.

	Average Distance	Variation
Mercury	36 million miles	7.5 million miles
Venus	67.2 million miles	0.5 million miles
Earth	92.75 million miles	1.75 million miles
Mars	141 million miles	13 million miles
Jupiter	484 million miles	24 million miles



a. Write and solve an inequality to represent the range of distances that can occur between the Sun and each planet.

b. Calculate the percentage variation (variation divided by average distance) in the orbit of each of the planets. Based on these percentages, which planet has the most elliptical orbit?