

12.1 Creating Systems of Linear Equations



Resource
Locker

Essential Question: How do you use systems of linear equations to model and solve real-world problems?

Explore Creating Linear System Models by Changing Parameters

Investigate how a system of equations can help you compare and interpret situations where rates of change affect the outcome.

After leaving her hotel, a student is walking to a café to have breakfast and do some sightseeing. On the way, she passes two stores that rent bicycles. The first shop charges an initial fee of \$7.00 and \$2.75 for each hour. The second shop charges a flat fee of \$3.00 per hour. Over breakfast, the student needs to decide which rental agency to use. How should she start?



- (A) Begin by finding functions that represent the cost of each rental. Let $f(t)$ represent the cost of renting a bicycle for t hours from the first shop, and let $g(t)$ represent the cost of renting a bicycle from the second shop.
- (B) What is the initial cost of renting a bicycle from the first shop? _____
- (C) This represents the _____ of the model for the first business.
- (D) The slope of this linear model represents the _____ of the cost, as a function of time.
- (E) The slope of the first model is _____.
- (F) This makes $f(t) =$ _____.
- (G) Similarly, the function modeling the cost for renting a bicycle from the second shop is $g(t) =$ _____.
- (H) Once the student decides the length of time she plans to spend on her bike ride, she can

solve the linear system of the two functions, _____, to determine from which company she wants to rent a bicycle.

Reflect

1. **Discussion** Under what conditions would this type of real-world situation have no solution?

2. **Discussion** Under what conditions would this type of real-world situation have infinitely many solutions?



Explain 1

Creating Linear System Models from Verbal Descriptions

Often, a company will charge a start-up fee for its services, followed by a monthly or per unit cost. This can be written as a linear function in slope-intercept form.

When the costs of the same services from two different companies must be compared, the variable amount for each must represent the same thing, and both models should produce values with the same type of unit. For example, if one function models yearly income in terms of thousands of dollars and the other function models monthly income in terms of hundreds of dollars, the comparison will not be accurate.

Example 1 Determine when the cost of the two services will be the same amount, and what the price will be.

Video streaming service Atomic Stream charges \$10 for membership and \$1.00 for each movie download. Blitz Video charges \$15 for a membership and \$0.50 per movie download. How many movies would you need to download for the services to have identical costs? What is that cost?



Analyze Information

Identify the important information.

Atomic Stream has a _____ membership fee.

Atomic Stream has a _____ per download fee.

Blitz Video has a _____ membership fee.

Blitz Video has a _____ per download fee.



Formulate a Plan

Create two functions to model the cost of each service, $A(x)$ and $B(x)$, where x represents the _____.

The solution can be found by setting up an equation so that the function

$A(x)$ is _____ to the function $B(x)$ and then solving for _____.



Solve

The model for Atomic Stream is $A(x) = \boxed{}$.

The model for Blitz Video is $B(x) = \boxed{}$.

The two functions are $\left\{ \begin{array}{l} \\ \end{array} \right.$.

Solve using substitution. You can use substitution because you are solving for the value where $A(x) \boxed{} B(x)$.

$$\boxed{} = \boxed{}$$

$$0.5x = 5$$

$$x = \boxed{}$$

$$A(x) = \boxed{}$$

$$A(\boxed{}) = \boxed{}$$

$$= \boxed{}$$

The cost of each service is _____ when _____ movies are screened.



Justify and Evaluate

It is reasonable to expect the cost of the services to be the same after a number of uses. The businesses are in the same market but can appeal to different customers.

Atomic Stream is more affordable for customers who stream _____

movies a month, while Blitz Video is a better deal for people who stream _____ movies a month.

Your Turn

- One cable television provider has a \$60 setup fee and charges \$80 per month, and another cable provider has a \$160 equipment fee and charges \$70 per month.
- The Strauss family is deciding between two lawn-care services. Green Lawn charges a \$49 startup fee plus \$29 per month. Yard Guard charges a \$25 startup fee plus \$37 per month.



Explain 2 Creating Linear System Models from Tables

Sometimes, there is not enough information to model an equation. Businesses may have a table of rates posted to explain their pricing. To compare the cost of items or services from two or more businesses, table entries can be used to create and solve a linear system.

Example 2 Use the cost tables for two services to create a linear system of equations. Then solve the system to determine when the cost of the two services will be equal.

A Two garden supply companies deliver mulch according to the following table.

Mulch (Cubic Yards)	Yard Depot	Lawn & Garden
1	\$60	\$80
2	\$90	\$105
3	\$120	\$130
4	\$150	\$155

Yard Depot	Lawn & Garden
Use points (1, 60) and (2, 90). $m = \frac{90 - 60}{2 - 1} = 30$ Write the equation. $y - 60 = 30(x - 1)$ $y = 30x + 30$	Use points (1, 80) and (2, 105). $m = \frac{105 - 80}{2 - 1} = 25$ Write the equation. $y - 80 = 25(x - 1)$ $y = 25x + 55$

The system of equations is: $\begin{cases} f(x) = 30x + 30 \\ g(x) = 25x + 55 \end{cases}$

Solve for x when $f(x) = g(x)$ to find the amount of cubic yards, x , for which both companies charge the same amount.

$$30x + 30 = 25x + 55$$

$$x = 5 \Rightarrow f(5) = 30(5) + 30 = 180$$

Both companies charge \$180 for 5 cubic yards of mulch.

- B The table shows canoe rental prices for two companies.

Time t (in hours)	Canoe Depot	Paddle and Oar
1	\$14	\$20
2	\$19	\$23
3	\$24	\$26

Canoe Depot	Paddle & Oar
Use points $(1, \square)$ and $(2, \square)$.	Use points $(1, \square)$ and $(2, \square)$.
$m = \frac{\square - \square}{2 - 1} = \square$	$m = \frac{\square - \square}{2 - 1} = \square$
Write the equation.	Write the equation.
$y - \square = \square(x - 1)$	$y - \square = \square(x - 1)$
$y = \square x + \square$	$y = \square x + \square$

The system of equations is $\begin{cases} C(x) = \square x + \square \\ P(x) = \square x + \square \end{cases}$

Solve for x when $C(x) = P(x)$ to find the number of hours, x , for which both canoe rental places charge the same amount.

$$\square x + \square = \square x + \square$$

$$x = \square \Rightarrow C(\square) = \square$$

Both companies charge \$ \square for \square hours of canoe rental.

Your Turn

5. Two garden supply companies deliver pea stone according to the following table.

Pea Stone x (in cubic yards)	Yard Depot	Lawn & Garden
1	\$75	\$45
2	\$110	\$85
3	\$145	\$125

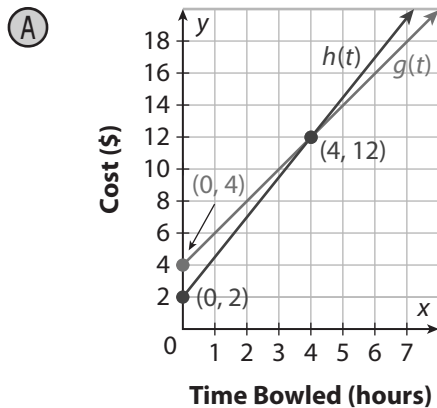
6. Two beachfront stores rent surfboards according to the following table.

Time t (in hours)	Hang Ten	Waverider
1	\$28	\$46
2	\$48	\$63
3	\$68	\$80
4	\$88	\$97

Explain 3 Creating Linear System Models from Graphs

In newspapers and magazines, information is often displayed in the form of a graph. You can use the graph of a linear system to write the function models that are represented.

Example 3 Use the graph to make a linear model of each function. Describe the meaning of the terms in the models. Then create the linear system, and state what the solution represents.

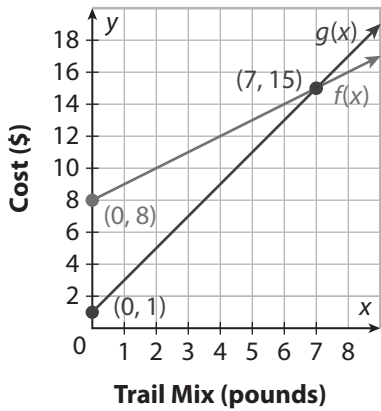


$g(x)$	$h(x)$
The y-intercept b is 4. Initial cost is \$4. Use $(0, 4)$ and $(4, 12)$:	The y-intercept b is 2. Initial cost is \$2. Use $(0, 2)$ and $(4, 12)$:
$m = \frac{12 - 4}{4 - 0} = 2$	$m = \frac{12 - 2}{4 - 0} = 2.5$
Charge per hour: \$2.00/h $y = 2x + 4$	Charge per hour: \$2.50/h $y = 2.5x + 2$

The system of equations is:
$$\begin{cases} g(x) = 2x + 4 \\ h(x) = 2.5x + 2 \end{cases}$$

The solution $(4, 12)$ represents the same charge of \$12 for 4 hours that both bowling alleys charge.

B



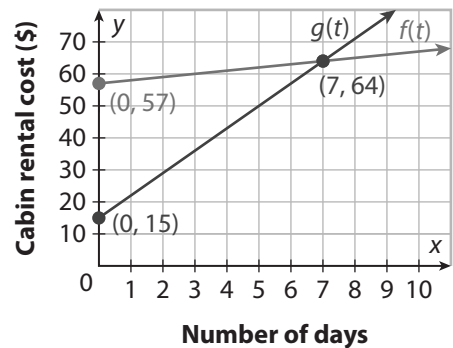
$f(x)$	$g(x)$
The y-intercept b is <input type="text"/> .	The y-intercept b is <input type="text"/> .
Initial cost is \$ <input type="text"/> .	Initial cost is \$ <input type="text"/> .
Use $(0, \text{>})$ and $(7, 15)$:	Use $(0, \text{>})$ and $(7, 15)$:
$m = \frac{15 - \text{>}}{\text{>} - 0} = \text{>}$	$m = \frac{15 - \text{>}}{\text{>} - 0} = \text{>}$
Rate of change: \$ <input type="text"/> /lb	Rate of change: \$ <input type="text"/> /lb
$y = \text{>}x + \text{>}$	$y = \text{>}x + \text{>}$

The system of equations is $\begin{cases} f(x) = \text{>}x + \text{>} \\ g(x) = \text{>}x + \text{>} \end{cases}$

The solution $(7, 15)$ represents the same charge of \$ for pounds of trail mix.

Your Turn

7. Use the graph to make a linear model of each function. Describe the meaning of terms in the models. Then create the linear system and state what the solution represents.



Elaborate

8. When writing a linear model of a situation, what does the slope represent?

9. **Discussion** Compare and contrast the system of equations that can be determined from a verbal description of a relationship, a table of values, and a graph.

10. **Essential Question Check-In** How do you use systems of linear equations to model and solve real-world problems?

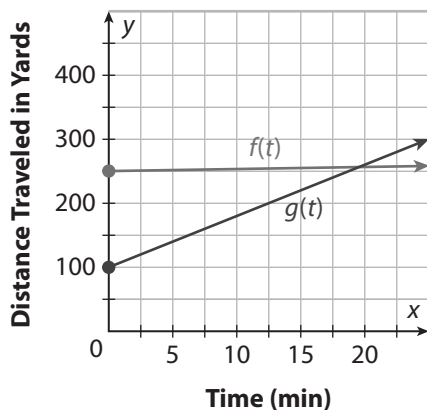


Evaluate: Homework and Practice



- Online Homework
- Hints and Help
- Extra Practice

1. In the graph shown, what do the parameters of each line represent?



Set up and solve a system of equations to solve the problem.

2. Casey wants to buy a gym membership. One gym has a \$150 joining fee and costs \$35 per month. Another gym has no joining fee and costs \$60 per month. When would Casey pay the same amount to be a member of either gym? How much would he pay?

3. A jar contains n nickels and d dimes. There are 20 coins in the jar, and the total value of the coins is \$1.40. How many nickels and how many dimes are in the jar?

4. Helene invested a total of \$1000 in two simple-interest bank accounts. One account paid 5% annual interest; the other paid 6% annual interest. The total amount of interest she earned after 1 year was \$58. Find the amount invested in each account.

5. A local boys club sold 176 bags of mulch and made a total of \$520. It sold two types of mulch: hardwood for \$3.50 a bag and pine bark for \$2.75 a bag. How many bags of each kind of mulch did it sell?

6. The school band sells carnations on Valentine's Day for \$2 each. It buys the carnations from a florist for \$0.50 each, plus a \$16 delivery charge. When will the cost of the carnations be equal to the revenue from selling them? How many carnations does it need to sell to reach this point?



Use the given cost tables for the same product from two different companies to create a linear system. Then solve the system to determine when the cost of the product will be the same and what the price will be.

7. Two online spice retailers sell paprika by the pound using the following pricing chart.

Paprika(lb)	iSpice	Spice Magic
1	\$15.75	\$26.25
2	\$27.50	\$36.50
3	\$39.25	\$46.75
4	\$51	\$57

8. Two online retailers sell organic vanilla extract by the ounce using the following pricing chart.

Vanilla Extract (oz)	Chef Mate	Grocery Gourmet
2	\$12.50	\$17
3	\$17.25	\$21
4	\$22	\$25
5	\$26.75	\$29

9. Two dry cleaning companies offer a home pick-up and delivery service. The monthly cost depends on the number of garments laundered, and is shown in the following table.

Number of Garments	Company 1	Company 2
5	\$55.25	\$31.25
10	\$75.50	\$57.50
15	\$95.75	\$83.75

10. A small town in the mountains needs to buy road salt for the coming winter. It has found two companies that use the following pricing table.

Road Salt (tons)	Company 1	Company 2
5	\$1775	\$2750
10	\$3350	\$4000
15	\$4925	\$5250



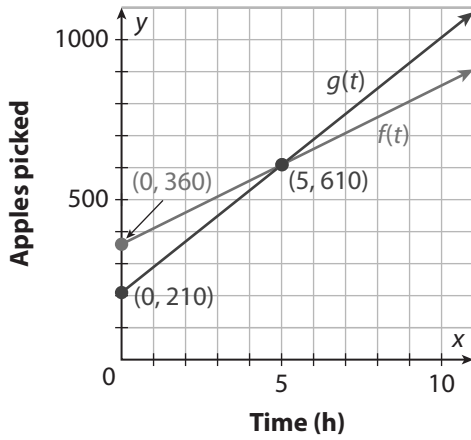
11. A restaurant needs to stock paper towels in its kitchen and bathrooms. It has found two vendors using the following case price chart.

Paper Towels (cases)	Restaurant Warehouse	Supply Side
5	\$300.20	\$220.20
10	\$480.15	\$420.15
15	\$660.10	\$620.10

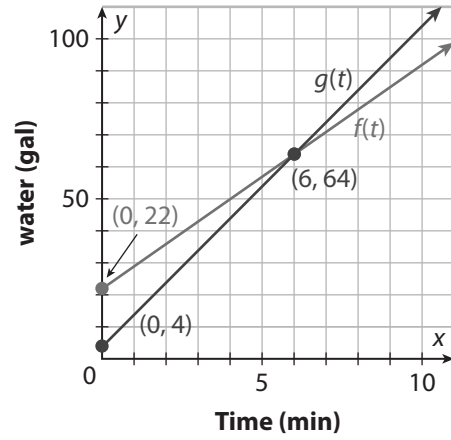
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Use the graph to make a linear model of each function. Describe the meaning of the terms in the models. Then create the linear system, and state what the solution represents.

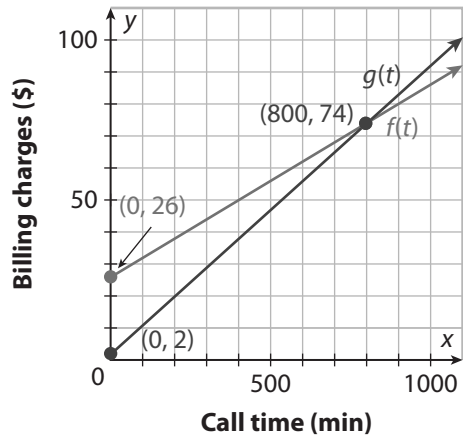
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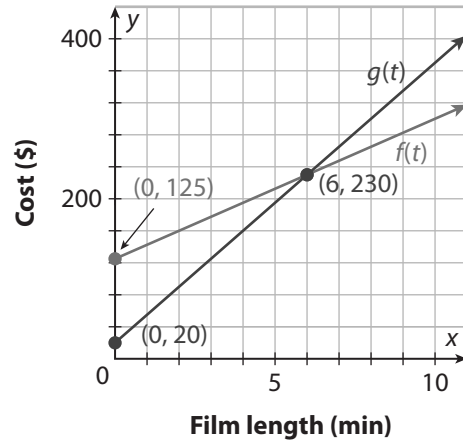
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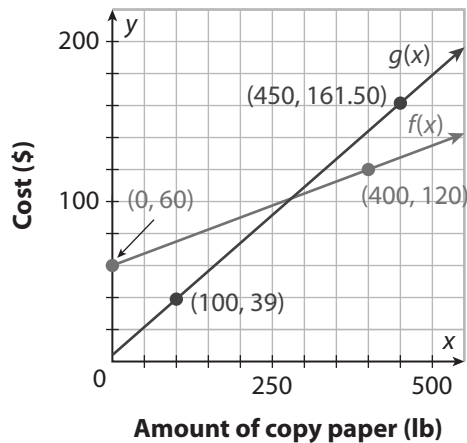
14.



15.



16. Two office supply stores sell their brand of copy paper by the pound. One company offers a flat rate shipping charge and the other offers free shipping. Use the graph provided to construct a linear system to model this situation. Solve the system to determine the amount of copy paper for which the cost is the same at both stores. Use the graph to verify that your answer is reasonable.



17. **Find the Error** A student is given the following problem:

A painter can buy 5-gallon containers of paint from two different stores based on the following pricing table.

Containers of Paint	Company A	Company B
2	\$512.00	\$422.00
4	\$904.00	\$834.00
6	\$1296.00	\$1246.00
8	\$1688.00	\$1658.00

The student's work is shown.

$$\text{Company A: } m = \frac{904 - 512}{4 - 2} = \frac{392}{2} = 196 \quad \text{Company B: } m = \frac{1246 - 834}{6 - 2} = \frac{412}{4} = 103$$

$$A(x) = 196x + b$$

$$B(x) = 103x + b$$

$$512 = 196(2) + b$$

$$834 = 103(2) + b$$

$$120 = b$$

$$628 = b$$

$$\begin{cases} A(x) = 196x + 120 \\ B(x) = 103x + 628 \end{cases}$$

$$196x + 120 = 103x + 628$$

$$93x = 508$$

$$x = 5.5$$

He concludes that the answer is unreasonable because both of the prices for Company A at $x = 4$ and $x = 6$ are greater than those of Company B. After the two functions intersect, the prices for Company B should be lower than the prices for Company A.

Is the student correct? If not, explain his error and find the correct solution.

H.O.T. Focus on Higher Order Thinking

- 18.** Two friends, Jorge and Mark, are taking a trip to the mountains for a camping trip, but they are not leaving together. Both friends separately record the distance they each have traveled from home every hour on the second day. The distance each friend is from his home can be modeled by a linear function of the hours spent traveling. Determine if the linear system created by the models for each situation has a unique solution with a positive value for time.
- a. Jorge and Mark each leave at the same time. Yes No
- b. Jorge travels 25 miles on the first day and drives 65 miles per hour on the second day. Mark travels 100 miles on the first day and also drives 65 miles per hour on the second day. Yes No
- c. Jorge travels 25 miles on the first day and drives 65 miles per hour on the second day. Mark travels 100 miles on the first day and drives 45 miles per hour on the second day. Yes No
- d. Jorge traveled 45 miles on the first day and drives 55 miles per hour on the second day. Mark arrived at the campsite on the first day after traveling 300 miles. Yes No
- e. Jorge gets sick before the trip and doesn't get in touch with Mark. Mark travels 40 miles on the first day and drives 67 miles per hour on the second day. Yes No
- 19. Communicate Mathematical Ideas** Given a set of data measuring the distance two planes have traveled after takeoff as a function of when they both passed over the same point, how would you find when they have both traveled the same distance since takeoff?
- 20. Analyze Relationships** How can you use the slope and the y -intercept of each model in a linear system to determine whether or not there will be a solution?

Lesson Performance Task

A family is going on vacation and they need to bring their dog to a kennel. Alpha Kennel charges an initial fee of \$75 and a daily rate of \$30. Beta Kennel charges a flat fee of \$34.95 a day. Find linear functions modeling the cost of boarding a dog for n days in each kennel. Set up and solve a system of linear equations. Then interpret the solution.

What if the family has two dogs? Alpha Kennel runs a special where you receive a 10% discount if you board more than one pet. Modify the linear models to give the price of boarding two dogs. Set up, solve, and interpret the linear system covering this case.



12.2 Graphing Systems of Linear Inequalities



Resource
Locker

Essential Question: How do you solve a system of linear inequalities?

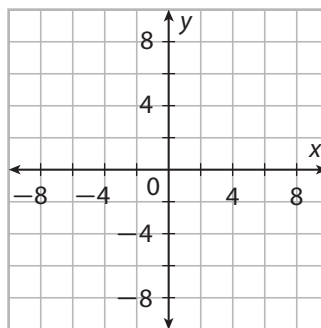
Explore Determining Solutions of Systems of Linear Inequalities

A **system of linear inequalities** consists of two or more linear inequalities that have the same variables. The **solutions of a system of linear inequalities** are all the ordered pairs that make all the inequalities in the system true.

Solve the system of equations by graphing.

$$\begin{cases} x + 3y > 3 \\ -x + y \leq 6 \end{cases}$$

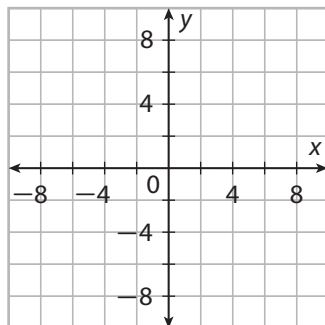
- (A) First look at $x + 3y > 3$. The equation of the boundary line is _____.
- (B) What are the x - and y -intercepts?
- (C) The inequality symbol is $>$ so use a _____ line.
- (D) Shade _____ the boundary line for solutions that are greater than the inequality.
- (E) Graph $x + 3y > 3$.



- (F) Look at $-x + y \leq 6$. The equation of the boundary line is _____.
- (G) What are the x - and y -intercepts?
- (H) The inequality symbol is \leq so use a _____ line.

I Shade _____ the boundary line for solutions that are less than the inequality.

J Graph $-x + y \leq 6$ on the same graph as $x + 3y > 3$.



K Identify the solutions. They are represented by the _____ shaded regions.

L Check your answer by using a point in each region. Complete the table.

Ordered Pair	Satisfies $x + 3y > 3$?	Satisfies $-x + y \leq 6$?	In the overlapping shaded regions?
(0, 0)			
(2, 3)			
(-8, 2)			
(-4, 6)			

Reflect

1. **Discussion** Why is (0, 0) a good point to use for checking the answer to this system of linear inequalities?

Explain 1 Solving Systems of Linear Inequalities by Graphing

You can use a graph of a system of linear inequalities to determine and identify solutions to the system of linear inequalities.

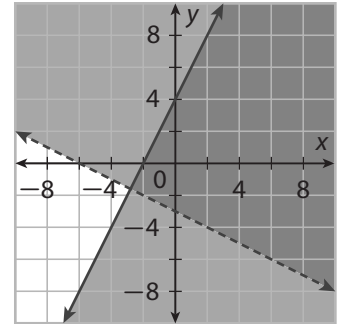
Example 1 Graph the system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

A
$$\begin{cases} -6x + 3y \leq 12 \\ y > \frac{1}{2}x - 3 \end{cases}$$

Solve the first inequality for y . Graph the system.

$$\begin{aligned} -6x + 3y &\leq 12 \\ 3y &\leq 6x + 12 \\ y &\leq 2x + 4 \end{aligned} \quad \begin{cases} y \leq 2x + 4 \\ y > \frac{1}{2}x - 3 \end{cases}$$

$(0, 0)$ and $(2, 8)$ are solutions. $(-6, -4)$ and $(-4, 4)$ are not solutions.

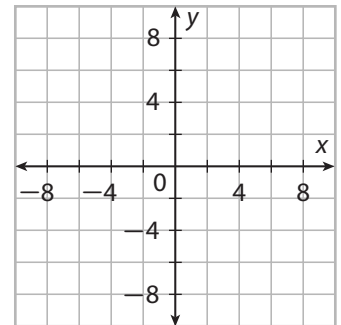


B
$$\begin{cases} 3x + y \leq 1 \\ y > \frac{2}{3}x - 2 \end{cases}$$

Solve the first inequality for y . Graph the system.

$$\begin{aligned} 3x + y &\leq 1 \\ y &\leq \boxed{} \end{aligned} \quad \begin{cases} y \leq \boxed{} \\ y > \frac{2}{3}x - 2 \end{cases}$$

_____ and _____ are solutions. _____ and _____ are not solutions.



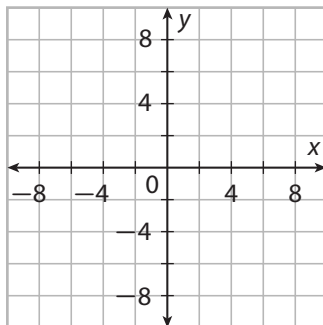
Reflect

2. Is $(-6, -6)$ a solution of the system?

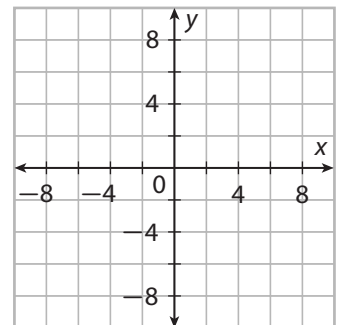
Your Turn

Graph the system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

3.
$$\begin{cases} y \leq x + 3 \\ y < -3 \end{cases}$$



4.
$$\begin{cases} y > x - 8 \\ 2x + 4y < 16 \end{cases}$$





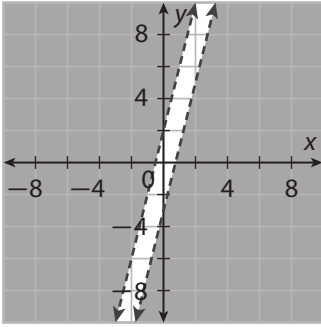
Explain 2

Graphing Systems of Inequalities with Parallel Boundary Lines

If the lines in a system of linear equations are parallel, there are no solutions. However, if the boundary lines in a system of linear inequalities are parallel, the system may or may not have solutions.

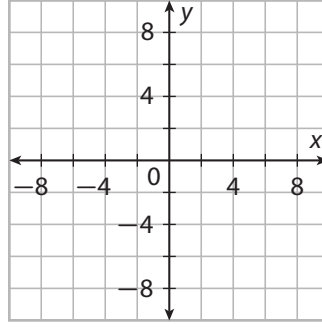
Example 2 Graph each system of linear inequalities. Describe the solutions.

(A)
$$\begin{cases} y < 4x - 3 \\ y > 4x + 2 \end{cases}$$



This system has no solution.

(B)
$$\begin{cases} y > x - 2 \\ y \leq x + 4 \end{cases}$$

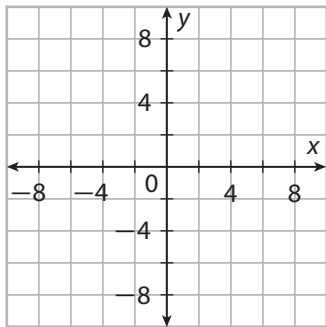


The solutions are all points _____ the parallel lines and on the _____ line.

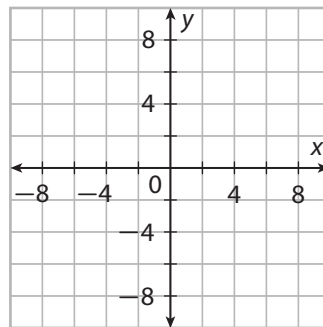
Your Turn

Graph each system of linear inequalities. Describe the solutions.

5.
$$\begin{cases} y \leq -2x - 3 \\ y \leq -2x + 1 \end{cases}$$



6.
$$\begin{cases} y < \frac{1}{3}x - 6 \\ y \geq \frac{1}{3}x + 5 \end{cases}$$



Elaborate

7. Is it possible for a system of two linear inequalities to have every point in the plane as solutions? Why or why not?

8. **Discussion** How would you write a system of linear inequalities from a graph?

9. **Essential Question Check-In** How does testing specific ordered pairs tell you that the solution you graphed is correct?



Evaluate: Homework and Practice



- Online Homework
- Hints and Help
- Extra Practice

1. Match the inequality with the correct boundary line. Answers may be used more than once.

- | | | |
|---------------------------|-------|-------------------------------------|
| a. $y = 3x$ | _____ | $-x + 3y \leq 0$ |
| b. $y = \frac{1}{3}x$ | _____ | $y > -x + \frac{1}{2}$ |
| c. $y = x - 0.5$ | _____ | $y \leq \frac{1}{3}x$ |
| d. $y = -x + \frac{1}{2}$ | _____ | $\frac{2}{3} + \frac{1}{3}y \geq x$ |
| e. $y = 3x - 2$ | _____ | $-y > x - 0.5$ |
| f. $y = x$ | _____ | $\frac{1}{3}y \geq x$ |

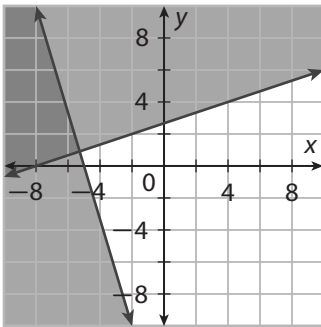
Determine if the given point satisfies either equation and is a solution of the system of inequalities.

2.
$$\begin{cases} 4y - 20x < 6 \\ \frac{5}{2}y \geq 5x - 10 \end{cases}; (0, 0)$$

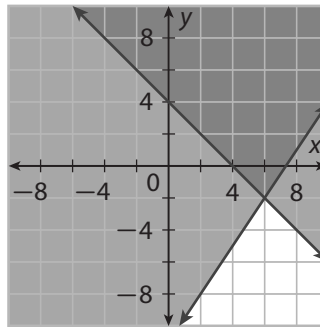
3.
$$\begin{cases} x + 5y > -10 \\ x - y \leq 4 \end{cases}; (2.5, -1.5)$$

Determine if the given point is a solution of the system of inequalities. If not, find a point that is.

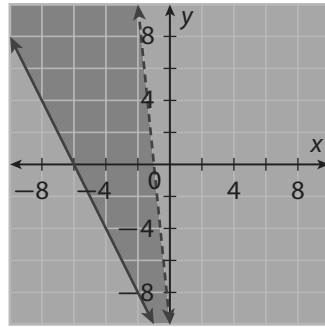
4. $(-9, 4)$



5. $(6, -2)$

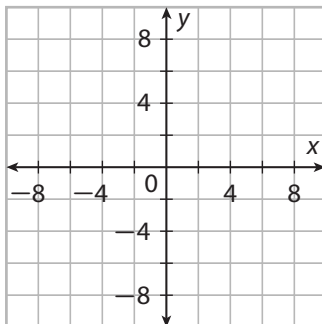


6. $(0, -4)$

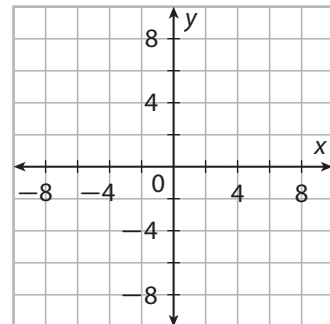


Graph the system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

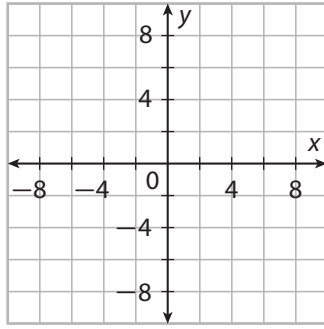
7.
$$\begin{cases} x > 2 \\ y \leq -\frac{1}{2}x - 2 \end{cases}$$



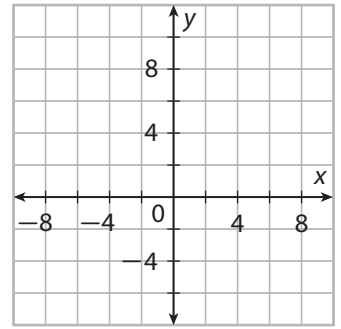
8.
$$\begin{cases} y > -x \\ y \geq x \end{cases}$$



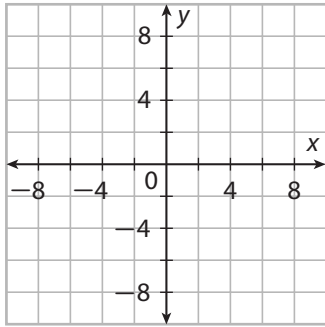
9.
$$\begin{cases} y < -x + 10 \\ y < \frac{1}{10}x + 7 \end{cases}$$



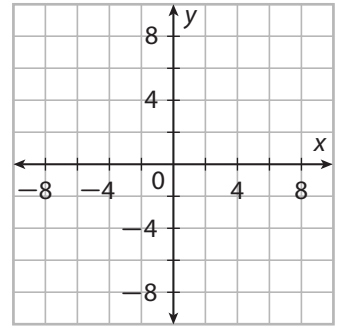
10.
$$\begin{cases} y \leq \frac{1}{2}x - 5 \\ y \geq -2x + 12 \end{cases}$$



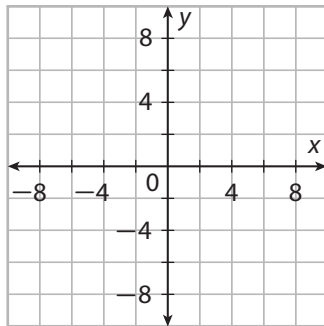
11.
$$\begin{cases} y \leq -\frac{3}{5}x \\ y > -x - 4 \end{cases}$$



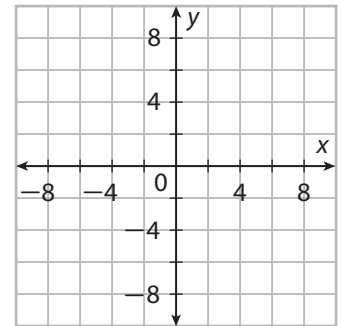
12.
$$\begin{cases} y \geq 2x + 6 \\ y < -\frac{1}{2}x - 1 \end{cases}$$



13.
$$\begin{cases} y \leq \frac{4}{5}x - 4 \\ y < 2x - 8 \end{cases}$$

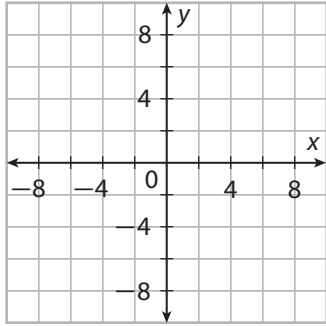


14.
$$\begin{cases} x \geq -6 \\ y < 3 \end{cases}$$

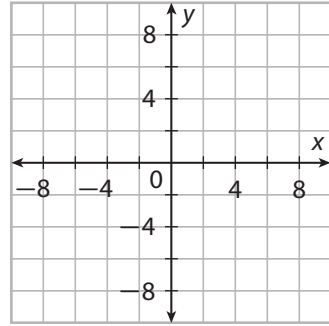


Graph each system of linear inequalities. Describe the solutions.

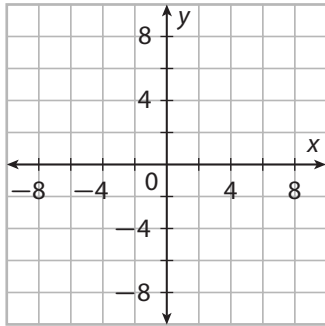
15.
$$\begin{cases} y \leq 3x + 6 \\ y < 3x - 8 \end{cases}$$



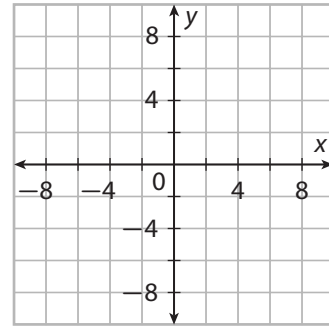
16.
$$\begin{cases} y \geq \frac{2}{5}x + 4 \\ y \leq \frac{2}{5}x - 6 \end{cases}$$



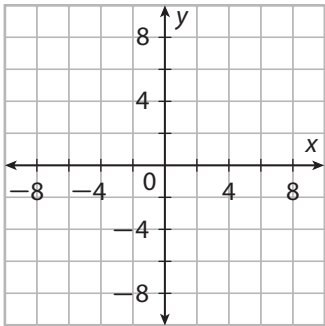
17.
$$\begin{cases} y \geq \frac{5}{4}x - 6 \\ y \geq \frac{5}{4}x \end{cases}$$



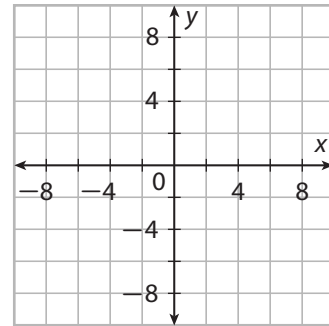
18.
$$\begin{cases} y \geq -\frac{3}{2}x - 3 \\ y \leq -\frac{3}{2}x + 10 \end{cases}$$



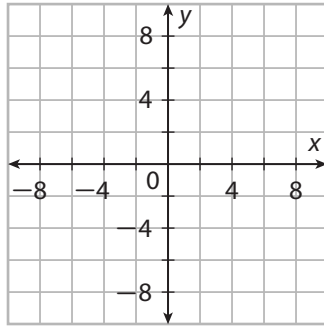
19.
$$\begin{cases} x < 6 \\ x \geq -3 \end{cases}$$



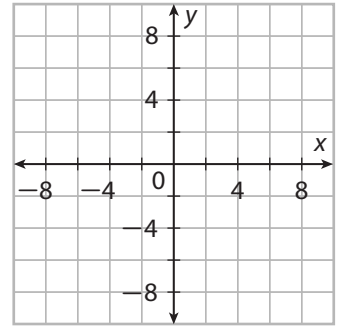
20.
$$\begin{cases} y \geq \frac{9}{4}x - 1 \\ y < \frac{9}{4}x - 9 \end{cases}$$



21.
$$\begin{cases} y < -\frac{3}{5}x + 3 \\ y \geq -\frac{3}{5}x - 4 \end{cases}$$

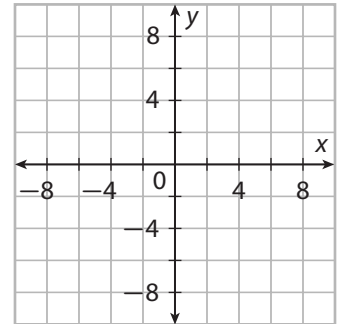


22.
$$\begin{cases} y > -\frac{1}{2}x + 5 \\ y > -\frac{1}{2}x - 1 \end{cases}$$



H.O.T. Focus on Higher Order Thinking

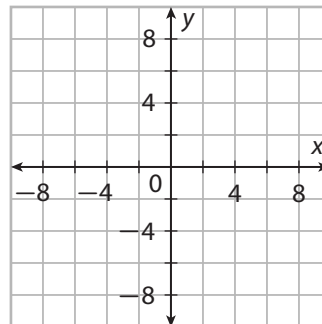
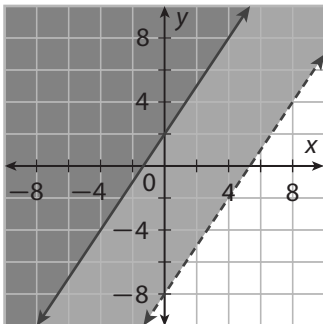
23. **Persevere in Problem Solving** Write and graph a system of linear inequalities for which the solutions are all the points in the second quadrant, not including points on the axes.



24. **Critical Thinking** Can the solutions of a system of linear inequalities be the points on a line? Explain.

25. **Explain the Error** A student was asked to graph the system $\begin{cases} y < \frac{3}{2}x - 8 \\ y \leq \frac{3}{2}x + 2 \end{cases}$ and describe the solution set. The student gave the following answer. Explain what the student did wrong, then give the correct answer.

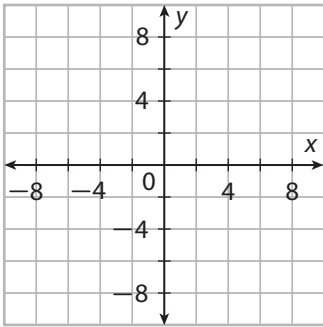
The solutions are the same as the solutions of $y \leq \frac{3}{2}x + 2$.



Lesson Performance Task

Successful stock market investors know a lot about inequalities. They know up to what point they are willing to accept losses, and at what point they are willing to “lock in” their profits and not subject their investments to additional risk. They often have these inequalities all mapped out at the time they purchase a stock, so they can tell instantly if they are sticking to their investment strategy. Graph the system of linear inequalities. Then describe the solution set and give two ordered pairs that are solutions and two that are not. Is there anything particular to note about the shape of this system?

$$\begin{cases} y < -\frac{3}{5}x + 4 \\ y \leq \frac{3}{2}x + 8 \\ y > -\frac{3}{5}x - 8 \\ y > \frac{3}{2}x - 6 \end{cases}$$



12.3 Modeling with Linear Systems



Resource Locker

Essential Question: How can you use systems of linear equations or inequalities to model and solve contextual problems?

Explore Modeling Real-World Constraints with Systems

Real-world situations can often be modeled by systems of equations. Usually, information about prices and the total number of items purchased is given, and the system is solved to find the number of each item purchased.

Joe goes to the store to buy jeans and some T-shirts. The jeans cost \$40 each and the T-shirts cost \$20 each. If Joe spends \$160 on 5 items, how many pairs of jeans and how many T-shirts did he buy?



- (A) Write an expression to represent the amount that Joe spent on x pairs of jeans. _____
- (B) Write an expression to represent the amount that Joe spent on y T-shirts. _____
- (C) Now write an equation that represents the total amount spent on jeans and T-shirts.

Amount spent on jeans	+	Amount spent on T-shirts	=	Total amount spent
<input style="width: 40px; height: 20px;" type="text"/>	+	<input style="width: 40px; height: 20px;" type="text"/>	=	<input style="width: 40px; height: 20px;" type="text"/>

- (D) What variable represents the number of jeans purchased? _____
- (E) What variable represents the number of T-shirts purchased? _____
- (F) Write an equation to represent the total number of items purchased. _____
- (G) Write the system that represents the situation.

	= 160
	= 5

Reflect

1. What units are associated with the two expressions that you wrote in steps A and B?

2. When you add the units for the expressions representing the amounts spent on jeans and T-shirts, what units do you get for the total amount spent?

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Explain 1

Modeling Real-World Constraints with Systems of Linear Equations

You can model real-world constraints, such as the number of items needed and the amount of money one has to spend, with systems of linear equations.

Example 1 Write a system of equations to represent the situation, and then solve the system.

- A** Bobby will buy coffee and hot chocolate for his co-workers. Each cup of coffee costs \$2.25 and each cup of hot chocolate costs \$1.50. If he pays a total of \$15.75 for 8 cups, how many of each did he buy?

Create a table to organize the information.

	Coffee	Hot Chocolate	Total
Number of Cups	c	h	8
Cost	$\$2.25c$	$\$1.50h$	$\$15.75$



Use the information to write a system of equations.

$$2.25c + 1.50h = 15.75 \quad \text{Total amount spent on } c \text{ cups of coffee and } h \text{ cups of hot chocolate}$$

$$c + h = 8 \quad \text{Total number of cups bought}$$

Multiply the second equation by -2.25 to get opposite coefficients for c .

$$-2.25(c + h = 8)$$

$$-2.25c - 2.25h = -18$$

Add the new equation to the first equation.

$$\begin{array}{r} 2.25c + 1.50h = 15.75 \\ +(-2.25c - 2.25h = -18) \\ \hline -0.75h = -2.25 \end{array}$$

Solve for h .

$$-0.75h = -2.25$$

$$h = 3$$

Substitute the value found for h back into one of the original equations and solve for c .

$$c + h = 8$$

$$c + 3 = 8$$

$$c = 5$$

So Bobby bought 5 cups of coffee and 3 cups of hot chocolate.

- B** A student is buying pens and markers for school. Packs of pens cost \$2.75 each and packs of markers cost \$3.25 each. If she bought a total of 6 packs and spent \$17.50, how many of each did she buy?

Create a table to organize the information.

	Pens	Markers	Total
Number of packs	p	m	
Cost			\$17.50

Use the information to write a system of equations.

$$\boxed{} + \boxed{} = 17.50 \quad \text{Total amount spent on } p \text{ packs of pens and } m \text{ packs of markers}$$

$$p + m = \boxed{} \quad \text{Total number of packs bought}$$

Multiply the second equation by _____ to get opposite coefficients for p .

$$\boxed{} (p + m = \boxed{})$$

$$\boxed{} p + \boxed{} m = \boxed{}$$

Add the new equation to the first equation.

$$\begin{array}{r} \boxed{} p + \boxed{} m = 17.50 \\ + \boxed{} p + \boxed{} m = \boxed{} \\ \hline \boxed{} m = \boxed{} \end{array}$$

Solve for m , the number of markers.

$$m = \boxed{}$$

Substitute the value found for m back into one of the original equations and solve for p .

$$\begin{aligned} p + m &= \boxed{} \\ p + \boxed{} &= \boxed{} \\ p &= \boxed{} \end{aligned}$$

So the student bought _____ packs of pens and _____ packs of markers.

Reflect

3. What's another possible way to solve the problem?

Your Turn

Write a system of equations to represent the situation, and then solve the system.

4. A company has to buy computers and printers. Each computer costs \$550 and each printer costs \$390. If the company spends \$8160 and buys a total of 16 machines, how many of each did it buy?

Explain 2 **Modeling Real-World Constraints with Systems of Linear Inequalities**

You can use a system of linear inequalities and its graph to model many real-world situations.

Example 2 Set up and solve the system of linear equalities.

- A Sue is buying T-shirts and shorts. T-shirts cost \$14 and shorts cost \$21. She plans on spending no more than \$147 and buy at least 5 items. Show and describe all combinations of the number of T-shirts and shorts she could buy.

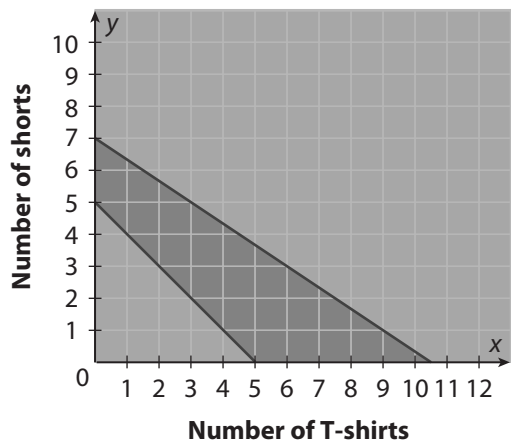
First write the system. Let x represent the number of T-shirts, and let y represent the number of shorts.

$x + y \geq 5$ She wants to buy at least 5 items.

$14x + 21y \leq 147$ She wants to spend no more than \$147.

Graph the system of inequalities: $\begin{cases} x + y \geq 5 \\ 14x + 21y \leq 147 \end{cases}$

T-shirts and Shorts



The possible solutions are where the shaded regions overlap. So, a possible solution is 5 T-shirts and 2 shorts. Substitute this value into the inequalities to make sure it is a reasonable solution.

$$\begin{cases} x + y \geq 5 \\ 14x + 21y \leq 147 \end{cases} \rightarrow \begin{cases} 5 + 2 \stackrel{?}{\geq} 5 \\ 14(5) + 21(2) \stackrel{?}{\leq} 147 \end{cases} \rightarrow \begin{cases} 7 \geq 5 \\ 112 \leq 147 \end{cases}$$

The result is two inequalities that are true, so this is a reasonable answer.

- B** John has to buy two different kinds of rope. Rope A costs \$0.60 per foot and Rope B costs \$0.90 per foot. John needs to buy at least 15 feet of rope, but he wants to spend no more than \$18. Show and describe all combinations of the number of feet of each type of rope John can buy.

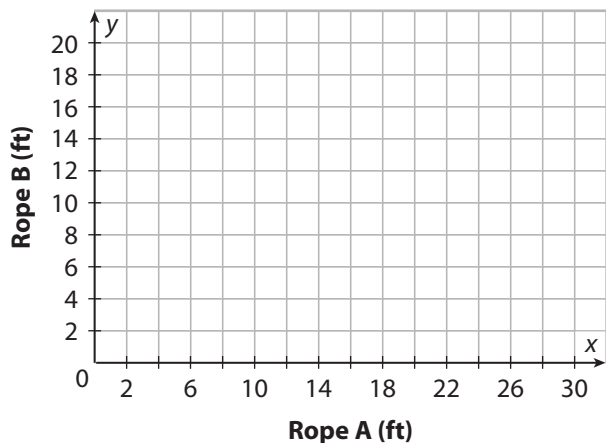
First write the system. Let x represent the amount of Rope A, and let y represent the amount of Rope B.

$$\boxed{} + \boxed{} \geq 15$$

$$\boxed{} + 0.9y \boxed{} \leq 18$$

Graph the system.

Buying Rope



Describe the solutions to the system.

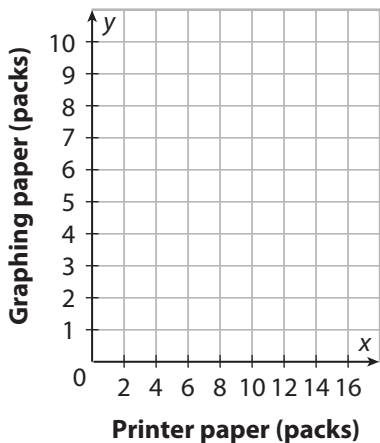
The possible solutions are _____

Your Turn

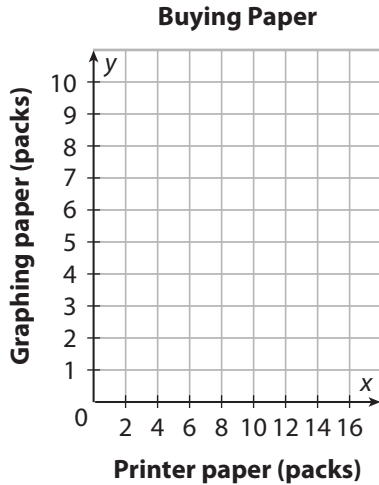
Write a system of inequalities for the given situation and graph the system. Then determine if the point $(8, 4)$ is a solution to the system.

5. A student has to buy graphing paper and printer paper. The printer paper costs \$2 a pack, while the graphing paper costs \$3 a pack. She wants to buy at least 6 packs of paper but wants to spend at most \$27.

Buying Paper



6. Now assume that she wants to buy at least 7 packs and will spend at most \$30.



Elaborate

7. Is it possible for a system of two linear inequalities to only have one solution?

8. Why can't a system of inequalities be solved using the same methods as solving systems of equations?

9. **Essential Question Check-In** When writing a system of equations or inequalities from a situation, how do you know that you have possibly written the system correctly?

Evaluate: Homework and Practice



- Online Homework
- Hints and Help
- Extra Practice

**Write a system of equations that corresponds to the situation.
Do not solve.**

1. Lisa spends part of her year as a member of a gym. She then finds a better deal at another gym, so she cancels her membership with the first gym and spends the rest of the year with the second gym. The membership to the first gym costs \$75 per month, while the membership for the second gym costs \$50 per month. She ends up spending a total of \$775 over the course of the year.

2. Jack is selling tickets to an event. Attendees can either buy a general admission ticket or a VIP ticket. The general admission tickets are \$60 and the VIP tickets are \$90. He doesn't know how many of each type he has sold, but he knows he sold a total of 29 tickets and made \$2100.

3. There are 200 adults and 300 children at a zoo. The zoo makes a total of \$7000 from the entrance fees, and the cost for an adult and a child to attend is \$30.

4. A local fish market is selling fish and lobsters by the pound. The fish costs \$4.50 a pound, while the lobster costs \$9.50 a pound. The fish market sells 25.5 pounds and makes \$189.75.



5. Jennifer has 12 nickels and dimes. The value of her coins is \$1.

6. The sum of 5 times one number and 2 times a second number is 57. The sum of the two numbers is 18.

7. Gary goes to the grocery store to buy hot dogs and hamburgers for a cookout. He buys a total of 8 packages for a total of \$28.52. A package of hot dogs costs \$2.29 and a package of hamburgers costs \$5.69.
8. The sum of two numbers is 28, and the sum of 6 times the first number and 3 times the second number is 105.

Find a system of equations that corresponds to the situation and then solve the resulting system.

9. Jan spends part of her year as a member of a gym. She then finds a better deal at another gym, so she cancels her membership with the first gym and spends the rest of the year with the second gym. The membership to the first gym costs \$80 per month, while the membership for the second gym costs \$45 per month. If she ends up spending a total of \$645 over the course of the year, how much time did she spend at each gym?
10. John is selling tickets to an event. Attendees can either buy a general admission ticket or a VIP ticket. The general admission tickets are \$70 and the VIP tickets are \$105. If he knows he sold a total of 33 tickets and made \$2730, how many of each type did he sell?

- 11.** There are 150 adults and 225 children at a zoo. If the zoo makes a total of \$5100 from the entrance fees, and the cost of an adult and a child to attend is \$31, how much does it cost each for a parent and a child?



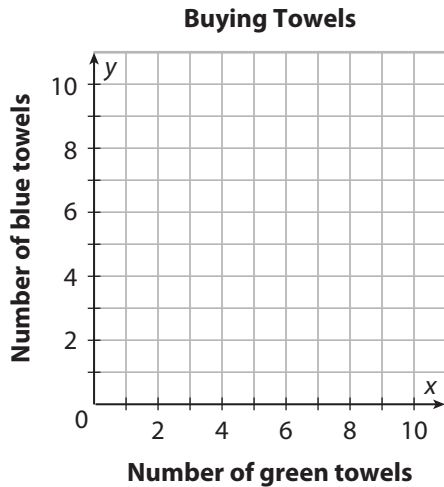
- 12.** A local fish market is selling fish and lobsters by the pound. The fish costs \$5.25 a pound, while the lobster costs \$10.50 a pound. The fish market sells 28.5 pounds and makes \$215.25.

- 13.** Nicole has 15 nickels and dimes. If the value of her coins is \$1.20, how many of each coin does she have?

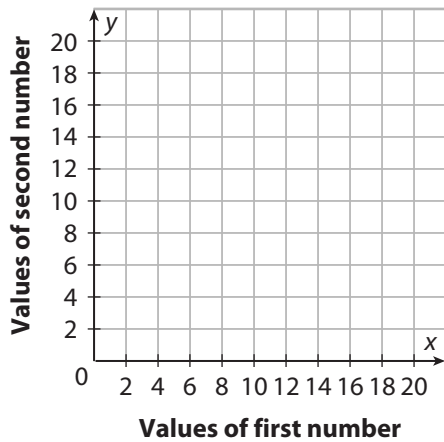
- 14.** The sum of 4 times one number and 3 times a second number is 64. If the sum of the two numbers is 19, find the two numbers.
- 15.** Meaghan goes to the grocery store to buy hot dogs and hamburgers for a cookout. She buys a total of 6 packages for a total of \$30.46. If a package of hot dogs costs \$2.65 and a package of hamburgers costs \$6.29, determine how many packages of each she bought.
- 16.** The sum of two numbers is 33, and the sum of 7 times the first number and 5 times the second number is 197.

Write the system of inequalities that represents the situation. Then graph the system and describe the solutions. Give one possible solution.

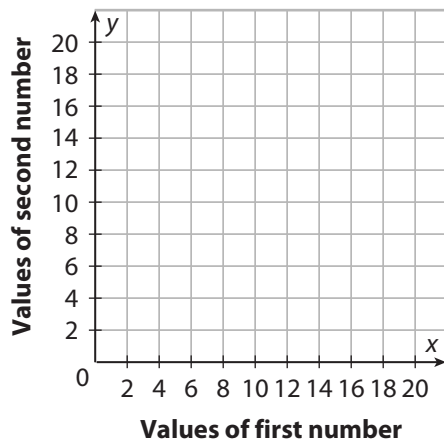
17. Angelique is buying towels for her apartment. She finds some green towels that cost \$8 each and blue towels that cost \$10 each. She wants to buy at least 4 towels but doesn't want to spend more than \$70. How many of each towel can she purchase?



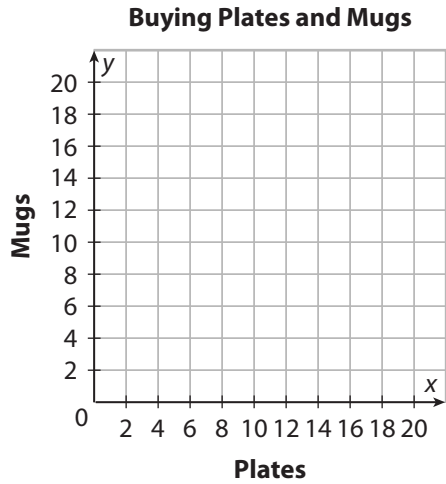
18. The sum of two numbers is at least 8, and the sum of one of the numbers and 3 times the second number is no more than 15.



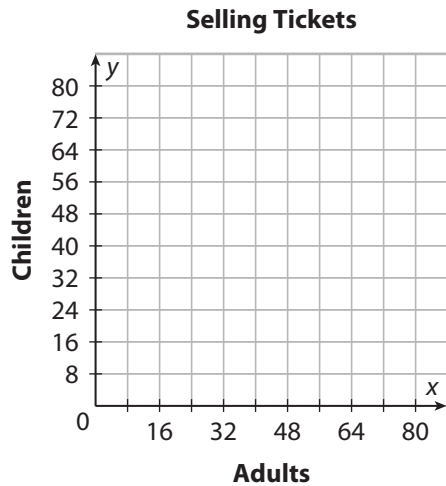
19. The sum of two numbers is at most 12, and the sum of 3 times the first number and 8 times the second number is at least 48.



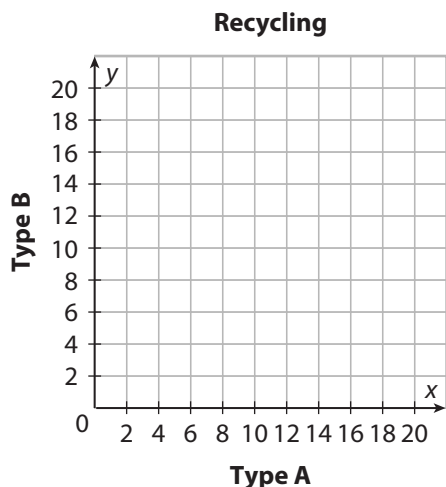
20. Katie is purchasing plates and mugs for her house. She would like to buy at least 8 items. Determine the possibilities if the plates cost \$8 each and the mugs cost \$7 each, and she plans to spend no more than \$112.



21. Christine is selling tickets at a museum. She knows that she has sold at least 40 tickets. The adult tickets cost 14 dollars and the children's tickets cost 12 dollars. If she knows she has sold no more than \$720 worth of tickets, what are the possible combinations?



22. Mike is bringing cans and bottles to a recycling center. For a type A can or bottle he gets 5 cents, and for a type B can or bottle he gets 10 cents. He knows that he has redeemed at least 11 cans but has no more than 95 cents. What are the possible combinations?

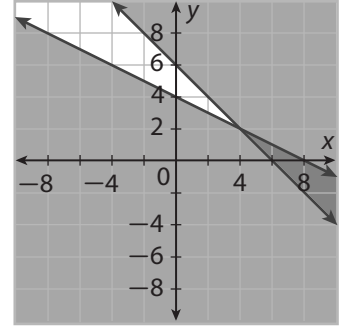


H.O.T. Focus on Higher Order Thinking

- 23. Explain the Error** A student is given the following system. He graphs the system as shown and determines that a solution is $(7, 0)$. Where did the student go wrong? What should the correct answer be?

$$x + y = 6$$

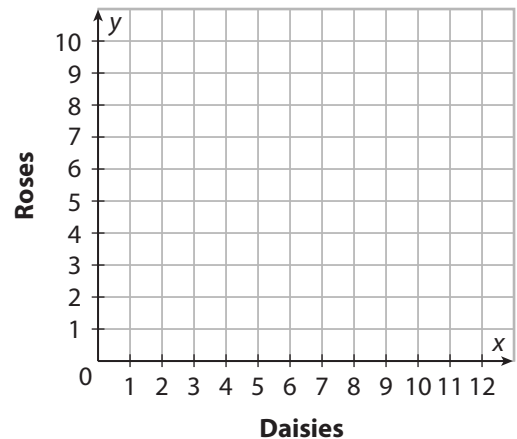
$$x + 2y = 8$$



- 24. Justify Reasoning** Molly went shopping to buy jewelry. All of the earrings cost \$15.25 and the necklaces cost \$40.75. If she spends \$127.25 and buys 5 items, how many necklaces and pairs of earrings did she buy? Justify your answer.

- 25. Check for Reasonableness** Chris is at the florist and has to buy flowers. Arrangements of daisies are \$4.25 and arrangements of roses are \$6.50. He wants to spend less than \$39 and wants to buy more than 4 arrangements. What are possible combinations that Chris can buy? Check to make sure your answer is reasonable.

Flower Arrangements



Lesson Performance Task

Amy is at the store to buy shirts and pants. The shirts cost \$40 each and the pants cost \$50 each. She plans to spend no more than \$400 and buy at least 5 items. Find a possible combination of shirts and pants she can buy. How do you know this is a solution? What are two possible ways to show that this is a solution?

