Name ______________________ Class ______________ Date ________

11.1 Solving Linear Systems by Graphing

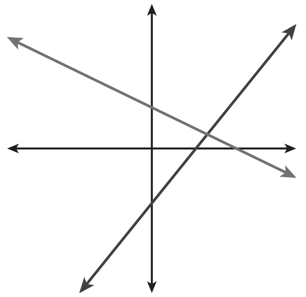
Essential Question: How can you find the solution of a system of linear equations by graphing?

Resource Locker

Explore Types of Systems of Linear Equations

A **system of linear equations,** also called a *linear system,* consists of two or more linear equations that have the same variables. A **solution of a system of linear equations** with two variables is any ordered pair that satisfies all of the equations in the system.

Graph A

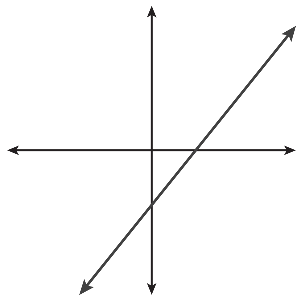


(A) Describe the relationship between the two lines in Graph A.

(B) What do you know about every point on the graph on a linear equation?

(C) How many solutions does a system of two equations have if the graphs of the two equations intersect at exactly one point?

Graph B

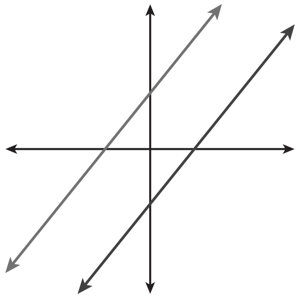


(D) Describe the relationship between the two lines that coincide in Graph B.

(E) How many solutions does a system of two equations have if the graphs of the two equations intersect at infinitely many points?

Ⓕ Describe the relationship between the two lines in Graph C.

Graph C



Ⓖ How many solutions does a system of two equations have if the graphs of the two equations do not intersect?

Reflect

1. **Discussion** Explain why the solution of a system of two equations is represented by any point where the two graphs intersect.

Explain 1 Solving Consistent, Independent Linear Systems by Graphing

A **consistent system** is a system with at least one solution. Consistent systems can be either independent or dependent.

An **independent system** has exactly one solution. The graph of an independent system consists of two lines that intersect at exactly one point. A **dependent system** has infinitely many solutions. The graph of a dependent system consists of two coincident lines, or the same line.

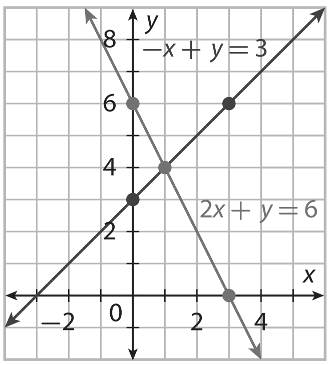
A system that has no solution is an **inconsistent system**.

Example 1 **Solve the system of linear equations by graphing. Check your answer.**

Ⓐ $\begin{cases} 2x + y = 6 \\ -x + y = 3 \end{cases}$

Find the intercepts for each equation, plus a third point for a check. Then graph.

$2x + y = 6$	$-x + y = 3$
x-intercept: 3	x-intercept: –3
y-intercept: 6	y-intercept: 3
third point: $(-1, 8)$	third point: $(3, 6)$



The two lines appear to intersect at $(1, 4)$. Check.

$2x + y = 6 \qquad -x + y = 3$

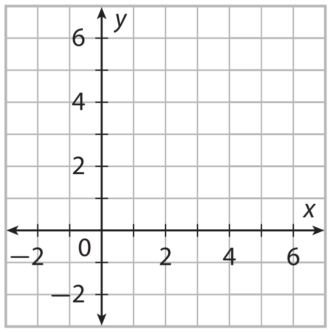
$2(1) + 4 \stackrel{?}{=} 6 \qquad -(1) + 4 \stackrel{?}{=} 3$

$6 = 6 \qquad 3 = 3$

The point satisfies both equations, so the solution is $(1, 4)$.

(B) $\begin{cases} y = 2x - 2 \\ 3y + 6x = 18 \end{cases}$

Find the intercepts for each equation, plus a third point for a check. Then graph.



$y = 2x - 2$

x-intercept: ____

y-intercept: ____

third point: $(3, \_\_\_\_)$

$3y + 6x = 18$

x-intercept: ____

y-intercept: ____

third point: $(1, \_\_\_\_)$

The two lines appear to intersect at ____. Check.

$y = 2x - 2$

$\_\_\_\_ \stackrel{?}{=} 2(\_\_\_\_) - 2$

$\_\_\_\_ = \_\_\_\_$

$y + 2x = 6$

$\_\_\_\_ + 2(\_\_\_\_) \stackrel{?}{=} 6$

$\_\_\_\_ = 6$

The point satisfies both equations, so the solution is ____.

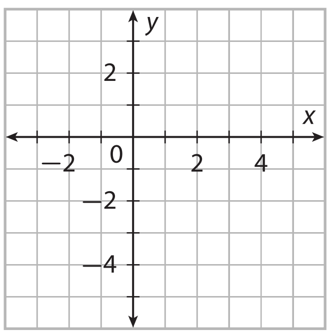
Reflect

2. How do you know that the systems of equations are consistent? How do you know that they are independent?

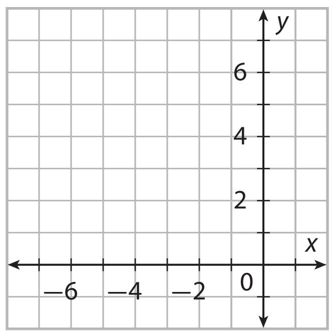
Your Turn

Solve the system of linear equations by graphing. Check your answer.

3. $\begin{cases} y = -2x - 2 \\ x + 2y = 2 \end{cases}$



4. $\begin{cases} y = 2x + 8 \\ -x + y = 6 \end{cases}$

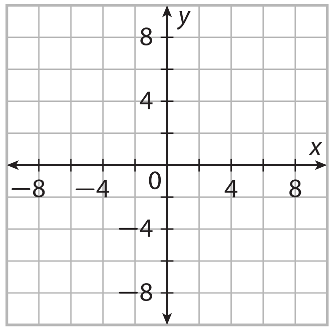


Explain 2 Solving Special Linear Systems by Graphing

Example 2 **Solve the special system of equations by graphing and identify the system.**

(A) $\begin{cases} y = 2x - 2 \\ -2x + y = 4 \end{cases}$

Find the intercepts for each equation, plus a third point for a check.

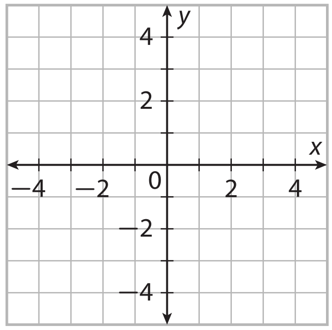


$y = 2x - 2$	$-2x + y = 4$
x-intercept: 1	x-intercept: -2
y-intercept: -2	y-intercept: 4
third point: (2, 2)	third point: (2, 8)

The two lines don't intersect, so there is no solution.
The two lines have the same slope and different y-intercepts so they will never intersect. This is an inconsistent system.

(B) $\begin{cases} y = 3x - 3 \\ -3x + y = -3 \end{cases}$

Find the intercepts for each equation, plus a third point for a check.



$y = 3x - 3$	$-3x + y = -3$
x-intercept: ☐	x-intercept: ☐
y-intercept: ☐	y-intercept: ☐
third point: (2, ☐)	third point: (2, ☐)

The two lines coincide, so there are ________ solutions.

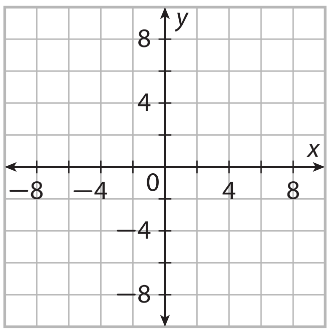
They have the same slope and y-intercept; therefore, they are ________ line(s) / equation(s).

This is a ________ and ________ system.

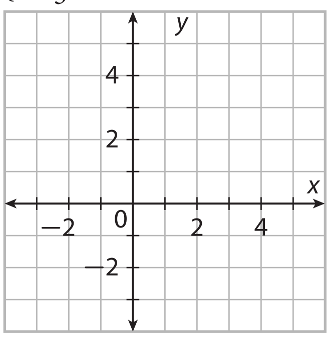
Your Turn

Solve the special system of linear equations by graphing. Check your answer.

5. $\begin{cases} y = -x - 2 \\ x + y + 2 = 0 \end{cases}$



6. $\begin{cases} y = \frac{2}{3}x - 1 \\ -\frac{2}{3}x + y = 1 \end{cases}$

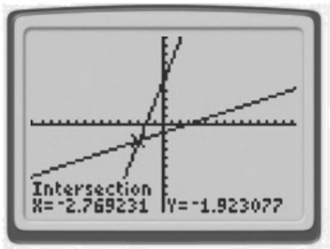


Explain 3 Estimating Solutions of Linear Systems by Graphing

You can estimate the solution of a linear system of equations by graphing the system and finding the approximate coordinates of the intersection point.

Example 3 **Estimate the solution of the linear system by graphing.**

Ⓐ $\begin{cases} x - 3y = 3 \\ -5x + 2y = 10 \end{cases}$



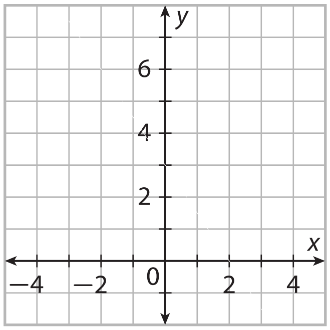
Graph the equations using a graphing calculator.

Y1 = (3 − X)/(−3) and Y2 = (10 + 5X)/2

Find the point of intersection.

The two lines appear to intersect at about $(-2.8, -1.9)$.

Ⓑ $\begin{cases} 6 - 2y = 3x \\ y = 4x + 8 \end{cases}$



Graph each equation by finding intercepts.

The two lines appear to intersect at about ______.

Check to see if ______ makes both equations true.

$6 - 2y = 3x$ $\qquad$ $y = 4x + 8$

$6 - 2(\_\_) \stackrel{?}{=} 3(\_\_)$ $\qquad$ $\_\_ \stackrel{?}{=} 4(\_\_) + 8$

$\_\_ \approx \_\_$ $\qquad$ $\_\_ \approx \_\_$

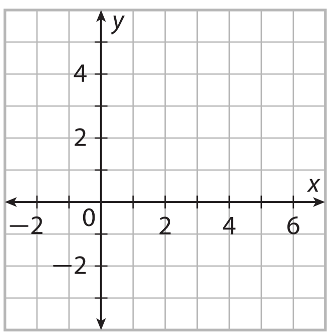
The point does not satisfy both equations, but the results are close.

So, ______ is an approximate solution.

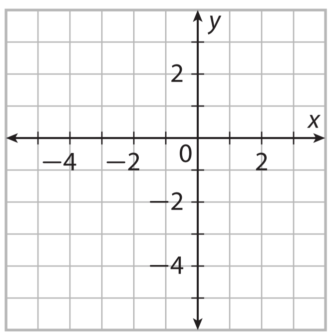
Your Turn

Estimate the solution of the linear system of equations by graphing.

7. $\begin{cases} 2y = -5x + 10 \\ -15 = -3x + 5y \end{cases}$



8. $\begin{cases} 3x + 3y = -9 \\ y = \frac{1}{2}x - 1 \end{cases}$



Explain 4 Interpreting Graphs of Linear Systems to Solve Problems

You can solve problems with real-world context by graphing the equations that model the problem and finding a common point.

Example 4 **Rock and Bowl charges $2.75 per game plus $3 for shoe rental. Super Bowling charges $2.25 per game and $3.50 for shoe rental. For how many games will the cost to bowl be approximately the same at both places? What is that cost?**

Analyze Information

Identify the important information.

- Rock and Bowl charges $ ☐ per game plus $ ☐ for shoe rental.
- Super Bowling charges $ ☐ per game and $ ☐ for shoe rental.
- The answer is the number of games played for which the total cost is approximately the same at both bowling alleys.

Formulate a Plan

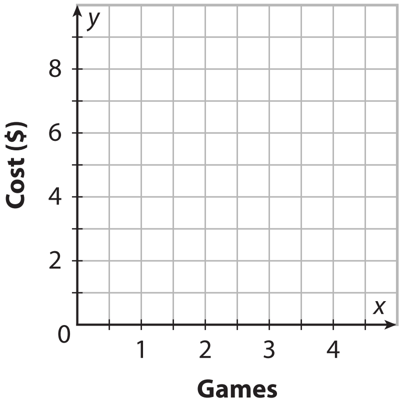
Write a system of linear equations, where each equation represents the price at each bowling alley.

____

Solve

Graph $y = 2.75x + 3$ and $y = 2.25x + 3.50$.

The lines appear to intersect at ________. So, the cost at both places will be the same for ________ game(s) bowled and that cost will be ________.



Justify and Evaluate

Check ________ using both equations.

$2.75(\square) + 3 = \square$ $\qquad$ $2.25(\square) + 3.5 = \square$

Reflect

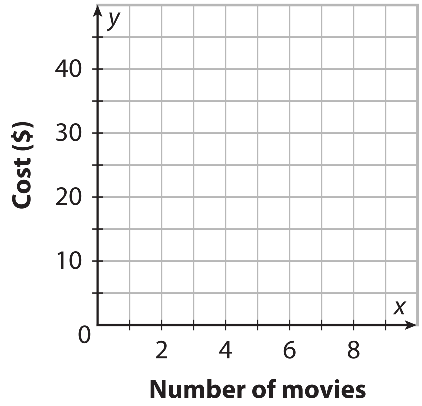
9. Which bowling alley costs more if you bowl more than 1 game? Explain how you can tell by looking at the graph.

____

____

Your Turn

10. Video club A charges $10 for membership and $4 per movie rental. Video club B charges $15 for membership and $3 per movie rental. For how many movie rentals will the cost be the same at both video clubs? What is that cost? Write a system and solve by graphing.



Elaborate

11. When a system of linear equations is graphed, how is the graph of each equation related to the solutions of that equation?

____

12. **Essential Question Check-In** How does graphing help you solve a system of linear equations?

____

____

____

Evaluate: Homework and Practice

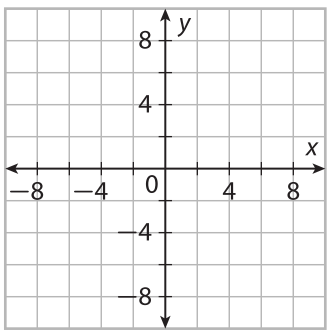
Personal Math Trainer
- Online Homework
- Hints and Help
- Extra Practice

1. Is the following statement correct? Explain.

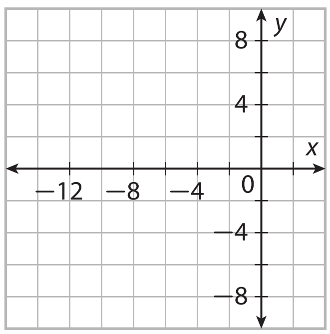
A system of two equations has no solution if the graphs of the two equations are coincident lines.

Solve the system of linear equations by graphing. Check your answer.

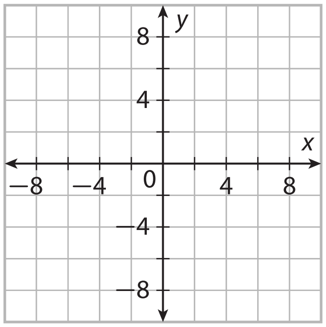
2. $\begin{cases} y = 2x - 4 \\ x + 2y = 12 \end{cases}$



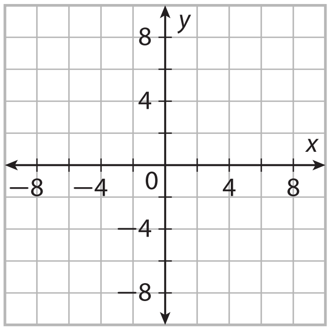
3. $\begin{cases} y = -\frac{1}{3}x + 2 \\ y + 4 = -\frac{4}{3}x \end{cases}$



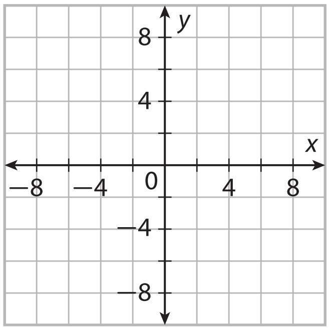
4. $\begin{cases} y = -x - 6 \\ y = x \end{cases}$



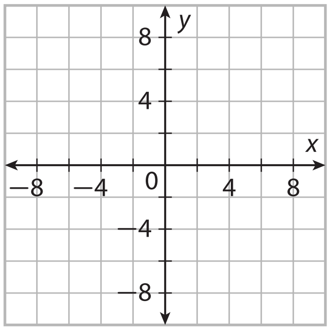
5. $\begin{cases} y = \frac{4}{3}x - 4 \\ y = 4 \end{cases}$



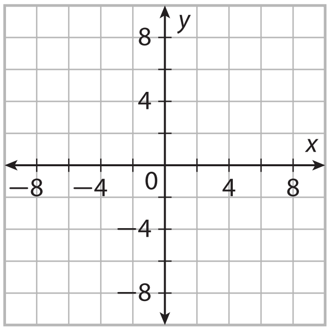
6. $\begin{cases} y = -2x + 2 \\ y + 2 = 2x \end{cases}$



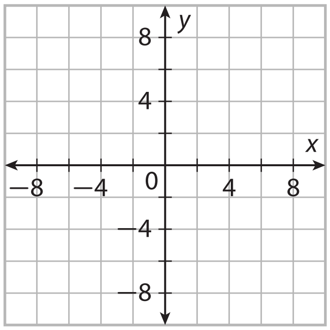
7. $\begin{cases} y = \frac{1}{2}x + 5 \\ \frac{2}{3}x + y = -2 \end{cases}$



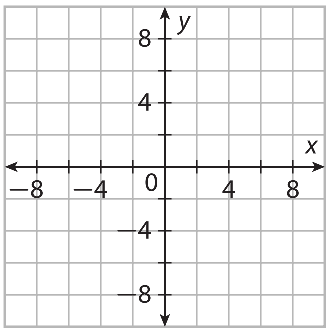
8. $\begin{cases} y = \frac{1}{2}x - 2 \\ -\frac{1}{2}x + y - 3 = 0 \end{cases}$



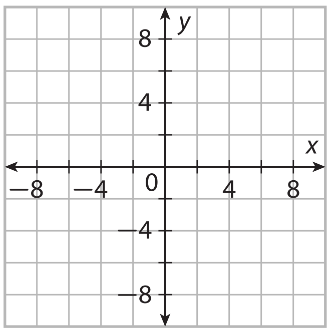
9. $\begin{cases} y = 2x + 4 \\ -4x + 2y = 8 \end{cases}$



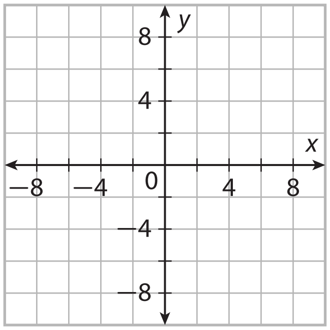
10. $\begin{cases} y = -3x + 1 \\ 12x + 4y = 4 \end{cases}$



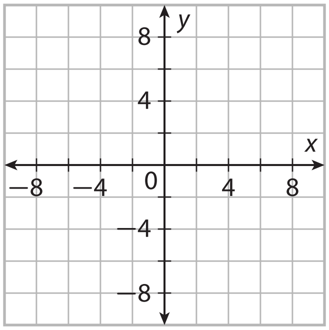
11. $\begin{cases} y = 4x + 4 \\ -4x + y + 4 = 0 \end{cases}$



12. $\begin{cases} y = -\frac{3}{4}x + \frac{1}{4} \\ \frac{3}{4}x + y - 2 = 0 \end{cases}$

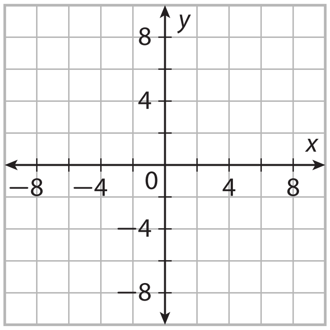


13. $\begin{cases} y = 5x - 1 \\ -5x + y + 4 = 3 \end{cases}$

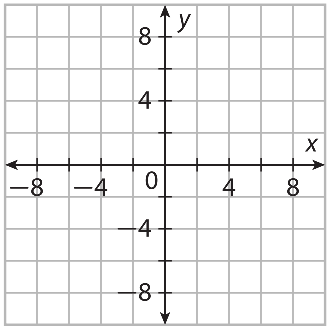


Estimate the solution of the linear system of equations by graphing.

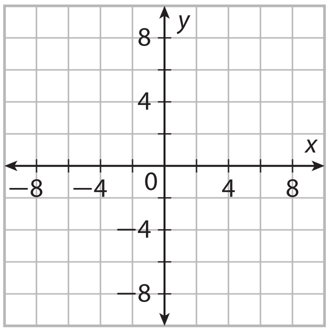
14. $\begin{cases} 3y = -5x + 15 \\ -14 = -2x + 7y \end{cases}$



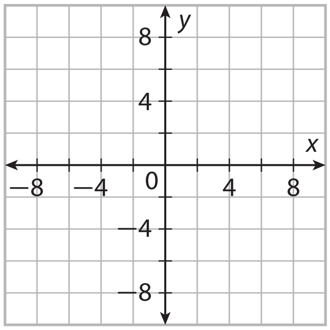
15. $\begin{cases} 2y + 5x = 14 \\ -35 = -5x + 7y \end{cases}$



16. $\begin{cases} \frac{4}{7}y = x + 4 \\ 2x - 5y = 10 \end{cases}$

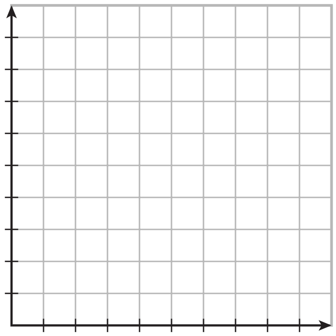


17. $\begin{cases} 6y = 5x + 30 \\ 2 = -\frac{2}{7}x + y \end{cases}$

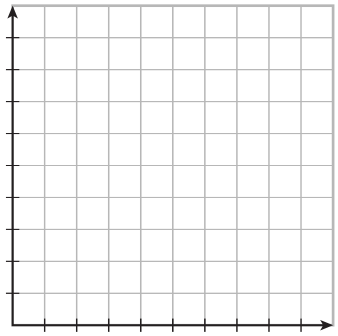


Solve by graphing. Give an approximate solution if necessary.

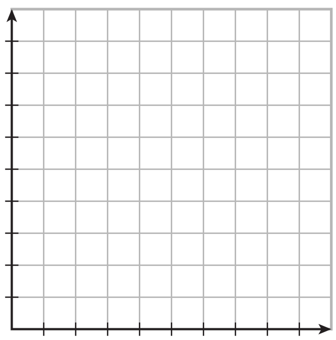
18. Wren and Jenni are reading the same book. Wren is on page 12 and reads 3 pages every night. Jenni is on page 7 and reads 4 pages every night. After how many nights will they have read the same number of pages? How many pages will that be?



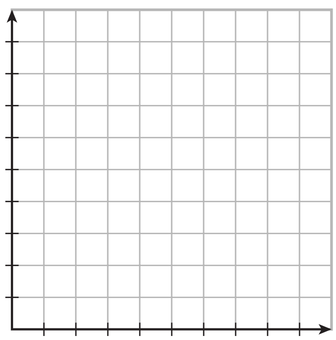
19. Rusty burns 6 calories per minute swimming and 10 calories per minute jogging. In the morning, Rusty burns 175 calories walking and swims for x minutes. In the afternoon, Rusty will jog for x minutes. How many minutes must he jog to burn at least as many calories y in the afternoon as he did in the morning? Round your answer up to the next whole number of minutes.



20. A gym membership at one gym costs $10 every month plus a one-time membership fee of $15, and a gym membership at another gym costs $4 every month plus a one-time $40 membership fee. After about how many months will the gym memberships cost the same amount?



21. Malory is putting money in two savings accounts. Account A started with $150 and Account B started with $300. Malory deposits $16 in Account A and $12 in Account B each month. In how many months will Account A have a balance at least as great as Account B? What will that balance be?



22. Critical Thinking Write *sometimes*, *always*, or *never* to complete the following statements.

a. If the equations in a system of linear equations have the same slope, there are ______________ infinitely many solutions for the system.

b. If the equations in a system of linear equations have different slopes, there is ___________ one solution for the system.

c. If the equations in a system of linear equations have the same slope and a different y-intercept, there is _________ any solution for the system.

H.O.T. Focus on Higher Order Thinking

23. Critique Reasoning Brad classifies the system below as inconsistent because the equations have the same y-intercept. What is his error?

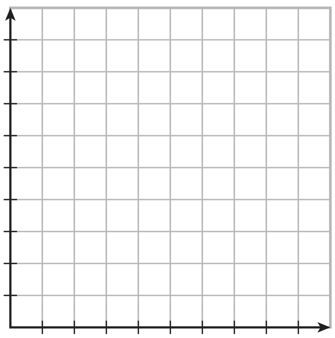
$$\begin{cases} y = 2x - 4 \\ y = x - 4 \end{cases}$$

24. Explain the Error Alexa solved the system

$$\begin{cases} 5x + 2y = 6 \\ x - 3y = -4 \end{cases}$$

by graphing and estimated the solution to be about $(1.5, 0.6)$.
What is her error? What is the correct answer?

25. Represent Real-World Problems Cora ran 3 miles last week and will run 7 miles per week from now on. Hana ran 9 miles last week and will run 4 miles per week from now on. The system of linear equations $\begin{cases} y = 7x + 3 \\ y = 4x + 9 \end{cases}$ can be used to represent this situation. Explain what x and y represent in the equations. After how many weeks will Cora and Hana have run the same number of miles? How many miles? Solve by graphing.

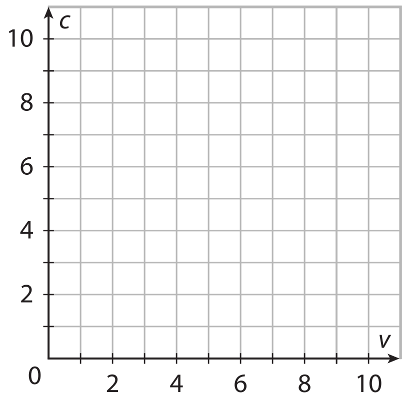


Lesson Performance Task

A boat takes 7.5 hours to make a 60-mile trip upstream and 6 hours on the 60-mile return trip. Let v be the speed of the boat in still water and c be the speed of the current. The upstream speed of the boat is $v - c$ and the downstream boat speed is $v + c$.

a. Use the distance formula to write a system of equations relating boat speed and time to distance, one equation for the upstream part of the trip and one for the downstream part.

b. Graph the system to find the speed of the boat in still water and the speed of the current.



c. How long would it take the boat to travel the 60 miles if there were no current?

Name ____________________ Class ____________ Date ________

11.2 Solving Linear Systems by Substitution

Essential Question: How can you solve a system of linear equations by using substitution?

Resource Locker

Explore — Exploring the Substitution Method of Solving Linear Systems

Another method to solve a linear system is by using the substitution method.

In the system of linear equations shown, the value of y is given. Use this value of y to find the value of x and the solution of the system.

$$\begin{cases} y = 2 \\ x + y = 6 \end{cases}$$

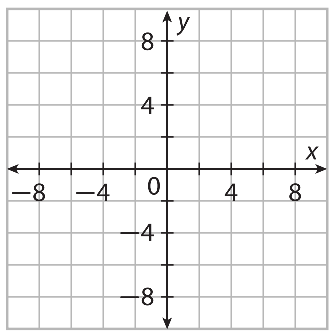
(A) Substitute the value of y in the second equation and solve for x.

$$x + y = 6$$
$$x + \square = 6$$
$$x = \square$$

(B) The values of x and y are known. What is the solution of the system?

Solution: $(\square, \square)$

(C) Graph the system of linear equations. How do your solutions compare?



(D) Use substitution to find the values of x and y in this system of linear equations. Substitute $4x$ for y in the second equation and solve for x. Once you find the value for x, substitute it into either original equation to find the value for y.

$$\begin{cases} y = 4x \\ 5x + 2y = 39 \end{cases}$$

Solution: $(\square, \square)$

Reflect

1. **Discussion** For the system in Step D, what equation did you get after substituting $4x$ for y in $5x + 2y = 39$ and simplifying?

____

2. **Discussion** How could you check your solution in part D?

____

____

Explain 1 Solving Consistent, Independent Linear Systems by Substitution

The **substitution method** is used to solve a system of equations by solving an equation for one variable and substituting the resulting expression into the other equation. The steps for the substitution method are as shown.

1. Solve one of the equations for one of its variables.
2. Substitute the expression from Step 1 into the other equation and solve for the other variable.
3. Substitute the value from Step 2 into either original equation and solve to find the value of the other variable.

Example 1 **Solve each system of linear equations by substitution.**

(A) $\begin{cases} 3x + y = -3 \\ -2x + y = 7 \end{cases}$

Solve an equation for one variable.

$3x + y = -3$	Select one of the equations.
$y = -3x - 3$	Solve for y. Isolate y on one side.

Substitute the expression for y in the other equation and solve.

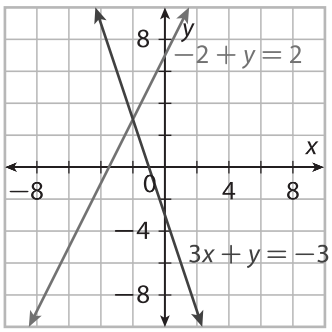
$-2x + (-3x - 3) = 7$	Substitute the expression for y.
$-5x - 3 = 7$	Combine like terms.
$-5x = 10$	Add 3 to both sides.
$x = -2$	Divide each side by −5.

Substitute the value for x into one of the equations and solve for y.

$3(-2) + y = -3$	Substitute the value of x into the first equation.
$-6 + y = -3$	Simplify.
$y = 3$	Add 6 to both sides.

So, $(-2, 3)$ is the solution of the system.

Check the solution by graphing.



$3x + y = -3$	$-2x + y = 7$
x-intercept: -1	x-intercept: $-\frac{7}{2}$
y-intercept: -3	y-intercept: 7

The point of intersection is $(-2, 3)$.

(B) $\begin{cases} x - 3y = 9 \\ x + 4y = 2 \end{cases}$

Solve an equation for one variable.

$x - 3y = 9$	Select one of the equations.
$x = \square$	Solve for x. Isolate x on one side.

Substitute the expression for ____ in the other equation and solve.

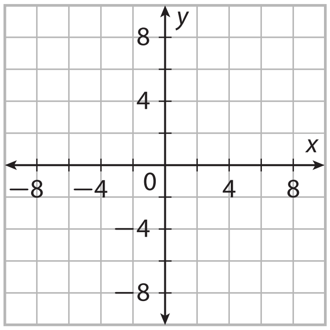
$(\square) + 4y = 2$	Substitute the expression for ____.
$\square = 2$	Combine like terms.
$\square = \square$	Subtract $\square$ from both sides.
$y = \square$	Divide each side by $\square$.

Substitute the value for y into one of the equations and solve for x.

$x - 3(\square) = 9$	Substitute the value of y into the first equation.
$\square = 9$	Simplify.
$x = \square$	Subtract $\square$ from both sides.

So, $(\square, \square)$ is the solution by graphing.

Check the solution by graphing.



$x - 3y = 9$	$x + 4y = 2$
x-intercept: $\square$	x-intercept: $\square$
y-intercept: $\square$	y-intercept: $\square$

The point of intersection is $(\square, \square)$.

Reflect

3. Explain how a system in which one of the equations if of the form $y = c$, where c is a constant is a special case of the substitution method.

4. Is it more efficient to solve $-2x + y = 7$ for x than for y? Explain.

Your Turn

5. Solve the system of linear equations by substitution.

$$\begin{cases} 3x + y = 14 \\ 2x - 6y = -24 \end{cases}$$

Explain 2 Solving Special Linear Systems by Substitution

You can use the substitution method for systems of linear equations that have infinitely many solutions and for systems that have no solutions.

Example 2 **Solve each system of linear equations by substitution.**

(A) $\begin{cases} x + y = 4 \\ -x - y = 6 \end{cases}$

Solve $x + y = 4$ for x.

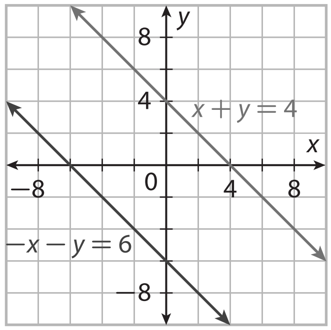
$$x = -y + 4$$

Substitute the resulting expression into the other equation and solve.

$-(-y + 4) - y = 6$ Substitute.

$-4 = 6$ Simplify.

The resulting equation is false, so the system has no solutions.



The graph shows that the lines are parallel and do not intersect.

(B) $\begin{cases} x - 3y = 6 \\ 4x - 12y = 24 \end{cases}$

Solve $x - 3y = 6$ for ____.

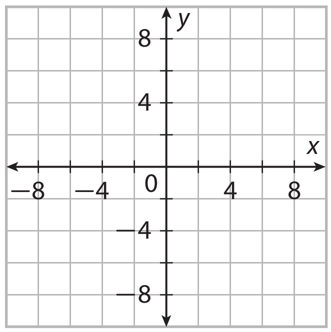
$x =$ [ ]

Substitute the resulting expression into the other equation and solve.

$4\left(\boxed{}\right) - 12y = 24$ Substitute.

$\boxed{} = 24$ Simplify.

The resulting equation is ______, so the system has ____________________.



The graphs are ____________________,

so the system has ____________________.

Reflect

6. Provide two possible solutions of the system in Example 2B. How are all the solutions of this system related to one another?

Your Turn

Solve each system of linear equations by substitution.

7. $\begin{cases} -2x + 14y = -28 \\ x - 7y = 14 \end{cases}$

8. $\begin{cases} -3x + y = 12 \\ 6x - 2y = 18 \end{cases}$

Explain 3 Solving Linear System Models by Substitution

You can use a system of linear equations to model real-world situations.

Example 3 **Solve each real-world situation by using the substitution method.**

(A) Fitness center A has a \$60 enrollment fee and costs \$35 per month. Fitness center B has no enrollment fee and costs \$45 per month. Let t represent the total cost in dollars and m represent the number of months. The system of equations $\begin{cases} t = 60 + 35m \\ t = 45m \end{cases}$ can be used to represent this situation. In how many months will both fitness centers cost the same? What will the cost be?

$60 + 35m = 45m$	Substitute $60 + 35m$ for t in the second equation.
$60 = 10m$	Subtract $35m$ from each side.
$6 = m$	Divide each side by 10.
$t = 45m$	Use one of the original equations.
$= 45(6) = 270$	Substitute 6 for m.
$(6, 270)$	Write the solution as an ordered pair.

Both fitness centers will cost \$270 after 6 months.

(B) High-speed Internet provider A has a \$100 setup fee and costs \$65 per month. High-speed internet provider B has a setup fee of \$30 and costs \$70 per month. Let t represent the total amount paid in dollars and m represent the number of months. The system of equations $\begin{cases} t = 100 + 65m \\ t = 30 + 70m \end{cases}$ can be used to represent this situation. In how many months will both providers cost the same? What will that cost be?

___ $= 30 + 70m$	Substitute ___ for t in the second equation.
$100 =$ ___	Subtract ___ m from each side.
___ $=$ ___ m	Subtract ___ from each side.
___ $= m$	Divide each side by ___.

$t = 30 + 70m$ Use one of the original equations.

$t = 30 + 70(\square)$ Substitute $\square$ for m.

$t = \square$

$(\square, \square)$ Write the sulotion as an ordered pair.

Both Internet providers will cost $______ after ______ months.

Reflect

9. If the variables in a real-world situation represent the number of months and cost, why must the values of the variables be greater than or equal to zero?

Your Turn

10. A boat travels at a rate of 18 kilometers per hour from its port. A second boat is 34 kilometers behind the first boat when it starts traveling in the same direction at a rate of 22 kilometers per hour to the same port. Let d represent the distance the boats are from the port in kilometers and t represent the amount of time in hours. The system of equations $\begin{cases} d = 18t + 34 \\ d = 22t \end{cases}$ can be used to represent this situation. How many hours will it take for the second boat to catch up to the first boat? How far will the boats be from their port? Use the substitution method to solve this real-world application.

Elaborate

11. When given a system of linear equations, how do you decide which variable to solve for first?

12. How can you check a solution for a system of equations without graphing?

13. **Essential Question-Check-In** Explain how you can solve a system of linear equations by substitution.

Evaluate: Homework and Practice

Personal Math Trainer

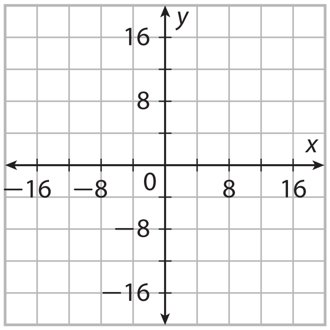
- Online Homework
- Hints and Help
- Extra Practice

1. In the system of linear equations shown, the value of y is given. Use this value of y to find the value of x and the solution of the system.

$$\begin{cases} y = 12 \\ 2x - y = 4 \end{cases}$$

a. What is the solution of the system?

b. Graph the system of linear equations. How do the solutions compare?



Solve each system of linear equations by substitution.

2. $\begin{cases} 5x + y = 8 \\ 2x + y = 5 \end{cases}$

3. $\begin{cases} x - 3y = 10 \\ x + 5y = -22 \end{cases}$

4. $\begin{cases} 5x - 3y = 22 \\ -4x + y = -19 \end{cases}$

5. $\begin{cases} x + 7y = -11 \\ -2x - 5y = 4 \end{cases}$

6. $\begin{cases} 2x + 6y = 16 \\ 3x - 5y = -18 \end{cases}$

7. $\begin{cases} 7x + 2y = 24 \\ -6x + 3y = 3 \end{cases}$

Solve each system of linear equations by substitution.

8. $\begin{cases} x + y = 3 \\ -4x - 4y = 12 \end{cases}$

9. $\begin{cases} 3x - 3y = -15 \\ -x + y = 5 \end{cases}$

10. $\begin{cases} x - 8y = 17 \\ -3x + 24y = -51 \end{cases}$

11. $\begin{cases} 5x - y = 18 \\ 10x - 2y = 32 \end{cases}$

12. $\begin{cases} -2x - 3y = 12 \\ -4x - 6y = 24 \end{cases}$

13. $\begin{cases} 3x + 4y = 36 \\ 6x + 8y = 48 \end{cases}$

Solve each real-world situation by using the substitution method.

14. The number of DVDs sold at a store in a month was 920 and the number of DVDs sold decreased by 12 per month. The number of Blu-ray discs sold in the same store in the same month was 502 and the number of Blu-ray discs sold increased by 26 per month. Let *d* represent the number of discs sold and *t* represent the time in months.

The system of equations $\begin{cases} d = 920 - 12t \\ d = 502 + 26t \end{cases}$ can be used to represent this situation. If this trend continues, in how many months will the number of DVDs sold equal the number of Blu-ray discs sold? How many of each is sold in that month?

15. One smartphone plan costs \$30 per month for talk and messaging and \$8 per gigabyte of data used each month. A second smartphone plan costs \$60 per month for talk and messaging and \$3 per gigabyte of data used each month. Let *c* represent the total cost in dollars and *d* represent the amount of data used in gigabytes. The system of equations $\begin{cases} c = 30 + 8d \\ c = 60 + 3d \end{cases}$ can be used to represent this situation. How many gigabytes would have to be used for the plans to cost the same? What would that cost be?

16. A movie theater sells popcorn and fountain drinks. Brett buys 1 popcorn bucket and 3 fountain drinks for his family, and pays a total of \$9.50. Sarah buys 3 popcorn buckets and 4 fountain drinks for her family, and pays a total of \$19.75. If p represents the number of popcorn buckets and d represents the number of drinks, then the system of equations $\begin{cases} 9.50 = p + 3d \\ 19.75 = 3p + 4d \end{cases}$ can be used to represent this situation. Find the cost of a popcorn bucket and the cost of a fountain drink.

17. Jen is riding her bicycle on a trail at the rate of 0.3 kilometer per minute. Michelle is 11.2 kilometers behind Jen when she starts traveling on the same trail at a rate of 0.44 kilometer per minute. Let d represent the distance in kilometers the bicyclists are from the start of the trail and t represent the time in minutes. The system of equations $\begin{cases} d = 0.3t + 11.2 \\ d = 0.44t \end{cases}$ can be used to represent this situation. How many minutes will it take Michelle to catch up to Jen? How far will they be from the start of the trail? Use the substitution method to solve this real-world application.

18. **Geometry** The length of a rectangular room is 5 feet more than its width. The perimeter of the room is 66 feet. Let L represent the length of the room and W represent the width in feet. The system of equations $\begin{cases} L = W + 5 \\ 66 = 2L + 2W \end{cases}$ can be used to represent this situation. What are the room's dimensions?

19. A cable television provider has a \$55 setup fee and charges \$82 per month, while a satellite television provider has a \$160 setup fee and charges \$67 per month. Let c represent the total cost in dollars and t represent the amount of time in months. The system of equations $\begin{cases} c = 55 + 82t \\ c = 160 + 67t \end{cases}$ can be used to represent this situation.

a. In how many months will both providers cost the same? What will that cost be?

b. If you plan to move in 12 months, which provider would be less expensive? Explain.

20. Determine whether each of the following systems of equations have one solution, infinitely many solutions, or no solution. Select the correct answer for each lettered part.

a. $\begin{cases} x + y = 5 \\ -6y - 6y = 30 \end{cases}$

b. $\begin{cases} x + y = 7 \\ 5x + 2y = 23 \end{cases}$

c. $\begin{cases} 3x + y = 5 \\ 6x + 2y = 12 \end{cases}$

d. $\begin{cases} 2x + 5y = -12 \\ x + 7y = -15 \end{cases}$

e. $\begin{cases} 3x + 5y = 17 \\ -6x - 10y = -34 \end{cases}$

21. **Finance** Adrienne invested a total of $1900 in two simple-interest money market accounts. Account A paid 3% annual interest and account B paid 5% annual interest. The total amount of interest she earned after one year was $83. If *a* represents the amount invested in dollars in account A and *b* represents the amount invested in dollars in account B, the system of equations $\begin{cases} a + b = 1900 \\ 0.03a + 0.05b = 83 \end{cases}$ can represent this situation. How much did Adrienne invest in each account?

H.O.T. Focus on Higher Order Thinking

22. **Real-World Application** The Sullivans are deciding between two landscaping companies. Evergreen charges a $79 startup fee and $39 per month. Eco Solutions charges a $25 startup fee and $45 per month. Let *c* represent the total cost in dollars and *t* represent the time in months. The system of equations $\begin{cases} c = 39t + 79 \\ c = 45t + 25 \end{cases}$ can be used to represent this situation.

a. In how many months will both landscaping services cost the same? What will that cost be?

b. Which landscaping service will be less expensive in the long term? Explain.

23. Multiple Representations For the first equation in the system of linear equations below, write an equivalent equation without denominators. Then solve the system.

$$\begin{cases} \frac{x}{5} + \frac{y}{3} = 6 \\ x - 2y = 8 \end{cases}$$

24. Conjecture Is it possible for a system of three linear equations to have one solution? If so, give an example.

25. Conjecture Is it possible to use substitution to solve a system of linear equations if one equation represents a horizontal line and the other equation represents a vertical line? Explain.

Lesson Performance Task

A company breaks even from the production and sale of a product if the total revenue equals the total cost. Suppose an electronics company is considering producing two types of smartphones. To produce smartphone A, the initial cost is \$20,000 and each phone costs \$150 to produce. The company will sell smartphone A at \$200. Let $C(a)$ represent the total cost in dollars of producing a units of smartphone A. Let $R(a)$ represent the total revenue, or money the company takes in due to selling a units of smartphone A. The system of equations $\begin{cases} C(a) = 20{,}000 + 150a \\ R(a) = 200a \end{cases}$ can be used to represent the situation for phone A.

To produce smartphone B, the initial cost is \$44,000 and each phone costs \$200 to produce. The company will sell smartphone B at \$280. Let $C(b)$ represent the total cost in dollars of producing b units of smartphone B and $R(b)$ represent the total revenue from selling b units of smartphone B. The system of equations $\begin{cases} C(b) = 44{,}000 + 200b \\ R(b) = 280b \end{cases}$ can be used to represent the situation for phone B.

Solve each system of equations and interpret the solutions. Then determine whether the company should invest in producing smartphone A or smartphone B. Justify your answer.

Name ______________________ Class ____________ Date ________

11.3 Solving Linear Systems by Adding or Subtracting

Essential Question: How can you solve a system of linear equations by adding and subtracting?

Resource Locker

Explore — Exploring the Effects of Adding Equations

Systems of equations can be solved by graphing, substitution, or by a third method, called **elimination.**

(A) Look at the system of linear equations.

$$\begin{cases} 2x - 4y = -10 \\ 3x + 4y = 5 \end{cases}$$

What do you notice about the coefficients of the y-terms?

____

(B) What is the sum of $-4y$ and $4y$? How do you know?

____

(C) Find the sum of the two equations by combining like terms.

$$\begin{array}{rcrcr} 2x & & -4y & = & -10 \\ +3x & & +4y & = & +5 \\ \hline \square & + & \square & = & \square \end{array}$$

(D) Use the equation from Step C to find the value of x.

$x = \square$

(E) Use the value of x to find the value of y. What is the solution of the system?

$y = \square$

Solution: ________

Reflect

1. **Discussion** How do you know that when both sides of the two equations were added, the resulting sums were equal?

____

____

2. **Discussion** How could you check your solution?

____

____

Explain 1 Solving Linear Systems by Adding or Subtracting

The **elimination method** is a method used to solve systems of equations in which one variable is eliminated by adding or subtracting two equations in the system.

Steps in the Elimination Method

1. Add or subtract the equations to eliminate one variable, and then solve for the other variable.
2. Substitute the value into either original equation to find the value of the eliminated variable.
3. Write the solution as an ordered pair.

Example 1 **Solve each system of linear equations using the indicated method. Check your answer by graphing.**

(A) Solve the system of linear equations by adding.

$$\begin{cases} 4x - 2y = 12 \\ x + 2y = 8 \end{cases}$$

Add the equations.

$$\begin{array}{r} 4x - 2y = 12 \\ x + 2y = 8 \\ \hline 5x + 0 = 20 \\ 5x = 20 \\ x = 4 \end{array}$$

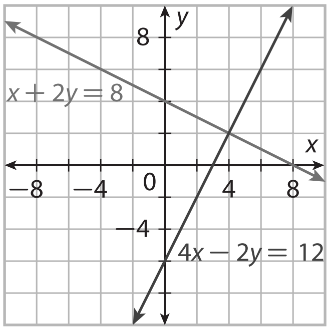
Substitute the value of x into one of the equations and solve for y.

$$\begin{aligned} x + 2y &= 8 \\ 4 + 2y &= 8 \\ 2y &= 4 \\ y &= 2 \end{aligned}$$

Write the solution as an ordered pair.

$(4, 2)$

Check the solution by graphing.



(B) Solve the system of linear equations by subtracting.

$$\begin{cases} 2x + 6y = 6 \\ 2x - y = -8 \end{cases}$$

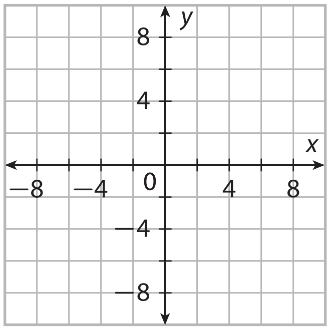
Subtract the equations.

$$\begin{array}{r} 2x + 6y = 6 \\ 2x - y = -8 \\ \hline \end{array}$$

$\square \; \square = \square$

Substitute the value of y into one of the equations and solve for x.

Write the solution as an ordered pair.



Check the solution by graphing.

Reflect

3. Can the system in part A be solved by subtracting one of the original equations from the other? Why or why not?

4. In part B, what would happen if you added the original equations instead of subtracting?

Your Turn

Solve each system of linear equations by adding or subtracting.

5. $\begin{cases} 2x + 5y = -24 \\ 3x - 5y = 14 \end{cases}$

6. $\begin{cases} 3x + 2y = 5 \\ x + 2y = -1 \end{cases}$

Explain 2 Solving Special Linear Systems by Adding or Subtracting

Example 2 **Solve each system of linear equations by adding or subtracting.**

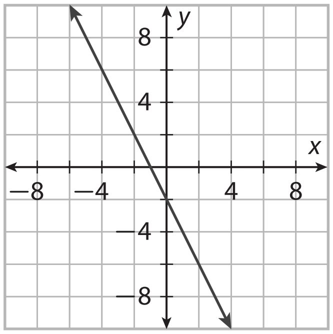
(A) $\begin{cases} -4x - 2y = 4 \\ 4x + 2y = -4 \end{cases}$

Add the equations.

$$\begin{aligned} -4x - 2y &= 4 \\ +4x + 2y &= -4 \\ \hline 0 + 0 &= 0 \\ 0 &= 0 \end{aligned}$$

The resulting equation is true, so the system has infinitely many solutions.

Graph the equations to provide more information.



The graphs are the same line, so the system has infinitely many solutions.

(B) $\begin{cases} x + y = -2 \\ x + y = 4 \end{cases}$

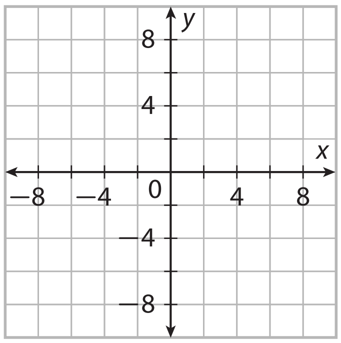
Subtract the equations.

$$\begin{aligned} x + y &= -2 \\ -(x + y &= 4) \\ \hline \end{aligned}$$

The resulting equation is ________ ,

so the system has ______ solutions.

Graph the equations to provide more information.



The graph shows that the lines are __________

and ____________________.

Your Turn

Solve each system of linear equations by adding or subtracting.

7. $\begin{cases} 4x - y = 3 \\ 4x - y = -2 \end{cases}$

8. $\begin{cases} x - 6y = 7 \\ -x + 6y = -7 \end{cases}$

Explain 3 Solving Linear System Models by Adding or Subtracting

Example 3 **Solve by adding or subtracting.**

(A) Perfect Patios is building a rectangular deck for a customer. According to the customer's specifications, the perimeter should be 40 meters and the difference between twice the length and twice the width should be 4 meters.

The system of equations $\begin{cases} 2\ell + 2w = 40 \\ 2\ell - 2w = 4 \end{cases}$ can be used to represent this situation, where ℓ is the length and w is the width. What will be the length and width of the deck?

Add the equations.

$$\begin{aligned} 2\ell + 2w &= 40 \\ 2\ell - 2w &= \ \ 4 \\ \hline 4\ell + \ \ 0 &= 44 \\ 4\ell &= 44 \\ \ell &= 11 \end{aligned}$$

Substitute the value of ℓ into one of the equations and solve for w.

$$\begin{aligned} 2\ell + 2w &= 40 \\ 2(11) + 2w &= 40 \\ 22 + 2w &= 40 \\ 2w &= 18 \\ w &= 9 \end{aligned}$$

Write the solution as an ordered pair.

$(\ell, w) = (11, 9)$

The length of the deck will be 11 meters and the width will be 9 meters.

(B) A video game and movie rental kiosk charges \$2 for each video game rented, and \$1 for each movie rented. One day last week, a total of 114 video games and movies were rented for a total of \$177. The system of equations $\begin{cases} x + y = 114 \\ 2x + y = 177 \end{cases}$ represents this situation, where x represents the number of video games rented and y represents the number of movies rented. Find the numbers of video games and movies that were rented.

Subtract the equations.

$$\begin{array}{r} x + y = 114 \\ \underline{-(2x + y = 177)} \end{array}$$

Substitute the value of x into one of the equations and solve for y.

Write the solution as an ordered pair.

______ video games and ______ movies were rented.

Your Turn

9. The perimeter of a rectangular picture frame is 62 inches. The difference of the length of the frame and twice its width is 1. The system of equations $\begin{cases} 2\ell + 2w = 62 \\ \ell - 2w = 1 \end{cases}$ represents this situation, where ℓ represents the length in inches and w represents the width in inches. What are the length and the width of the frame?

Elaborate

10. How can you decide whether to add or subtract to eliminate a variable in a linear system? Explain your reasoning.

11. **Discussion** When a linear system has no solution, what happens when you try to solve the system by adding or subtracting?

12. **Essential Question Check-In** When you solve a system of linear equations by adding or subtracting, what needs to be true about the variable terms in the equations?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. Which method of elimination would be best to solve the system of linear equations? Explain.

$$\begin{cases} \frac{1}{2}x + \frac{3}{4}y = -10 \\ -x - \frac{3}{4}y = 1 \end{cases}$$

Solve each system of linear equations by adding or subtracting.

2. $\begin{cases} 3x + 2y = 10 \\ 3x - y = 22 \end{cases}$

3. $\begin{cases} -2x + y = 3 \\ 3x - y = -2 \end{cases}$

4. $\begin{cases} x + y = 5 \\ x - 3y = 3 \end{cases}$

5. $\begin{cases} 7x + y = -4 \\ 2x - y = 1 \end{cases}$

6. $\begin{cases} -5x + y = -3 \\ 5x - 3y = -1 \end{cases}$

7. $\begin{cases} 2x + y = -6 \\ -5x + y = 8 \end{cases}$

8. $\begin{cases} 6x - 3y = 15 \\ 4x - 3y = -5 \end{cases}$

9. $\begin{cases} 8x - 6y = 36 \\ -2x + 6y = 0 \end{cases}$

10. $\begin{cases} \frac{1}{2}x - \frac{7}{9}y = -\frac{20}{3} \\ -\frac{1}{2}x + \frac{7}{9}y = 6\frac{2}{3} \end{cases}$

11. $\begin{cases} -10x + 2y = -7 \\ -10x + 2y = -2 \end{cases}$

12. $\begin{cases} -2x + 5y = 7 \\ 2x - 5y = -7 \end{cases}$

13. $\begin{cases} x + y = 0 \\ -x - y = 0 \end{cases}$

14. $\begin{cases} -5x - y = -3 \\ -5x - y = -2 \end{cases}$

15. $\begin{cases} ax - by = c \\ ax - by = c \end{cases}$

16. The sum of two numbers is 65, and the difference of the numbers is 27. The system of linear equations $\begin{cases} x + y = 65 \\ x - y = 27 \end{cases}$ represents this situation, where x is the larger number and y is the smaller number. Solve the system to find the two numbers.

17. A rectangular garden has a perimeter of 120 feet. The length of the garden is 24 feet greater than twice the width. The system of linear equations $\begin{cases} 2\ell + 2w = 120 \\ \ell - 2w = 24 \end{cases}$ represents this situation, where ℓ is the length of the garden and w is its width. Find the length and width of the garden.

18. The sum of two angles is 90°. The difference of twice the larger angle and the smaller angle is 105°. The system of linear equations $\begin{cases} x + y = 90 \\ 2x - y = 105 \end{cases}$ represents this situation where x is the larger angle and y is the smaller angle. Find the measures of the two angles.

19. Max and Sasha exercise a total of 20 hours each week. Max exercises 15 hours less than 4 times the number of hours Sasha exercises. The system of equations $\begin{cases} x + y = 20 \\ x - 4y = -15 \end{cases}$ represents this situation, where x represents the number of hours Max exercises and y represents the number of hours Sasha exercises. How many hours do Max and Sasha exercise per week?

20. The sum of the digits in a two-digit number is 12. The digit in the tens place is 2 more than the digit in the ones place. The system of linear equations $\begin{cases} x + y = 12 \\ x - y = 2 \end{cases}$ represents this situation, where x is the digit in the tens place and y is the digit in the ones place. Solve the system to find the two-digit number.

21. A pool company is installing a rectangular pool for a new house. The perimeter of the pool must be 94 feet, and the length must be 2 feet more than twice the width.

The system of linear equations $\begin{cases} 2\ell + 2w = 94 \\ \ell = 2w + 2 \end{cases}$ represents this situation, where ℓ is the length and w is the width. What are the dimensions of the pool?

22. Use one solution, no solutions, or infinitely many solutions to complete each statement.

a. When the solution of a system of linear equations yields the equation $4 = 4$, the system has ______________________.

b. When the solution of a system of linear equations yields the equation $x = 4$, the system has ______________________.

c. When the solution of a system of linear equations yields the equation $0 = 4$, the system has ______________________.

H.O.T. Focus on Higher Order Thinking

23. **Multiple Representations** You can use subtraction to solve the system of linear equations shown.

$$\begin{cases} 2x + 4y = -4 \\ 2x - 2y = -10 \end{cases}$$

Instead of subtracting $2x - 2y = -10$ from $2x + 4y = -4$, what equation can you add to get the same result? Explain.

24. Explain the Error Liang's solution of a system of linear equations is shown. Explain Liang's error and give the correct solution.

$$\begin{cases} 3x - 2y = 12 \\ -x - 2y = -20 \end{cases}$$

$$\begin{aligned} 3x - 2y &= 12 \\ \underline{-x - 2y} &\underline{= -20} \\ 2x &= -8 \\ x &= -4 \\ 3x - 2y &= 12 \\ 3(-4) - 2y &= 12 \\ -12 - 2y &= 12 \\ -2y &= 24 \\ y &= -12 \end{aligned}$$

Solution: $(-4, -12)$

25. Represent Real-World Problems For a school play, Rico bought 3 adult tickets and 5 child tickets for a total of \$40. Sasha bought 1 adult ticket and 5 child tickets for a total of \$25.

The system of linear equations $\begin{cases} 3x + 5y = 40 \\ x + 5y = 25 \end{cases}$ represents this situation, where x is the cost of an adult ticket and y is the cost of a child ticket. How much will Julia pay for 5 adult tickets and 3 child tickets?

Lesson Performance Task

A local charity run has a Youth Race for runners under the age of 12. The entry fee is $5 for an individual or $4 each for two runners from the same family. Carter is collecting the registration forms and fees. After everyone has registered, he picks up the cash box and finds a dollar on the ground. He checks the cash box and finds that it contains $200 and the registration slips for 47 runners. Does the dollar belong in the cash box or not? Explain your reasoning. (Hint: You can use the system of equations $i + f = 47$ and $5i + 4f = 200$, where i equals the number of individual tickets and f equals the number of family tickets.)

Name ______________________ Class ______________ Date ________

11.4 Solving Linear Systems by Multiplying First

Essential Question: How can you solve a system of linear equations by using multiplication and elimination?

Resource Locker

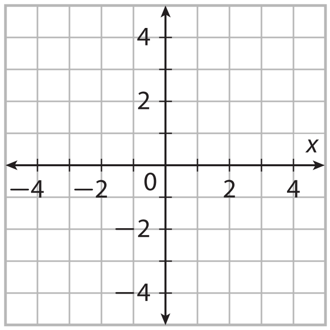
Explore 1 Understanding Linear Systems and Multiplication

A system of linear equations in which one of the like terms in each equation has either the same or opposite coefficients can that be readily solved by elimination.

How do you solve the system if neither of the pairs of like terms in the equations have the same or opposite coefficients?

(A) Graph and label the following system of equations.

$$\begin{cases} 2x - y = 1 \\ x + y = 2 \end{cases}$$



(B) The solution to the system is ____________.

(C) When both sides of an equation are multiplied by the same value, the equation [is/is not] still true.

(D) Multiply both sides of the second equation by 2.

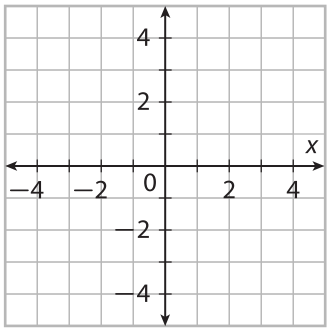
____

(E) Write the resulting system of equations.

$$\begin{cases} \boxed{} \\ 2x - y = 1 \end{cases}$$

(F) Graph and label the new system of equations.

Solution: ____________



(G) Can the new system of equations be solved using elimination now that $2x$ appears in each equation?

____

Reflect

1. **Discussion** How are the graphs of $x + y = 2$ and $2x + 2y = 4$ related?

____

2. **Discussion** How are the equations $x + y = 2$ and $2x + 2y = 4$ related?

____

Explore 2 Proving the Elimination Method with Multiplication

The previous example illustrated that rewriting a system of equations by multiplying a constant term by one of the equations does not change the solutions for the system. What happens if a new system of equations is written by adding this new equation to the untouched equation from the original system?

(A) Original System → New System

$$\begin{cases} 2x - y = 1 \\ x + y = 2 \end{cases} \rightarrow \begin{cases} 2x - y = 1 \\ 2x + 2y = 4 \end{cases}$$

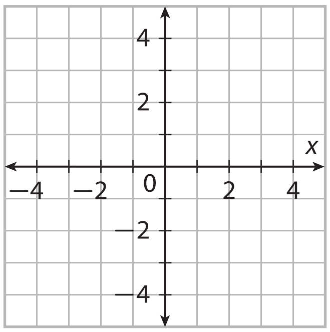
Add the equations in the new system.

$$\begin{array}{r} 2x - y = 1 \\ \underline{2x + 2y = 4} \\ \boxed{} \end{array}$$

(B) Write a new system of equations using this new equation.

$$\begin{cases} 2x - y = 1 \\ \boxed{} \end{cases}$$

(C) Graph and label the equations from this new system of equations.



(D) Is the solution to this new system of equations the same as the solution to the original system of equations? Explain.

____

____

____

(E) If the original system is $Ax + By = C$ and $Dx + Ey = F$, where A, B, C, D, E, and F are constants, then multiply the second equation by a nonzero constant k to get $kDx + kEy = kF$. Add this new equation to $Ax + By = C$.

$$\begin{array}{rcrcrcc} & & Ax & + & By & = & C \\ + & & kDx & + & kEy & = & kF \\ \hline & & \boxed{} & + & \boxed{} & = & \boxed{} \end{array}$$

(F) So, the original system is $\begin{cases} Ax + By = C \\ Dx + Ey = F \end{cases}$, and the new system is

$$\begin{cases} Ax + By = C \\ \underline{}. \end{cases}$$

(G) Let (x_1, y_1) be the solution to the original system. Fill in the missing parts of the following proof to show that (x_1, y_1) is also the solution to the new system.

(H) $Ax_1 + By_1 = \square$ Given.

(I) $Dx_1 + Ey_1 = \square$ Given.

(J) $\square(Dx_1 + Ey_1) = kF$ ________ Property of Equality

(K) $kDx_1 + kEy_1 = kF$ ________

(L) $\square + kDx_1 + kEy_1 = C + kF$ ________ Property of Equality

(M) $Ax_1 + \square + kDx_1 + kEy_1 = C + kF$ Substitute $Ax_1 + \square$ for $\square$ on the left.

(N) $Ax_1 + \square + By_1 + kEy_1 = C + kF$ ________ Property of Addition

(O) $(Ax_1 + kDx_1) + (By_1 + kEy_1) = C + kF$ ________ Property of Addition

(P) $(A + kD)x_1 + (\square)y_1 = C + kF$ ________

(Q) Therefore, (x_1, y_1) is the solution to the new system.

Reflect

3. Discussion Is a proof required using subtraction? What about division?

Explain 1 Solving Linear Systems by Multiplying First

In some systems of linear equations, neither variable can be eliminated by adding or subtracting the equations directly. In these systems, you need to multiply one or both equations by a constant so that adding or subtracting the equations will eliminate one or more of the variables.

Steps for Solving a System of Equations by Multiplying First

1. Decide which variable to eliminate.
2. Multiply one or both equations by a constant so that adding or subtracting the equations will eliminate the variable.
3. Solve the system using the elimination method.

Example 1 **Solve each system of equations by multiplying. Check the answers by graphing the systems of equations.**

(A) $\begin{cases} 3x + 8y = 7 \\ 2x - 2y = -10 \end{cases}$

Multiply the second equation by 4.

$4(2x - 2y = -10) \Rightarrow 8x - 8y = -40$

Add the result to the first equation.

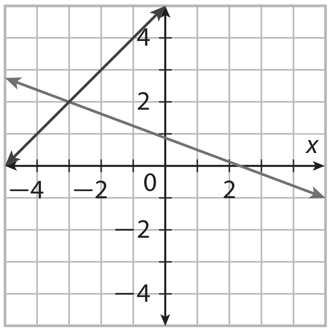
$$\begin{array}{r} 3x + 8y = 7 \\ +\ 8x - 8y = -40 \\ \hline 11x \qquad = -33 \end{array}$$

Solve for x.

$11x = -33$

$x = -3$

Substitute -3 for x in one of the original equations, and solve for y.



$3x + 8y = 7$

$3(-3) + 8y = 7$

$-9 + 8y = 7$

$8y = 16$

$y = 2$

The solution to the system is $(-3, 2)$.

(B) $\begin{cases} -3x + 2y = 4 \\ 4x - 13y = 5 \end{cases}$

Multiply the first equation by ______ and multiply the second equation by ______ so the x terms in the system have coefficients of -12 and 12 respectively.

$\square(-3x + 2y = 4) \qquad -12x + \square y = \square$

$\Rightarrow$

$\square(4x - 13y = 5) \qquad 12x - \square y = \square$

Add the resulting equations.

$$\begin{array}{r} -12x + \square y = \square \\ +12x - \square y = \square \\ \hline \square y = \square \end{array}$$

Solve for y.

$\square y = \square$

$y = \square$

Solve the first equation for x when $y = \square$.

$$-3x + 2y = 4$$

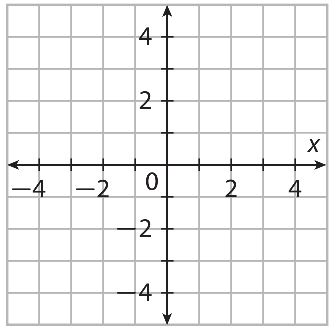
$$-3x + 2(\square) = 4$$

$$-3x + \square = 4$$

$$-3x = \square$$

$$x = \square$$

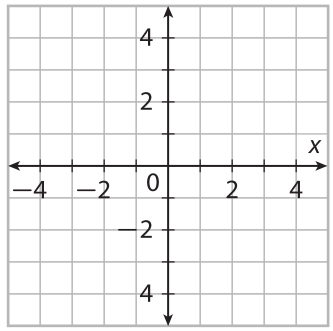
The solution to the system is $\square$.



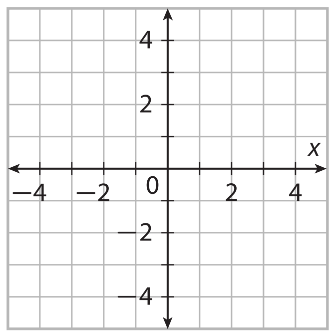
Your Turn

Solve each system of equations by multiplying. Check the answers by graphing the systems of equations.

4. $\begin{cases} -3x + 4y = 12 \\ 2x + y = -8 \end{cases}$



5. $\begin{cases} 2x + 3y = -1 \\ 5x - 2y = -12 \end{cases}$



Explain 2 Solving Linear System Models by Multiplying First

You can solve a linear system of equations that models a real-world example by multiplying first.

Example 2 **Solve each problem by multiplying first.**

(A) Jessica spent \$16.30 to buy 16 flowers. The bouquet contained daisies, which cost \$1.75 each, and tulips, which cost \$0.85 each. The system of equations $\begin{cases} d + t = 16 \\ 1.75d + 0.85t = 16.30 \end{cases}$ models this situation, where d is the number of daisies and t is the number of tulips. How many of each type of flower did Jessica buy?

Multiply the first equation by −0.85 to eliminate t from each equation. Then, add the equations.

$$\begin{cases} -0.85(d + t) = -0.85(16) \\ 1.75d + 0.85t = 16.30 \end{cases} \Rightarrow \begin{array}{rcr} -0.85d - 0.85t & = & -13.60 \\ +1.75d + 0.85t & = & 16.30 \\ \hline 0.9d & = & 2.70 \\ d & = & 3 \end{array}$$

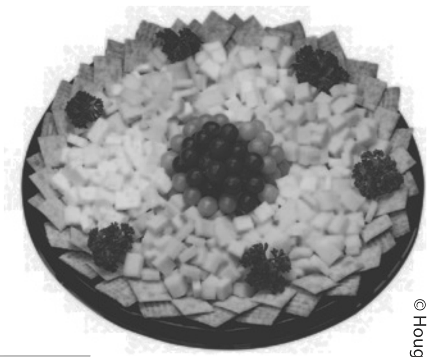
Find t.

$$\begin{aligned} d + t &= 16 \\ 3 + t &= 16 \\ t &= 13 \end{aligned}$$

The solution is (3, 13).

Jessica bought 3 daisies and 13 tulips.

(B) The Tran family is bringing 15 packages of cheese to a group picnic. Cheese slices cost \$2.50 per package. Cheese cubes cost \$1.75 per package. The Tran family spent a total of \$30 on cheese. The system of equations $\begin{cases} s + c = 15 \\ 2.50s + 1.75c = 30 \end{cases}$ represents this situation, where s is the number of packages of cheese slices and c is the number of packages of cheese cubes. How many packages of each type of cheese did the Tran family buy?



Multiply the first equation by a constant so that c can be eliminated from both equations, and then subtract the equations.

$$\begin{cases} \boxed{}(s + c) = \boxed{}(15) \\ 2.50s + 1.75c = 30 \end{cases} \Rightarrow \begin{array}{rcr} \boxed{}s + \boxed{}c & = & \boxed{} \\ -(2.50s + 1.75c) & = & -30 \\ \hline \boxed{}s & = & \boxed{} \\ s & = & \boxed{} \end{array}$$

Find c.

$$\begin{aligned} s + c &= 15 \\ \boxed{} + c &= 15 \\ c &= \boxed{} \end{aligned}$$

The solution is $\boxed{}$.

The Tran family bought ________ packages of sliced cheese and ________ packages of cheese cubes.

Your Turn

6. Jacob's family bought 4 adult tickets and 2 student tickets to the school play for \$64. Tatianna's family bought 3 adult tickets and 3 students tickets for \$60. The system of equations $\begin{cases} 4a + 2s = 64 \\ 3a + 3s = 60 \end{cases}$ models this situation, where a is the cost of an adult ticket and s is the cost of a student ticket. How much does each type of ticket cost?

Elaborate

7. When would you solve a system of linear equations by multiplying?

8. How can you use multiplication to solve a system of linear equations if none of the coefficients are multiples or factors of any of the other coefficients?

9. **Essential Question Check-In** How do you solve a system of equations by multiplying?

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

For each linear equation,

a. **find the product of 3 and the linear equation;**

b. **solve both equations for *y*.**

1. $2y - 4x = 8$

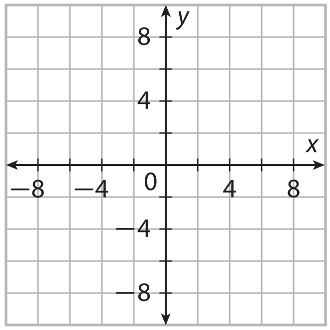
2. $-5y + 7x = 12$

3. $4x + 7y = 18$

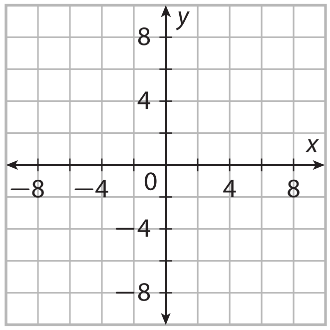
4. $x - 2y = 13$

For each linear system, multiply the first equation by 2 and add the new equation to the second equation. Then, graph this new equation along with both of the original equations.

5. $\begin{cases} 2x + 4y = 24 \\ -12x + 8y = -16 \end{cases}$

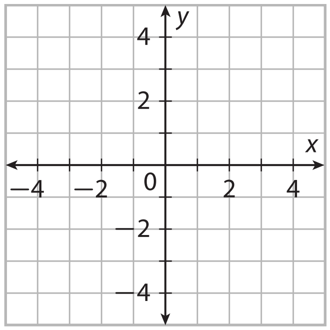


6. $\begin{cases} 2x + 2y = 16 \\ -15x + 3y = -12 \end{cases}$

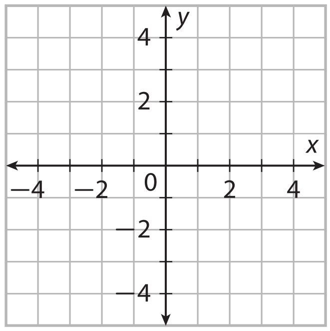


Solve each system of linear equations by multiplying. Verify each answer by graphing the system of equations.

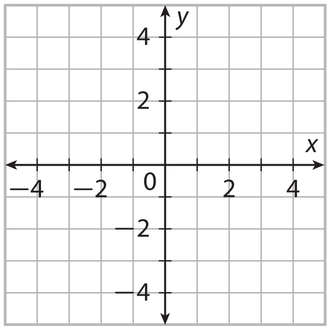
7. $\begin{cases} 5x - 2y = 11 \\ 3x + 5y = 19 \end{cases}$



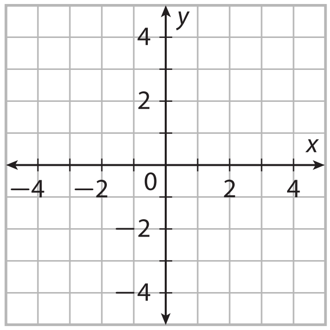
8. $\begin{cases} -2x + 2y = 2 \\ -4x + 7y = 16 \end{cases}$



9. $\begin{cases} 3x + 4y = 13 \\ 2x - 2y = -10 \end{cases}$



10. $\begin{cases} x - 4y = -1 \\ 5x + 2y = 17 \end{cases}$



Solve each system of linear equations using multiplication.

11. $\begin{cases} -3x + 2y = 4 \\ 5x - 3y = 1 \end{cases}$

12. $\begin{cases} 3x + 3y = 12 \\ 6x + 11y = 14 \end{cases}$

Solve each problem by multiplying first.

13. The sum of two angles is 180°. The difference between twice the larger angle and three times the smaller angle is 150°. The system of equations $\begin{cases} x + y = 180 \\ 2x - 3y = 150 \end{cases}$ models this situation, where x is the measure of the larger angle and y is the measure of the smaller angle. What is the measure of each angle?

14. The perimeter of a rectangular swimming pool is 126 feet. The difference between the length and the width is 39 feet. The system of equations $\begin{cases} 2x + 2y = 126 \\ x - y = 39 \end{cases}$ models this situation, where x is the length of the pool and y is the width of the pool. Find the dimensions of the swimming pool.

15. Jamian bought a total of 40 bagels and donuts for a morning meeting. He paid a total of \$33.50. Each donut cost \$0.65 and each bagel cost \$1.15. The system of equations $\begin{cases} b + d = 40 \\ 1.15b + 0.65d = 33.50 \end{cases}$ models this situation, where b is the number of bagels and d is the number of donuts. How many of each did Jamian buy?

16. A clothing store is having a sale on shirts and jeans. 4 shirts and 2 pairs of jeans cost \$64. 3 shirts and 3 pairs of jeans cost \$72. The system of equations $\begin{cases} 4s + 2j = 64 \\ 3s + 3j = 72 \end{cases}$ models this situation, where s is the cost of a shirt and j is the cost of a pair of jeans. How much does one shirt and one pair of jeans cost?

17. Jayce bought 5 bath towels and returned 2 hand towels. His sister Jayna bought 3 bath towels and returned 4 hand towels. Jayce paid a total of \$124 and Jayna paid a total of \$24. The system of equations $\begin{cases} 5b - 2h = 124 \\ 3b - 4h = 24 \end{cases}$ models this situation, where b is the price of a bath towel and h is the price of a hand towel. How much does each kind of towel cost?

18. Apples cost \$0.95 per pound and bananas cost \$1.10 per pound. Leah bought a total of 8 pounds of apples and bananas for \$8.05. The system of equations $\begin{cases} a + b = 8 \\ 0.95a + 1.10b = 8.05 \end{cases}$ models this situation, where a is the number of pounds of apples and b is the number of pounds of bananas. How many pounds of each did Leah buy?

19. Which of the following are possible ways to eliminate a variable by multiplying first? $\begin{cases} -x + 2y = 3 \\ 4x - 5y = -3 \end{cases}$

a. Multiply the first equation by 4.

b. Multiply the first equation by 5 and the second equation by 2.

c. Multiply the first equation by 4 and the second equation by 2.

d. Multiply the first equation by 5 and the second equation by 4.

e. Multiply the first equation by 2 and the second equation by 5.

f. Multiply the second equation by 4.

20. **Explain the Error** A linear system has two equations $Ax + By = C$ and $Dx + Ey = F$. A student begins to solve the equation as shown. What is the error?

$$\begin{array}{r} Ax + By = C \\ \underline{+\ k(Dx + Ey) = F} \\ (A + kD)x + (B + kE)y = C + F \end{array}$$

21. **Critical Thinking** Suppose you want to eliminate y in this system: $\begin{cases} 2x + 11y = -3 \\ 3x + 4y = 8 \end{cases}$

By what numbers would you need to multiply the two equations in order to eliminate y? Why might you choose to eliminate x instead?

H.O.T. Focus on Higher Order Thinking

22. **Justify Reasoning** Solve the following system of equations by multiplying.

$\begin{cases} x + 3y = -14 \\ 2x + y = -3 \end{cases}$ Would it be easier to solve the system by using substitution? Explain your reasoning.

23. **Multi-Step** The school store is running a promotion on school supplies. Different supplies are placed on two shelves. You can purchase 3 items from shelf A and 2 from shelf B for $16. Or you can purchase 2 items from shelf A and 3 from shelf B for $14. This can be represented by the following system of equations.

a. Solve the system of equations $\begin{cases} 3A + 2B = 16 \\ 2A + 3B = 14 \end{cases}$ by multiplying first.

b. If the supplies on shelf A are normally $6 each and the supplies on shelf B are normally $3 each, how much will you save on each package plan from part A?

Lesson Performance Task

A chemist has a bottle of 1% acid solution and a bottle of 5% acid solution. She wants to mix the two solutions to get 100 mL of a 4% acid solution.

a. Complete the table to write the system of equations.

	1% Solution	+	5% Solution	=	4% Solution
Amount of Solution (mL)	x	+	y	=	
Amount of Acid (mL)	$0.01x$	+		=	0.04(100)

b. Solve the system of equations to find how much she will use from each bottle to get 100 mL of a 4% acid solution.